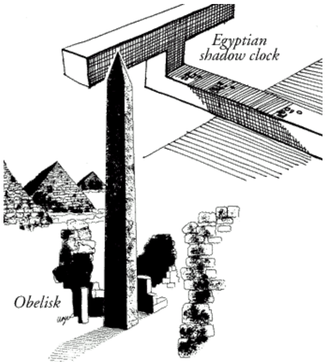
NATIONAL UNIVERSITY of SINGAPORE
Founded 1905

# GEK 1506
# Heavenly Mathematics: Cultural Astronomy

## Group 32
## METHODS OF TELLING TIME



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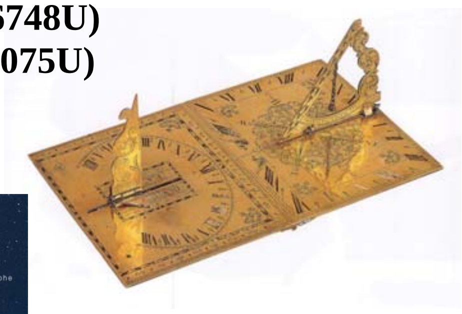

# CONTENT PAGE

# CHAPTER 1: INTRODUCTION

Men have always been conscious of the passing of time, constantly seeking to invent instruments to tell time accurately. As people in the past started following the Sun's movement in a day, they observed its motion across the sky as it rises in the East, climbs to its highest point in the sky (due South in the Northern hemisphere and North in the Southern hemisphere) at the time called noon and sets in the West. Recognizing dawn, noon and dusk then gave people the broad division of a day into morning, afternoon and night.

Primitive men, agricultural farmers and later, astronomers alike from different civilizations realised the importance and practical need to devise methods and invent instruments to divide the day into intervals and to tell time of a day in a predictable manner. With that, men began the attempts to accurately mark the passage of time.

This paper looks at different time-keeping instruments and offers various methods of telling time in an astronomical, mathematical and cultural perspective. The first two chapters give a brief introduction to some historical methods of telling time, as well as the different types of the instruments and clockworks used in ancient history. The bulk of this paper will focus on instruments for telling time (particular hours of a day) as alternatives to the modern watch, pervasively used in the modern era. For telling time in the day, the various types of sundials and their methods of usage shall be examined. Following that, the usage of astrolabes to tell time shall also be discussed. For telling of time at night, the focus will be on the usage of the Moon dial and constellations. Several concepts which influence the marking of time shall also be discussed. These include the difference between solar time and standard time, the Equation of Time, declination of the Sun and apparent motion of the Sun and the Moon. We shall begin by setting down some definitions of the two basic terminology conventionally used in this paper: The definitions of the Hour and Time.

## Common Definition of the Hour

There are numerous ways in the division of the day into intervals.

**Planetary hour** measures the time needed for 15º of the celestial equator to rise above the horizon since sunrise. This means that there is about 180º/15º = 12 planetary hours. Planetary hour is sometimes also defined as the time needed for the 15º of the ecliptic to rise above the horizon. **Temporal hour** is defined as the time elapsed from sunset to sunrise. It is usually counted as12 daytime hours and 12 night-time hours but these are unequal hours, where the duration of an hour depends on the date. This is similar to **seasonal hour,** which varies throughout the year as with the seasons. All these time systems have unequal length of hours throughout the year as time is counted starting from sunrise each day. A sundial is an instrument used in the daytime, measuring **unequal hour**.

**Italian Hour** is the time starting from sunrise (Hour 0) to sunset. It has 24 equal hours each day and this 1- 24 hour time system is used even till today. The most commonly used time system today is the **Modern** Hour, which is a timekeeping system also called the French, European or German Hour. One hour is equivalent to 60 minutes or 3600 seconds.

For the purpose of this paper, we shall be looking at the unequal hour and the modern hour.

## Common definitions of Time

Likewise, there are different variations for the concept of time. This paper shall mainly look at the following conventional definitions of time: Local mean time, Standard mean time and Civil time. **Local mean time** is the true solar time (read off from the sundial) derived from the real Sun. It is the local apparent time corrected for the Equation of Time, but ignores the longitude of the particular location. **Standard Mean Time** is instead based conceptually on the diurnal motion of the fictitious mean Sun. This is the time shown on the clock at the central meridian of a given time zone, and the Earth's rate of rotation is assumed to be constant. **The civil time** (shown on the clock) is based on the Greenwich Mean Time (GMT) and is the region's legally-accepted time scale based on the standard time for that particular standard time zone. It is measured in modern hours from the most recent midnight.

# CHAPTER 2: BRIEF HISTORY OF TIMEKEEPING

Timekeeping is as essential in the past as it is now. Several activities are affected by time, some of which are nomadic, farming and religious activities. In the past, people observed celestial bodies, seasonal changes, day and night periods to determine the time. Time measuring instruments were later invented to tell time. The sundial and the water clocks are perhaps the two of the most prominent and earliest tools used in ancient times for timekeeping. Ancient civilisations, namely Egypt, Greece and Roman, China and Europe had a long history of timekeeping.

## Egypt

The Egyptians were well-known historically for their mathematics and astronomical capabilities. They realized the importance of celestial bodies in telling of time and fixed a year duration to 365 days.

### -Sundial

Egyptians were probably the first to begin using a T-shaped "time stick", consisting of a vertical stick and a crossbar. Gnomons (the shadow stick), in the form of obelisks were used in Egypt as early as 3500 BC for the measurements of time and the setting up of calendars. They built obelisks in the form of tall, slender, tapering, four-sided stone towers, sometimes called "Cleopatra's Needles". The moving shadow of the obelisk formed a type of sundial, and markers arranged about the base separated the day into divisions, as well as indicating the longest and shortest days of the year.

Around 1500 BC, smaller Egyptian timepieces were created. One of them is the sundial, also known as the shadow clock, which is a smaller version of the obelisk. The sundial tells the length of the day and the direction of the sun by the shadow cast by the gnomon on the sundial. Sundials were made not only for timekeeping also as votive offerings placed in temples. The sundial separates the day-night period into 24 equal hours. However, the length of each day actually varies due to the seasonal change. Summer will have longer days and winter will have shorter days.

