The Mathematics of the Public Holidays in Singapore

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Public Holidays

There are 11 public holidays in Singapore. Three of them are secular.

1. New Year's Day

2. Labour Day

3. National Day

The remaining eight cultural, racial or religious holidays consist of two Chinese, two Muslim, two Indian and two Christian.

Cultural, Racial or Religious Holidays

1. Chinese New Year and day after

2. Good Friday

3. Vesak Day

4. Deepavali

5. Christmas Day

6. Hari Raya Puasa

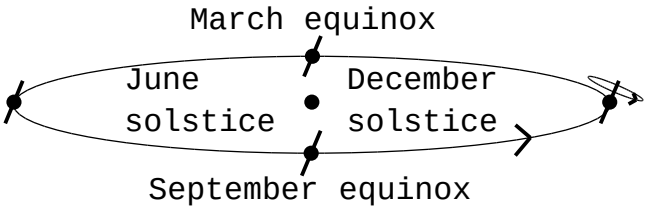
7. Hari Raya Haji

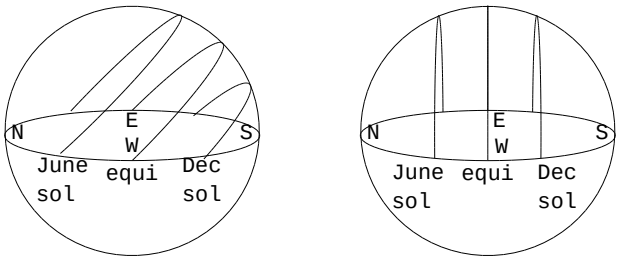
Listed in order, except for the Muslim holidays, which can occur anytime during the year.

Christmas Day falls on a fixed date, but all the others move.

A Quick Course in Astronomy

The Earth revolves counterclockwise around the Sun in an elliptical orbit. The Earth rotates counterclockwise around an axis that is tilted 23.5 degrees.





Beijing Singapore

In the northern hemisphere, the day will be longest at the June solstice and shortest at the December solstice. At the two equinoxes day and night will be equally long. The equinoxes and solstices are called the seasonal markers.

The Year

The tropical year (or solar year) is the time from one March equinox to the next. The mean value is 365.2422 days.

The synodic month is the time from one new Moon to the next. It ranges from 29.27 days to 29.84 days with a mean of 29.53 days.

$12 \times 29.5 = 354$, so a lunar year consisting of 12 lunar months is about 11 days short of a solar year.

$$365 - 12 \times 29.5 = 11$$

$$365/11 \approx 33$$

The Metonic Cycle

19 solar years is almost exactly 235 lunar months.

$$\begin{aligned} 235 \times 29.53 &= 6939.6884, \\ 19 \times 365.2422 &= 6939.6018. \end{aligned}$$

The difference is about two hours. This is called the Metonic cycle (432 BCE). It was known in China by about 600 BCE and was called the zhāng (章) cycle.

The Metonic cycle is used in the Jewish calendar, in the computation of Easter, and was used in the Chinese calendar before 104 BCE.

Classification of Calendars

solar Gregorian calendar. Basic unit is day. Approximates the tropical year by adding leap days. Ignores the Moon. The year is 365 or 366 days.

lunar Islamic calendar. Basic unit is lunar month. Ignores the Sun. The year is 12 months or 354 (sometimes 353 or 355) days.

lunisolar Chinese and Jewish calendars. Basic unit is lunar month. Approximates the tropical year by adding leap months. The year is 12 or 13 months. A 12-month year is 354 (sometimes 353 or 355) days. A 13-month year is 384 (sometimes 383 or 385) days.

The Chinese calendar is NOT a lunar calendar!

Alternative Classification of Calendars

arithmetical Gregorian and Jewish calendars. Based on arithmetical formulas. Prediction and conversion between different arithmetical calendars is simple.

astronomical Islamic, Indian and Chinese calendars. Based on astronomical data. Prediction and conversion is hard.

The Gregorian Calendar

A normal year consists of 365 days, but leap years have 366 days. Year n is a leap year if n is divisible by 4, but not by 100 or if n is divisible by 400. 1900 is not a leap year, but 2000 is.

In the Julian calendar, every fourth year is a leap year.

The average length of the Gregorian year is 365.2425. The difference between this and the tropical year will cause an error of about one day in 2,500 years.