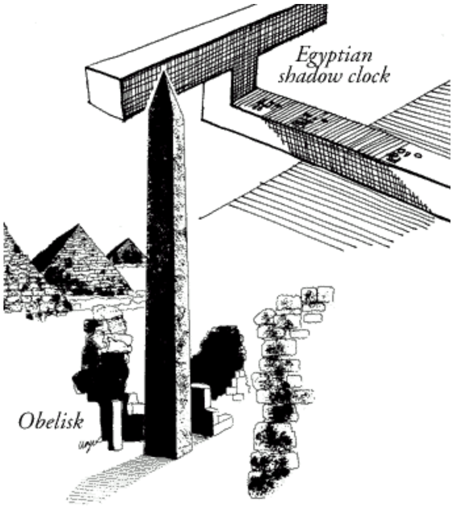

*Diagram 2.1: Obelisk used in early Egypt. The ancient Egyptian shadow clock on top measures the hours from sunrise till noon and is reversed for measurement until sunset.*

### -Merkhet

The merkhet is also one of the oldest astronomical tools. It was invented in around 600 BC in Egypt. Merkhets are used to form a north-south line, which is the meridian, to align with the Polaris. The night hours will then be marked when certain stars crossed the meridian.

### -Water Clock

The water clocks, also called the clepsydra, were believed to be first used by the Egyptians. The clepsydra is independent of the celestial bodies to measure time both during day and night. It consists of two containers filled with water, with one of them higher than the other and a tube connecting them together. Markings were inscribed on the containers at different heights in the containers so as to indicate time by the water level in the container.

## Greece and Roman

### -Sundial

The earliest sundial in ancient Greece is probably the "hemi-cycle" or hemisphere, created by the Chaldean astronomer Berosus. It was a hollow hemisphere, with its rim perfectly horizontal and a bead or globule fixed at its center. It is a typical concave type of sundial. The shadow of this bead moved and was marked on the inside of the instrument, done at various periods throughout the year. This curve (path of a circle) was divided into twelve equal parts, corresponding to the twelve hours of the day from sunrise to sunset. These, however, were temporal hours and not of equal length. The dial of Berosus remained in use for centuries since its invention in the fourth century.

The Romans was later introduced to the sundial by the Greeks and made changes to it. The dial was designed to divide the period of day light into equal parts. However, the Romans were unaware of that the sundial must be constructed differently to suit the latitude of the place where the sundial is situated in. A sundial was brought from Sicily to Rome (of different latitude) and gave incorrect time to the Romans for almost a century.

The sundial is also considered a prestigious item in the Roman and Greek cities. Donors of public sundials had their names inscribed onto it and Romans carried portable sundials that were a little over one inch in diameter. Varieties of sundials with different styles and sizes were built by the 1st century.

### -Water Clock

The Greeks began to use the clepsydra at around 325BC. Water was allowed to drip at an almost constant rate from a small hole near the bottom through stone vessels with steep sides. Some clepsydras were cylindrical in shape and were made to accumulate water at a regular rate. Interval markings were also

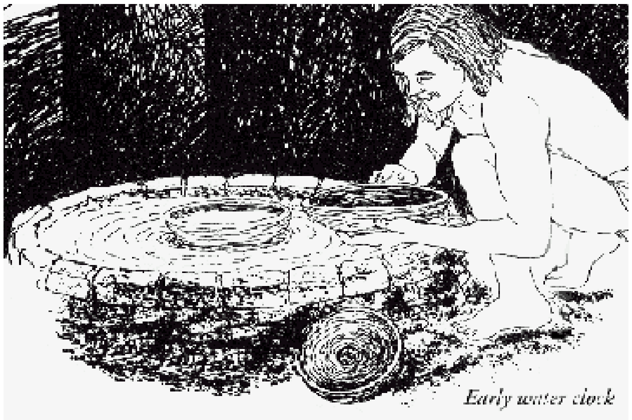

*Diagram 2.2 : Early water clock*

inscribed to the surface to measure when the water level reached a certain height, which correspond with the ‘hour’. However, the clepsydra is quite inaccurate and inconsistent as it depends on the rate of the flow of water.

In around 100 BC to 500 BC, more elaborated mechanized water clocks were developed by the Greeks and the Romans. They made the instrument more accurate, with constant and regular flow of water, adjustments of the pressure and more complex markings of the hourly interval. Sound alarms in the form of gongs and bells were included in some of the water clocks.

One of the more famous water clock is the Tower of the Winds, also known as the Horologion. It features a 24-hour mechanized clepsydra and indicators for the eight winds from which the tower got its name, and it displays the seasons of the year and astrological dates and periods.

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## China

### -Sundial

In early China, the gnomon was commonly used as an instrument for astronomical observations. The perforated gnomon and later, the pierced gnomon were invented in China. The assembly of calendar experts (dated about 104 BC) determined the true East and West points, set up sundials and gnomons and later designed the water clocks. Oriented in the meridian to record the direction or azimuthal position of the Sun, most sundials then determined time by indicating the direction of the shadow instead of the length. There were also others which measured the length of the shadow and recorded the altitude of the Sun. The use of sundials was very popular then but there was little written documentation or clear references to the sundial or sun-clock in Chinese writings.

### -Incense

During the Sung Dynasty in China (960-1279), calibrated candles and sticks of incense were used to measure time. In the incense clock, weights were connected to the incense stick by threads. When the incense stick is burnt, the weights will fall off one by one to a plate that sounds off when the weights hit the plate. The change of the time can also be marked by the change in the fragrance when different types of incense sticks are used at different time.

### -Water Clock

The water clock in China reached its peak when the emperor of the Sung dynasty ordered Su Song, a Buddhist monk, to build the most spectacular clock in history. The water clock tower is over 30 feet tall in height and is mechanized measure to time astronomically and astrologically. The water wheel at the bottom of the tower provides power to the clock. It has a device that enables the clock to stop the water flow every 15 minutes. The weight of the water drives the motion of the wheels and thus activates the mechanism inside this simple ‘machine’. The tower is able to measure quite accurately the hours of the day and the movements of the stars. They are indicated by the statues carrying inscribed tablets within the five doors that come out periodically to strike bells and gongs. In around 1094, the tower clock was stolen and destroyed by barbarians.

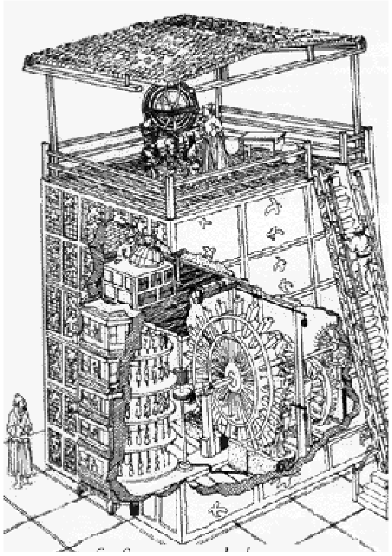


*Diagram2.3: Su Sung water clock tower*

## Europe

### -Sundial

People in medieval Europe used the hand dial as a device for telling time. The gnomon on this sundial was a stick held at angle by a person's left hand by his thumb. This was held in the left hand in the morning, horizontal to the ground, pointing West. In the afternoon the gnomon was held in the right hand, pointed East. In the Middle Ages peasants in Northern Europe even used sundials carved into the bottom of their wooden clogs! To tell time, the peasant would take off his shoe and stand it up facing the Sun using the shadow which the heel cast on the dial to tell the hour.

Europe uses sundials not far off in principles as of the Egyptian sundials. Simple sundials were place above doorways to indicate the midday and the important times of the sunlit day. By 10th century, several portable sundials were developed with an English model that could tell the seasonal changes in the Sun’s altitude.

### -Weight Driven Clock

Around the 14th century, weight driven clock revolutionized timekeeping. Large mechanical clocks appeared in towers of some Italian cities. The clock uses the principle of transferring gravitational energy acted on the weights to move the mechanisms of the clock to count time. However, the clocks had the same problem as the water clock (in which water flow rate was difficult to regulate). The problem of weight driven clock is mainly due to the friction between the moving parts which oscillates at their natural periods and causes inaccuracy in the measurements of time.

# CHAPTER3: DIFFERENT INSTRUMENT /CLOCKWORKS TO TELL TIME IN ANCIENT HISTORY

## 3500 BC Sundial

*Picture 3.1 : Sundial*

*Picture 3.2 : Timespace – built in 2000 at Gosport*

A vertical stick, gnomon, or obelisk that casts a shadow is a Sundial. They were used to tell time because the position of the Sun varies throughout the day and hence the shadow casts on the dial plate also changes. The shadow rotates "clockwise" from sunrise to sunset.

## 1400 BC Water Clocks - or Clepsydras in Egypt

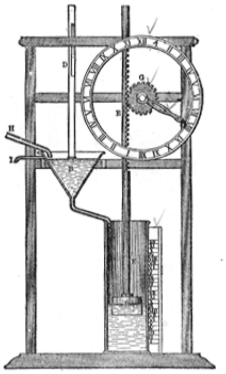


FIG. 29. — A COMMON FORM OF CLEPSYDRA IN GREEK AND ROMAN TIMES.

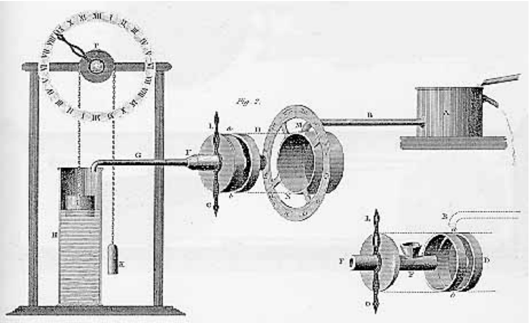


*Diagram 3.3 : water clocks from Egypt date back to 1400 BC.*

Clepsydra works in a way so that water dripping into a container raises a float that carries a pointer marks the hours on a scale. A float with a rack attached turns a toothed wheel with gadgets such as birds and ringing bells. The water-level falls at almost uniform rate. It is true that the water fall into the container at a faster rate when the vessel is nearly full but this is counterbalanced by the bigger area of cross-section of the vessel near the top. In the 16th century AD, Galileo used the Clepsydras to time his experimental falling objects. After that, an improved model was invented similar to the hourglass.

## AD 1000: Candles

*Picture 3.4 : Candle clock uses as alarm clock*

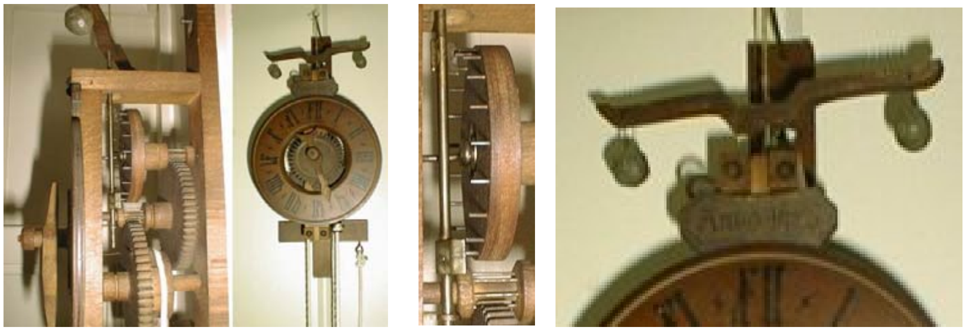
Candles were also used in ancient times as an instrument to measure time. It can be used to tell time indoors, at night, or on a cloudy day. Markings were made along the length of the candle to indicate the passage of periods of time. It can be used as an alarm clock by putting a nail into the wax. Whenever the wax which has melted down reaches the nail, then nail will fall down because there is no longer wax to support it. The nail then hits the tin pan and produces a noise.

## Hourglass or Sand Clock

*Picture 3.5 : Various types of hourglass*

The date of its invention is unknown. It is believed to have been invented in Europe in 14th century. It is made of glass vessel which consists of two compartments. The two compartments are connected by a narrow tube. The uppermost compartment has a quantity of sand, water, or mercury which runs into the lower compartment during a period of time. Navy in the past uses the sand clock as a timekeeper and to find the speed of the ship.

## AD1285: Mechanical Clocks



*Picture 3.6 : Replica of an early 17th-century foliot-and-verge clock*

Mechanical timekeepers make use of cyclic mechanical motion which repeats itself over a period of time. Mechanical clocks with an escapement came into use sometime around 1285. The escapement ticks in a steady rhythm. These mechanical timepieces had a verge and foliot which were used for the mechanism that sounded a bell. The foliot is a horizontal bar that has weights on each side. The vertical rod that supports it is called a verge.

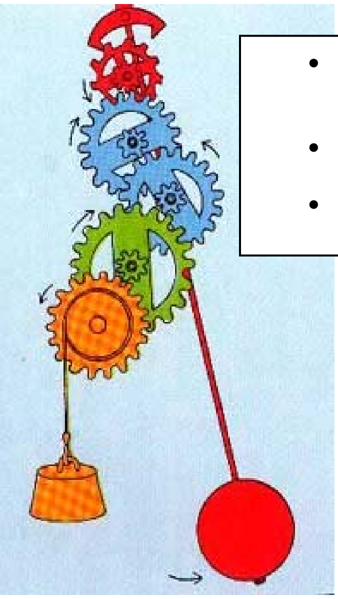
## AD1510: Spring-Powered Clock

*Picture 3.7 : Spring driven clock*

The spring-powered clock was invented in by Peter Henlein of Nuremberg, Germany. The main problem of the spring-powered clock is that it slowing down when the mainspring unwound. The force of the mainspring is greater when fully wound than when it is almost run down. Therefore Jacob Zech of Prague, at around 1525, used a *Fusee* or spiral pulley, to equalize the uneven pull of the spring. The fusee is a cone shaped grooved pulley used together with a barrel that contains the mainspring. The varying spring pull was also compensated by a very rough device known as a stackfreed.