Computation of Easter

The rule of thumb is that Easter Sunday is the first Sunday after the first full Moon on or after the day of the March equinox. It will fall between March 22 and April 25.

The actual rule is that Easter Sunday in year y falls on day d in month m where d and m are computed as follows (all remainders from division are dropped).

$$
\begin{aligned}
c &= y/100 \\
n &= y - 19 * \lfloor y/19 \rfloor \\
k &= \lfloor (c - 17)/25 \rfloor \\
i &= c - c/4 - \lfloor (c - k)/3 \rfloor + 19 * n + 15 \\
i &= i - 30 * \lfloor i/30 \rfloor \\
i &= i - \lfloor i/28 \rfloor * (1 - \lfloor i/28 \rfloor * \lfloor 29/(i + 1) \rfloor \\
&\quad * \lfloor (21 - n)/11 \rfloor) \\
j &= y + \lfloor y/4 \rfloor + i + 2 - c + \lfloor c/4 \rfloor \\
j &= j - 7 * \lfloor j/7 \rfloor \\
l &= i - j \\
m &= 3 + \lfloor (l + 40)/44 \rfloor \\
d &= l + 28 - 31 * \lfloor m/4 \rfloor
\end{aligned}
$$

The Islamic Calendar

No leap months. Muslim holiday move about 11 days backward each year.

New months are determined by sighting of the new Moon. A new Moon is normally not visible until it is more than 24 hours old. Muslim months start between one and four days after the Chinese months (usually on the third day). The Muslim month starts at sunset the previous day.

Hari Raya Puasa is the first day of the 10th month. Hari Raya Haji is the 10th day of the 12th month.

In 1999, Hari Raya Puasa fell on Jan. 19. In 2000, it will fall on Jan. 8 and on Dec. 27. Since $365/11 \approx 33$, we see that such double Hari Raya Puasa's will occur every 32 or 33 years.

The Chinese Calendar

The goal is to approximate the solar year by adding leap months. Sine 12 lunar months are 11 days too short we will need to add a leap month a little bit more than every third year. In ancient times, this was done by observing nature.

Since $235 = 19 \times 12 + 7$, we can use the Metonic cycle and get a decent lunisolar calendar by having 7 leap years in every 19-year cycle.

Chinese New Year

It can be shown that Chinese New Year will always fall between Jan. 21 and Feb. 21. Most of the time Chinese New Year will fall 11 (or sometimes 10 or 12) days earlier than the previous year, but if that would take us outside of the Chinese New Year range of Jan. 21 to Feb. 21, we must add a leap month, so Chinese New Year jumps 19 (or sometimes 18 or 20) days later.

1998	1999	2000	2001	2002	2003
28/1	16/2	5/2	24/1	12/2	1/2
19	11	12	19	11	

	2004	2005	2006	2007	2008	2009
	22/1	9/2	29/1	18/2	7/2	26/1
10	18	11	20	11	12	

Chinese New Year and Hari Raya Puasa

The Muslim holidays will always fall about 11 days earlier. Sometimes Hari Raya Puasa will coincide with Chinese New Year. They will then stay together for two or three years. After that, HRP will continue on its 33 year cycle backwards through the calendar, while CNY will jump forward because of a leap month. They will then meet up again in 30 or 31 years. This happened from 1964 to 1966, from 1996 to 1998, and will happen again from 2029 to 2031.

The 19-year Cycle

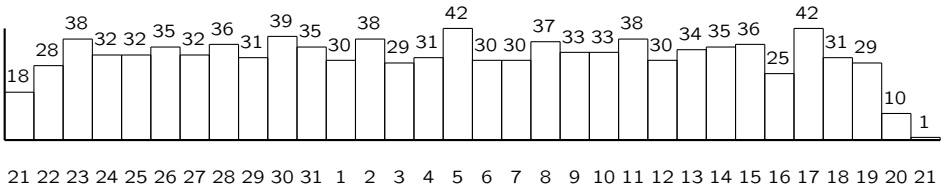
Because of the Metonic cycle, there is almost a 19-year cycle in the Chinese calendar. I was born on April 16, 1960. This was the 21st day in the 3rd month in the Chinese calendar. Normally my birthday will fall on different days in the Chinese calendar, but my 19th birthday fell on the 20th day in the third month. The same goes for my 38th and 57th birthday. So we see that the 19-year cycle is close but not exact.