## AD1656: Pendulum

In 1583, Galileo demonstrated that the period of a pendulum is the same, regardless of the distance through which the "pendulum do swing". He sketched out a design for a pendulum clock. However he did not constructed it. Christian Huygens, a Dutch Scientist is the first to design the weight-driven clock with a pendulum instead of a verge and foliot escapement or balance wheel, and this made it possible to have some accuracy in timekeeping.



- *Escapement at the top*
- *Weight lower left*
- *Pendulum lower right*

*Diagram 3.8 : Parts in pendulum clock*

*Picture 3.9 : Pendulum Clock*

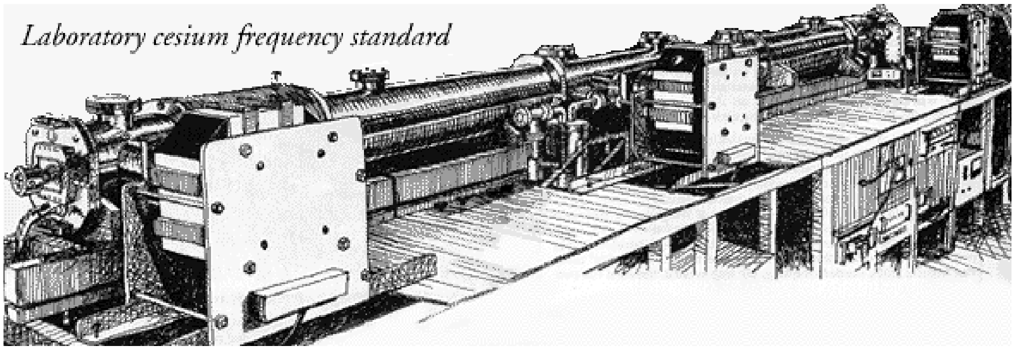
The use of the pendulum to replace foliot verge escapement is a better way in timekeeping. This is due to the fact that pendulum has a natural frequency that is independent of amplitude. The frequency does not depend on the weight of the pendulum. Its only depends on the length of the pendulum and the acceleration of gravity. The strength of gravity varies with latitude and elevation. The period of a pendulum will be greater on a mountain than at sea level. Another influence to the pendulum is caused by the room temperature. Metal alloy Invar can be used to reduce errors caused by temperature.

## AD1929: Quartz Crystal Clock

*Picture 3.10 : Quartz clock*

'Quartz clock' operation is based on the piezoelectric property of the quartz crystal discovered by the Curie brothers in 1880. The first Quartz Crystal was applied in a clock in 1929. W.A. Marrison and J.W. Horton invented the original quartz clock, which at that time was very large indeed. When an electric field is applied to a quartz crystal, it actually changes the shape of the crystal itself. If you then squeeze it or bend it, it generates an electric field. When put in an appropriate electronic circuit, this interaction between the mechanical stress and the electrical field causes the crystal to vibrate, generating a constant electric signal which can then be used for example on an electronic clock display. The magnet insides the quartz clock then changes back and forth and moves a small pinion that turns the crystal vibration to mechanical movement. The first wrist-watches that appeared in mass production used 'LED', 'Light Emitting Diode' displays. By the 1970's these were to be replaced by a 'LCD', 'Liquid Crystal Display'.

## AD 1949: Atomic Clock



*Picture 3.11 : Laboratory cesium frequency standard*

Scientists discovered some time ago that atoms and molecules have 'resonances' and that each chemical element and compound absorbs and emits electromagnetic radiation within its own characteristic frequencies. These resonances are stable over both time and space. The National Bureau of Standards (now the National Institute of Standards and Technology, or NIST) built the first atomic clock by using ammonia. By 1960, 'Cesium Time Standards' were incorporated as the official time keeping system at NIST. The ‘clock’, designed and constructed at the National Physical Laboratory in 1955, measure time by counting the vibration of cesium atoms. Each atom behaves as a small magnet, and as it passes along the tube from left to right it is deflected by two fixed magnets. Between these two it is subjected to a field alternating at a very high frequency, and if this is exactly equal to a natural frequency of the cesium, the atom turns over and when it reaches the second magnet it is deflected back on to the detector.

## AD2001 and Beyond .....

- Laser-Cooled Cesium Atomic Clock
- Primary Atomic Reference Clock in Space (PARCS)
- The Linear Ion Trap Frequency Standard
- Matchbox Atomic Clock .....

## TIMELINE SUMMARY

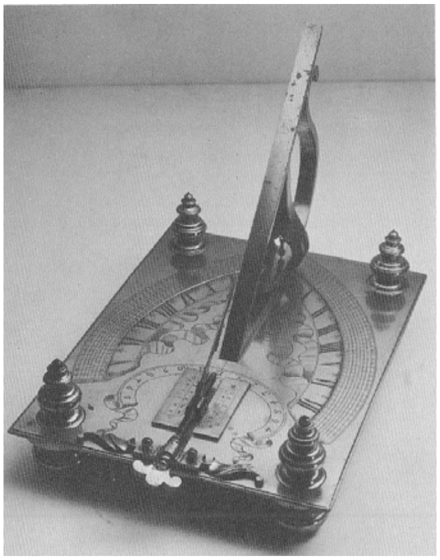
| | |
|---|---|
| 3500BC Sundials | Used in Egypt to measure time of a day due to the apparent motion of the Sun, using the shadow casting device called gnomon. |
| 1400 BC Water clocks | Believed to be first used in Egypt. The Greeks then named it Clepsydras or “water thieves”. Water flows in the device in almost constant pace to raise the float in the container which link to a pointer with scale beside it. |
| AD1000 Candles | Alfred the Great (a Saxon king) used burning candles to measure time. Candles were also used by the Egyptians and Romans. Markings were made along the length of the candle. |
| 14th century Hourglass | Hourglass or sandglass is believed to be invented in Europe around 14th century. Sandglasses were first used to measure cooking time. Sand or water flowed between the two compartments within a period of time. |
| AD1285 Mechanical clocks | Who or when these devices were invented is still unknown. Some believes they were built in Europe. These clocks were weight driven and make used of repeating cyclic mechanical motion. |
| AD1510 Spring Powered Clocks | Invented in by Peter Henlein of Nuremberg, Germany. These clocks used wounded springs to work. |
| AD1583 Pendulum Clocks | Invented by Christian Huygens. They were more accurate in telling time and capable to measure second. The frequency of the pendulums does not depend on their masses. |
| AD1929 Quartz Clocks | Invented by Warren A. Marrison and J.W Horton. The quartz crystal vibrates when it different electric fields. This vibration generates a constant electric signal. |
| AD1949 Atomic Clocks | National Bureau of Standards (now the National Institute of Standards and Technology, or NIST) builds the first atomic clock, using ammonia. After that The Britain’s National Physical Laboratory invented the cesium clock in 1955. |

# CHAPTER4: TELLING TIME IN THE DAY: SUNDIALS

## Origins and History of Sundials

The Sun is the first and most prominent timekeeper bestowed by nature, showing the time of the day with its passage across the sky. The earliest and simplest form of sundial is the use of a shadow stick, where men judged the time of day by the length and position/ direction of the Sun's shadow. By indication of the shadow path of the Sun through the course of the day, sundials are among the earliest and simplest devices for time measurement.

Appearing as early as 3500 B.C, the first gnomons in the form of obelisks were built by the ancient Egyptians. Early sundials were designed and adjusted for specific locations and in the 1500s, the invention of the "dual ring sundial" made it possible for usage of sundials at any geographic location. Sundials then became the standard, portable and accurate time measuring device for travelers, navigators and ordinary folk. During the Renaissance period, sundials were created with many various designs and other features to indicate the seasons, the calendar date, the times of sunrise and sunset, Zodiac signs, the azimuth and altitude of the Sun and the points of the compass. In the quest for accuracy, many types of sundials evolved, including some very complex portable sundials. The common sundials by the late 1700s were able to indicate the time of the day by the angle of the shadow and the time of the year by the shadow's length. At the beginning of the 1900s, sundials were commonly used at railway stations for trainmen to regulate their watches in France, as well as in parts of China, Greece and Europe.



*Picture 4.1 : An early brass sundial from 18th Century, Germany. The small mechanism allows correction for the Equation of Time.*

Despite being a reliable time indicator for a long time in history, the total dependence on the Sun makes the sundial less useful on a cloudy day. The invention of accurate mechanical clocks and the standardization of time using time zones had further made sundials obsolete. Today, most sundials are for ornamental purposes rather than a way of keeping or indicating time. However, the Sun will always rise in the morning and set at night without a doubt, and the sundial will remain as one of the reliable methods of telling accurate time in the day.

# The Basics of Sundial

## Introduction

A sundial consists of a **dial plate** and an upright shadow stick. The technical name for this stick is called ‘**gnomon**’, a Greek word meaning the ‘one who knows’, acting as a time indicator or pointer. This raised projection is usually a metallic rod, triangular plate or pin, casting a shadow on the table of the dial. The inclined edge of the gnomon is called the "style", referring to the shaft or triangular sloping edge of the gnomon which produces the working edge of the shadow used to tell the time. A gnomon is used to indicate a given moment in time, but not the interval or lapse of time.

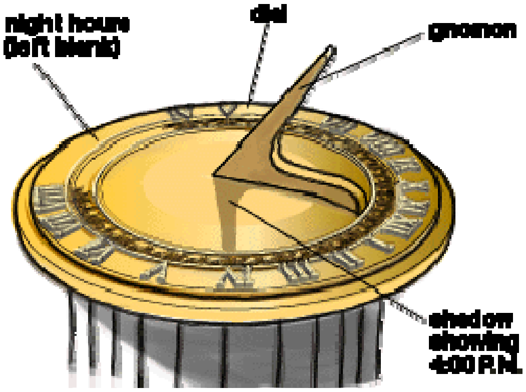


*Diagram 4.2: A typical Sundial*

As the Sun moves through the sky from sunrise to sunset, the gnomon indicates the time of day by casting its shadow on the marked surface of the dial. **Hour-lines** are marked out on the dial plane and the time can be read by observing the location of the shadow on this marked surface. Diagram 4.3 on the left illustrates the apparent motion of the Sun across the sky and the time can be read off the dial. Diagram 4.4 on the right shows how the shadow of the gnomon marks out a hyperbolic curve as the Sun makes its passage across the sky in daytime.

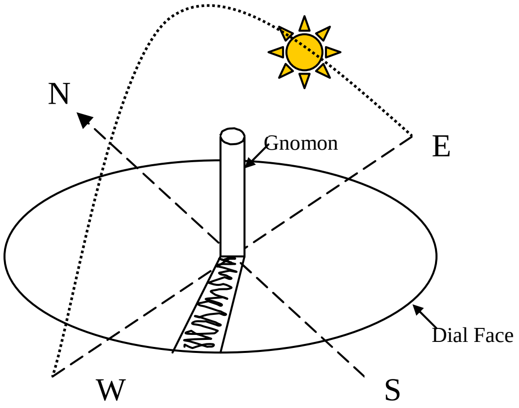


*Diagram 4.3 : Sun’s apparent motion across the sky, moving along the imaginary line called the ecliptic.*

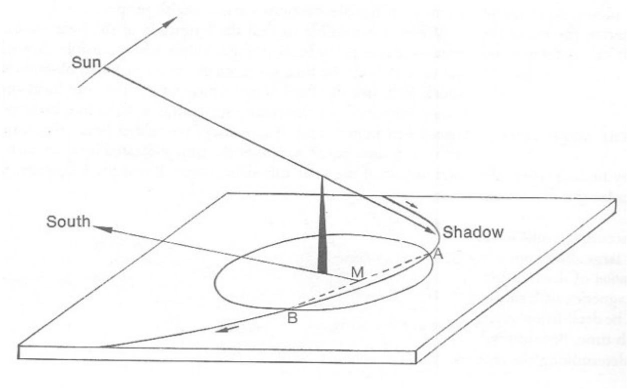


*Diagram 4.4: The shadow of a gnomon maps out a hyperbola, intersecting the circle at 2 points*

## Fixing a Sundial

The proper placement of the time marks/ hour lines is crucial to telling time accurately. A sundial has to be designed pointing in the right direction and take into consideration the **latitude** of its location. This means that adjustments has to be made for the latitude of its location and to compensate for the Earth's tilt, so that the alignment allows the hour marks to remain the same all year round.

Except for the analemmatic dials, the gnomon of a classical sundial is to be fixed to the dial plate placed on a flat, horizontal surface. The axis of the dial (gnomon) is then oriented parallel to the Earth’s true north-south axis, pointing toward the North Pole in the sky in the celestial sphere (imaginary globe).

The angle made by the style or the dial plate, with respect to the Earth's surface (horizon) has to be fixed equal to the local latitude. This ensures that the sundial is pointed to the proper elevation. As time lines are determined by the local latitude; they must be arrayed around the center of the dial differently in different places.