There are two reasons for this. First of all, the Metonic cycle is off by about two hours. But more importantly, we are now comparing the Chinese calendar not with the tropical year, but with the Gregorian calendar, which is just an approximation to the tropical year. In particular, since 19 is not a multiple of 4, different cycles will contain different numbers of leap years.

The 19-year Cycle and the Dates of Chinese New Year

1980: Feb. 16	1999: Feb. 16
1981: Feb. 5	2000: Feb. 5
1982: Jan. 25	2001: Jan. 24
1983: Feb. 13	2002: Feb. 12
1984: Feb. 2	2003: Feb. 1
1985: Feb. 20	2004: Jan. 22
1986: Feb. 9	2005: Feb. 9
1987: Jan. 29	2006: Jan. 29
1988: Feb. 17	2007: Feb. 18
1989: Feb. 6	2008: Feb. 7
1990: Jan. 27	2009: Jan. 26
1991: Feb. 15	2010: Feb. 14
1992: Feb. 4	2011: Feb. 3
1993: Jan. 23	2012: Jan. 23
1994: Feb. 10	2013: Feb. 10
1995: Jan. 31	2014: Jan. 31
1996: Feb. 19	2015: Feb. 19
1997: Feb. 7	2016: Feb. 8
1998: Jan. 28	2017: Jan. 28

Dates of Chinese New Year Between 1645 and 2644



The 24 Jié Qì

A fundamental concept in the Chinese calendar is the 24 solar terms or jié qì (节气). They are a generalization of the solstices and equinoxes. The even ones are called major solar terms or zhōng qì (中气).

A useful rule of thumb is that Chinese New Year is the new Moon closest to lì chūn (立春), the beginning of spring. This rule is correct most of the time, but it failed in 1985 and will fail again in 2015. Since lì chūn falls around Feb. 4, this helps explain why Chinese New Year will always fall between Jan. 21 and Feb. 21. It also helps explain why Chinese New Year is called the spring festival.

In Western astronomy, spring begins at spring equinox. In Chinese astronomy, spring begins midway between winter solstice and spring equinox.

The 24 Jié Qì

J1	Lì chūn	立春	Beginning of spring	February 4
Z1	Yǔ shuǐ	雨水	Rain water	February 19
J2	Jīng zhé	惊蛰	Waking of insects	March 6
Z2	Chūn fēn	春分	Spring equinox	March 21
J3	Qīng míng	清明	Pure brightness	April 5
Z3	Gǔ yǔ	谷雨	Grain rain	April 20
J4	Lì xià	立夏	Beginning of summer	May 6
Z4	Xiǎo mǎn	小满	Grain full	May 21
J5	Máng zhòng	芒种	Grain in ear	June 6
Z5	Xià zhì	夏至	Summer solstice	June 22
J6	Xiǎo shǔ	小暑	Slight heat	July 7
Z6	Dà shǔ	大暑	Great heat	July 23
J7	Lì qiū	立秋	Beginning of autumn	August 8
Z7	Chǔ shǔ	处暑	Limit of heat	August 23
J8	Bái lù	白露	White dew	September 8
Z8	Qiū fēn	秋分	Autumnal equinox	September 23
J9	Hán lù	寒露	Cold dew	October 8
Z9	Shuāng jiàng	霜降	Descent of frost	October 24
J10	Lì dōng	立冬	Beginning of winter	November 8
Z10	Xiǎo xuě	小雪	Slight snow	November 22
J11	Dà xuě	大雪	Great snow	December 7
Z11	Dōng zhì	冬至	Winter solstice	December 22
J12	Xiǎo hán	小寒	Slight cold	January 6
Z12	Dà hán	大寒	Great cold	January 20

The Chinese Meridian

Calculations are based on the meridian 120° East.

Before 1929 the computations were based on the meridian in Beijing (116°25′), but in 1928 China adopted a standard time zone based on 120° East.

The Length of a Chinese Month

The day on which a new Moon occurs is the first day of the new month.

The length of the months are determined astronomically. The lunar month can vary between about 29.25 and 29.75 with a mean of 29.53. Suppose a month is 29.5 days.

New Moon	Next new Moon	Length
May 1 13h	May 31 1h	30 days
May 1 1h	May 30 13h	29 days

There can be four long months or three short months in a row.

The Mid-Autumn Festival

If the 1st month marks the beginning of spring, autumn should start with the 7th month. This explains why the Mid-Autumn Festival is celebrated on the 15th day of the 8th month.

The Chinese Solar Calendar

It is important to understand that the Chinese calendar is a combination of two calendars, the usual lunisolar calendar and a solar calendar that follows the 24 jié qì. The solar calendar is traditionally called the farmer's calendar (农历). Unfortunately the term farmer's calendar has come to include the lunisolar calendar. The Chinese solar calendar follows the tropical year closely, so it is perfect for farming purposes, but the lunisolar calendar is not at all suitable for farmers.

There are two Chinese holidays that are determined by the solar calendar, namely qīng míng (清明) around Apr. 5 and winter solstice dōng zhì (冬至) around Dec. 22.

Notice that lunar dates can fall within a range of about one month in the solar calendar and conversely. Chinese New Year can fall between Jan. 21 and Feb. 21, while qīng míng can fall between the 13th day of the 2nd month and the 17th day of the 3rd month.

The Chinese Year

There are several years in the Chinese calendar. The most important are the suì (岁) and the nián (年).

A suì is the solstice year from one winter solstice to the next. This is the same as the tropical year.

A nián is the Chinese year from one Chinese New Year to the next. Since a Chinese year can contain 12 or 13 lunar months, and they can have 29 or 30 days, it can be shown that the length of a nián can be 353, 354, 355, 383, 384 or 385 days long.

Just like we can think of the Gregorian year as an approximation to the tropical year, we can think of the nián as an approximation to the suì.

The Chinese astrological year runs from lì chūn, the beginning of spring about Feb. 5, not from Chinese New Year.

Vesak Day

Traditionally, Buddhists have observed Vesak Day on the 8th or 15th day of the fourth month. Since the 1950's the Singapore Buddhist Federation celebrates it on the first full Moon in May.

Deepavali

Deepavali falls on the last day of the lunar month Asvina, called Purattasi in Tamil. Like the Chinese calendar, the Tamil calendar consists of a solar calendar and a lunisolar calendar. The solar month Asvina runs from around Sep. 16 to around Oct. 17. The lunar month Asvina starts with the first new Moon within this solar month. It follows that Deepavali can fall between Oct. 15 and Nov. 15.

The Sexagenary Cycle

Heavenly Stems	天干	tiān gān	Element
1	甲	jiǎ	Wood
2	乙	yǐ	Wood
3	丙	bǐng	Fire
4	丁	dīng	Fire
5	戊	wù	Earth
6	己	jǐ	Earth
7	庚	gēng	Metal
8	辛	xīn	Metal
9	壬	rén	Water
10	癸	guǐ	Water

Earthly Branches	地支	dì zhī	Animal
1	子	zǐ	Rat
2	丑	chǒu	Ox
3	寅	yín	Tiger
4	卯	mǎo	Rabbit
5	辰	chén	Dragon
6	巳	sì	Snake
7	午	wǔ	Horse
8	未	wèi	Goat
9	申	shēn	Monkey
10	酉	yǒu	Chicken
11	戌	xū	Dog
12	亥	hài	Pig

The Golden Dragon

Let us denote both the stems and the branches by their numbers. We denote 1 by $(1,1)$ or (甲, 子), 2 by $(2,2)$ or (乙, 丑) and so on up to $(10,10)$ or (癸, 酉). But now we have run out of stems, so we denote 11 by $(1,11)$ or (甲, 戌) and 12 by $(2,12)$ or (乙, 亥). Now we have run out of branches, too, so 13 becomes $(3,1)$ or (丙, 子). We continue in this way through 6 cycles of stems and 5 cycles of branches up to 60, which is $(10,12)$ or (癸, 亥). The next number is then $(1,1)$ or (甲, 子), which starts a new sexagesimal cycle.

Notice that each branch, or animal, occurs five times in each 60-year cycle. An animal corresponding to an odd number, will meet the stems that correspond to the odd numbers. Year 2000 is the 17th year in the current cycle (see below), so it corresponds to $(7,5)$ $(17 = 10 + 7 = 12 + 5)$ or (庚, 辰). So we see that it is a metal dragon year, or a "Golden Dragon".