## Reading from a Sundial.

The Sun’s shadows records the daily apparent motion of the Sun across the sky, where the Sun generally rises and moves across the Eastern horizon and sets in the Western horizon. The diagram below shows a sundial template labeled with the major features on the face of the sundial.

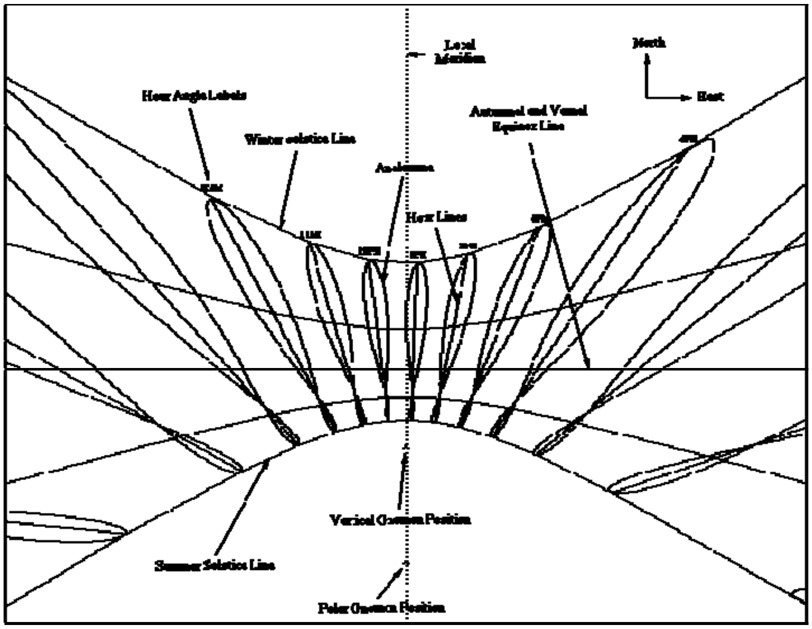


*Diagram 4.5 : A sundial plate representing the Hour lines.*

Each of the hour lines in this template indicates a time of the day, representing the set of points that the tip of the gnomon will fall on for a given time for every day of the year. However, this diagram does not account for the tilt of the Earth's axis and may only represent **mean solar time**.

Diagram4.6 on the right shows the hour line of the analemma. The Sun takes the path in the sky which looks like the figure 8, centered on each of the hour lines are called **analemma**. This is due to the fact that the Earth’s axis is tiled 23.5° in relation to the plane of its orbit around the Sun, and the path of orbit is an **ellipse**. The mean local noon can be obtained from this curve, which is the meridian of mean time. The analemma marks the difference between the standard solar time (mean local time) on the watch and the true solar time (time told by the Sun). This difference is known as the **‘Equation of time’** (which will be discussed in chapter6).

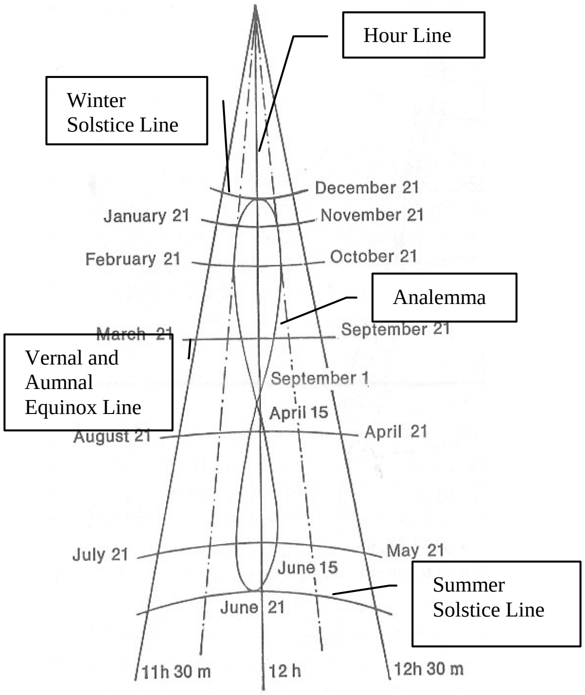


*Diagram 4.6: The analemma, part of the meridian of mean time*

## True Local Time vs. Standard Mean Time

Most sundials show what is known as **true local time.** The North "noon" line cast by the Sun at its zenith will not correspond to noon clock time, called **standard mean time.** The sundial hours (often referred to as temporary hours) consists of 12 parts of the daylight, different from the clock’s 12 equal hours of sixty minutes each. Ancient observations, tracing back to Greece and Babylon had recognized the irregular motion of the Sun through the sky. The inequality of the four seasons and the difference in times between the passages of solstitial and equinoctial points also indicates that the length of the apparent movement of the Sun across the sky varies. As the hours of true local time derived from the solar day and thus sidereal time are unequal throughout the year, it differs from the mean time, measured in hours of constant duration. Therefore there is a difference between the true local time and the standard mean time, accounted for by two reasons.

First, sundials tell the apparent Sun time and so measure true local time. The sundial reading will indicate the solar time which is defined as the hour-angle of the Sun starting from noon. This solar time, otherwise known as the true local time, is dependent on its **longitude**, which is the angular distance West of the Greenwich meridian. On the other hand, the mechanical clocks in the 24 standard time zones are set to the standard mean time. The Earth is divided into 24 zones, each 15º of longitude in width. This is because, the Earth turns 360º in about 24 hours and so the Sun's apparent position moves 360º/24hr = 15º per hour. The local placement of the sundial within the time zones will be different for every longitude. Therefore, the reading of the local solar time will differ from the international, standard mean time. Since:

$$60\,\text{min} = 15^{\circ}$$

$$1^{\circ} = 60/15 = 4\,\text{min}$$

We need to adjust by 4 minutes for each degree East or West from the central meridian. Let say we stand at a place of longitude 105° East and read the time and we get 11am. If at longitude 107°, we should get a reading of 11:08am because 1 degree is 4 minutes and since it is 2° more to the East, we will see 12:08pm. Thus if we are at a place of longitude 107° and standard mean time is using 105º East, we need to minus 8 minutes from the time we got. Now we try using Singapore as example. Singapore is at longitude 103º East but the time that they follow is 120ºEast. 120º minus 103º will give us 17º. Therefore if we get a true local time of 11am, we will need to add 68 minutes to it, making it 12:08pm.