The Eight Characters

This cycle is used for keeping track of years, months, days and (double) hours in Chinese astrology. Your date and time of birth is determined by the "Eight Characters" (八字) formed by the pair of cyclical characters for the year, month, day and hour. The 60-day cycle has been used for keeping track of days since ancient times. During the Hàn (汉) dynasty, the 60-year cycle was also introduced.

What is Year 2000 in the Chinese Calendar?

The Chinese do not have a continuous year count. They started counting from one again with each new emperor. However, from the Hàn dynasty, some scholars tried to reconstruct the ancient Chinese chronology, and it became customary to claim that the calendar was invented by the Yellow Emperor, Huáng Dì (黃帝), in 2637 BCE during the 61st year of his reign. However, many people prefer to start the count with the first year of his reign in 2697 BCE. Since these years are 60 years apart, it follows that 1984 was the first year of either the 78th or 79th 60-year cycle. Using this as a starting point, Chinese New Year in 2000 marks the beginning of the Chinese year 4637 or 4697. While Chinese chronology is fairly reliable going back to 841 BCE, and oracle bones with date inscription go back to the 13th century BCE, modern scholars consider the Yellow Emperor to be a mythological figure.

Why Was the Calendar Important?

With a lunar or lunisolar calendar, errors are much more obvious than with a solar calendar. A solar calendar can be off by a couple of weeks without anybody noticing. The reason why the Catholic church had to reform the Julian calendar was because the rules for computing Easter had frozen the March equinox to be March 21. That meant that Easter was drifting noticeably towards summer. Otherwise, few would have cared about the drift of the March equinox. But with a lunar calendar, an error of even a couple of days is a serious problem. Every peasant could each month see that the new Moon was visible near the end of the previous month or that the old Moon was visible in the next month.

Because of the importance the Chinese rulers placed on calendars, they were surprisingly open to incorporate foreign ideas into the making of calendars. The last three main calendar reforms have all been associated with foreign impulses.

The Main Calendar Reforms

Before 621 BCE, the start of the month was based on visibility of the crescent Moon. During the Zhōu (周) dynasty, the Metonic cycle was used for determining leap months and the leap months were always placed at the end of the year. After the Tài Chū (太初) calendar reform in 104 BCE, the "no zhōng qì" (无中气) rule was used for determining leap months, and the month containing the December solstice was fixed to be the 11th month.

The Táng (唐) dynasty calendar reform in 619 switched to following the true Moon. This was inspired by Indian Buddhist astronomers.

The Yuán (元) dynasty reform in 1280 was inspired by Muslim astronomers. It was the most accurate calendar in the world at that time.

The last calendar reform came in 1645 during the Qīng dynasty (清) and was implemented by Jesuit missionaries. It used the true Sun.

The Jesuits

In 1644, the German Adam Schall went to the new Qīng rulers and presented his calculations for an upcoming solar eclipse. He challenged the Chinese and the Muslim astronomers in the Bureau, and the Jesuits' calculations were best. Schall was appointed director of the Bureau. The next year, he formulated the current rules for the Chinese calendar.

The Trial of the Jesuits

A Chinese official, Yáng Guāng Xiān (杨光先), had as his slogan that it was "better to have a wrong calendar than to have foreigners in China". Yáng managed to have the Jesuits arrested in 1664. A solar eclipse was coming up and while in prison, the Jesuits predicted it would occur at 3 pm, Yáng predicted 2.15 pm, and the Muslim Wú Míng Xuǎn (吴明炫) predicted 2.30 pm. On the day of the eclipse, the Jesuits were brought into the palace in chains, and everybody watched as the eclipse occurred at 3pm sharp! Unfortunately, the regents were not impressed and the Jesuits were sentenced to death. However, the next day a strong earthquake struck Beijing. This was taken as a sign from Heaven that the sentence was unjust, and the sentence of the Jesuits was first converted to flogging and eventually to just house arrest.

The Kang Xi Emperor

In 1668, the Kāng Xī (康熙) emperor took over from the regents. The emperor ordered the Belgian Verbiest, Yáng and Wú to compute the length of the shadow of a pole on a certain day and the position of the Sun at noon on a certain day. They were to leave their instruments pointing towards the predicted spot in the emperor's garden two weeks in advance. Verbiest easily won and was appointed director of the Bureau, while Yáng and Wú were arrested. Verbiest became personal tutor to the Kāng Xī emperor, and even learned Manchu. Jesuits remained as directors of the Bureau until 1746 and it was run by other Westerners until 1826.