Second, the length of the day varies throughout the year, due to the movement of the Sun. The mechanical watch keeps constant, mean seconds (where each day is exactly 24hrs long) but the shadow of a gnomon does not move at a uniform rate during the day. The sundial traces the solar day and shadow of a gnomon indicates the sidereal time or solar time, defined as the hour-angle of the Sun, which measures the time elapsed between successive noons of two day. Noon is defined as the time where the Sun crosses the meridian. As the length of the day varies throughout the year, the measurement of hours in a solar day varies, while the number of hours in a day kept by the clock is constant.

The apparent solar time marked by the sundial can be adjusted to the standard mean time by correcting for the differences for every moment of the year. The local mean time can first be derived from the apparent solar time by applying the Equation of time through calculations. Adjusting the local mean time in relation to the longitude of its location will then give the standard mean time. The following section on the ‘Equation of Time’ discusses the two effects which causes the variation of the length of a day.

## Different Types of Sundials

Sundials can be classified according to the plane in which the dial plate lies. Most sundials have dial plates which lie parallel to the Earth's axis (equatorial plane). For example, the horizontal dial lies in the horizontal plane, and the direct vertical dials lies in a vertical plane, facing the cardinal points of the compass. The time scale of the dial will then be equiangular, meaning that all the hour lines are exactly 15° apart.

Direct vertical dials also belongs to a category called **reclining dials** (or inclined dials). Reclining dials face the cardinal points of the compass (exactly North or South). The equatorial and polar dials are examples of reclining dials. A meridional reclining dial is one which faces north-south direction, but is tilted at an angle to the horizon. **Declining dials** are vertical dials which has dial plates placed in a direction other than the equatorial plane and so do not face the cardinal points of the compass. The substyle, which is the line on which the style or gnomon stands, does not point to the meridian or the noon line, which lies vertically below the dial center. The time scale of the dial is not equiangular and the angles between the hour lines can be calculated using trigonometrical formulae. The declining-reclining dial is a sundial which does not face exactly north-south and is neither horizontal nor vertical/reclining.

Some examples of special types of sundials include the horizontal-vertical dial, the concave and convex dials and the conical dial.

A **horizontal-vertical dial** has a horizontal dial plate which has multiple hour lines centered with a solid or thread gnomon. This portable dial is usually in the form of two hinged tablets. **Concave dials** are dials inscribed on concave surfaces. Many equatorial dials are concave but the position of the dial plate determines the type. Equatorial dials with convex surfaces are then **convex dials**. The **conical dial** is then a combination of all dials in the form of a cone, either partial or full, usually with a vertical shadow-casting pin the center or edge of the rim. The time is read by recording the altitude of the Sun. Goblet or chalice dials are considered conical dials.

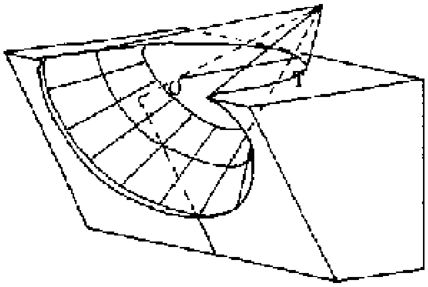


*Diagram 4.7 : Conical dial*

In the following sections, 4 common types of sundials shall be discussed, namely: **Equatorial dials, Horizontal dials, Vertical dials and Analemmatic sundials.**

## Equatorial Sundial

An equatorial sundial is a dial tilted at an angle equal to the latitude. Its base plane is aligned parallel to the equator and the gnomon lies in the centre of the dial perpendicular to it, pointing in the direction of the celestial axis. The equatorial disk dial consists of an upper dial face and a lower dial face, both marked with hour lines. These hour lines are equally spaced in the form of a star or a cross around the gnomon, making a 15º separation from adjacent lines.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*Picture 4.8 : An outdoor equatorial sundial*

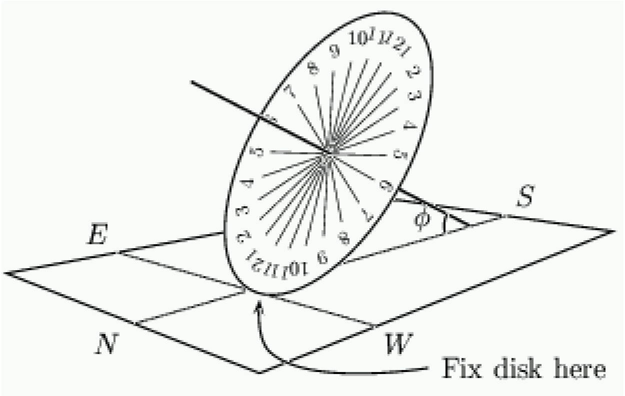


*Diagram 4.9 : Fixing an equatorial sundial*

*Picture 4.10 : An equatorial sundial used in the 19th Century, also known as the solar clock.*

For an equatorial sundial, the dial can be fixed with the gnomon set in its centre. The face of the dial is to be divided into 24 equal parts, representing 24 hours of the day and the number of hours between sunrise and sunset marked out. The equatorial sundial is useful as it is able to indicate time for as long as the Sun is above the horizon. The hour lines are to be drawn corresponding to the latitude where it is placed. If the location is far from the equator, there will be longer day during certain part of the year. In that case more hour lines have to be drawn. Whereas for location near the equator, the duration of the day and night are almost equal. Hours lines are marked on both sides of the dial plate, where the numbering on the top of the plate run clockwise and those at the bottom numbered in the other direction.

The equatorial sundial can be used at all latitude by ensuring that the gnomon of the sundial points to the celestial pole. The angle difference between the style and the horizontal is to be positioned corresponding to the same magnitude as the latitude. This means that the angle of the vertical gnomon and horizontal is equal to the latitude and the angle between the disc and horizontal is equal to the co-latitude (90º – latitude). An imaginary 'latitude triangle' can be used to arrange an equatorial sundial. A latitude triangle (shaded in the diagram) is a right-angled triangle where one of the angles is equal to the location's latitude the other is co-latitude. With its hypotenuse facing the horizontal in a north-south direction, one of the edges will point to the location's North Celestial Pole and the other pointing to the celestial equator, which the dial plate is to be aligned with.

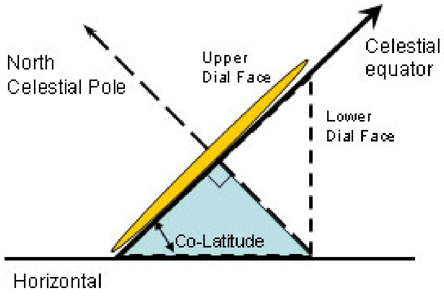


*Diagram 4.11: Setting the dial with a 'Latitude triangle'*

The dial can be useful in the Northern hemisphere, where the gnomon is pointing into the Northern sky at one end and the ground to the South at the other. The opposite applies to Southern hemisphere. On the other hand, the dial is not as useful at the equator. This is because the dial cannot make use of the angle of the Sun to indicate time. Placed on the equator, the gnomon points horizontally, the disc positioned vertically as the co-latitude is equal to 90º. (This is similar to a north dial which will be discussed later. We will see why north dial is not a very useful dial.) Nonetheless, the equatorial sundial is one of the easiest sundials to construct, and the dial plate can also be rotated for the Equation of time or longitude corrections. The time of the day is directly linked to the angle between the angle of the Sun, zenith and due north.

## How the sundial works:

An equatorial sundial resembles the Earth, with the dial plate representing the plane of the Earth's equator and the gnomon representing the Earth's axis of rotation. The upper dial face represents the Northern Hemisphere, receiving sunlight for half of the year, and the lower dial face represents the Southern Hemisphere, receiving sunlight for the other half, as the dial plate lies parallel to the equator.

Since the dial plate is fixed parallel to the celestial equator, the Sun will be above the dial plate for a period of time in a year and at other times, below. The Sun lies above the celestial equator plane from the vernal equinox (21st March) until the autumnal equinox (22nd September) and sunlight will fall on the upper dial face. For this period, the Sun is at a positive declination. The gnomon then casts a shadow only on the upper dial face. Conversely, the Sun will cast a shadow on the lower dial face from autumnal equinox (22nd September) to the vernal equinox (21st March). For this period, the Sun is at a negative declination. The shadow of the gnomon will then project only on the lower dial face.

On the day of the equinox, the declination of the Sun is 0º and the Sun moves across the sky in a course parallel to the dial plate. The gnomon will then cast shadows on both the upper and lower faces as the Sun is on the celestial equator.

## Horizontal Sundial

Horizontal sundials are the most prevalent of dials, commonly found in gardens. The flat dial plate is horizontal (hence it’s name), with multiple hour lines radiating outwards from the tip of the upright triangular plate gnomon. In the (less common) case of a horizontal sundial fixed parallel to the Earth’s axis, the hour marks will have to be corrected for the sundial’s latitude using trigonometric calculations. Horizontal sundials are usually fitted with a portable compass for adjustments.

*Picture 4.12 : A garden-variety horizontal dial*

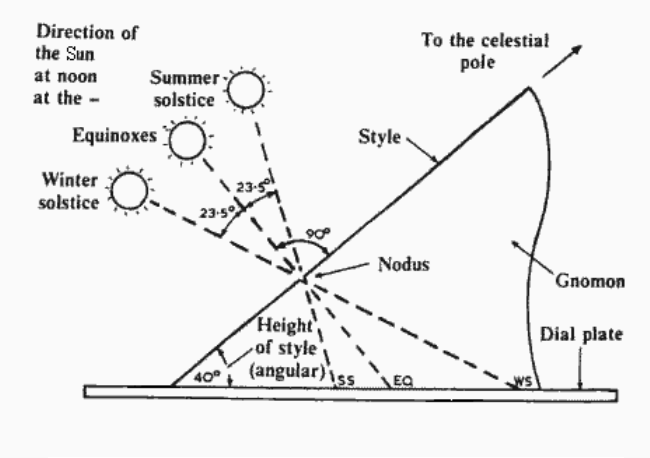


*Diagram 4.13: Side profile of a horizontal sundial*

Most horizontal sundials are constructed with a fixed or folding gnomon, slanted at an angle equal to the particular latitude of the location. This is done by ensuring that the angle which the gnomon makes with the dial face is equal to the latitude. Setting the dial in this way means that the horizontal sundials are designed for use in a particular location and so cannot be shifted very far North or South. For example, a sundial with a style/ gnomon set at 40° will tell time accurately at latitude 40° only (In Diagram 4.13). Therefore, the dial plate has to be adjusted when there is a difference between the angle at which the gnomon points (location of sundial) and the North Celestial Pole. The whole sundial is to be tipped (raised or lowered) for the correct magnitude, which is the angle difference between the gnomon and the latitude of its location.

## How the sundial works

The dial face is to be placed horizontally and the gnomon edge set parallel to the Earth's axis and thus pointing upwards to the celestial pole. The gnomon of the horizontal sundial (the thick, vertical line extended through the dial plates) is to be set in a true north-south line, along the meridian pointing to the true South pole in the Northern hemisphere and the true North pole in the Southern hemisphere. During the course of a day, the shadow of the upright, triangular gnomon will fall on the dial plate and among the hour lines, indicating the time of the day.

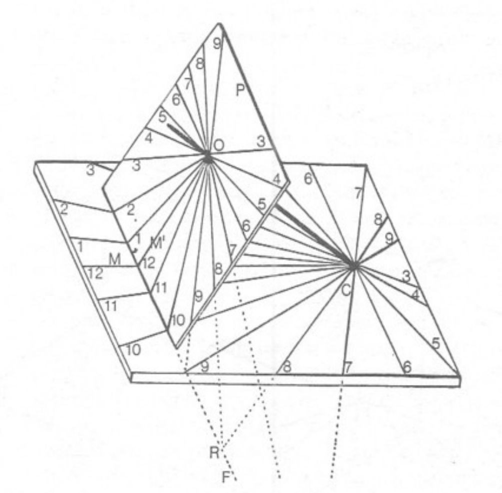


*Diagram 4.14 : Constructing a horizontal sundial*

Basic Trigonometry can be used to calculate the hour angle (the angle which the hour line makes with the 12 o’clock line) of a horizontal sundial. This hour angle can be converted to tell the time using the relation that the Earth rotates 360º in 24 hours or 15º in 1 hour.

Where:
X = the shadow angle, the angle the hour line makes with the 12 o'clock line.
h = time measured from noon in degrees
(Hour angle)
Ø = the latitude of the dial.

As illustrated in Diagram 4.15, the style is pointing to the celestial pole, P. X is the shadow angle and h is the hour angle. PNS is the meridian at that particular latitude and the shadow tip of the gnomon will fall on a point T. For PNT = 90º, NOP = Ø and TON = X, we can therefore deduce from spherical trigonometry that:

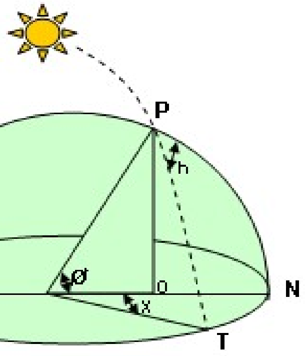


*Diagram 4.15 : Deriving the hour angle using a horizontal sundial*

$$\cos NP \cos PNT = \sin NP \cot TON - \sin PNT \cot NPT$$

$$\cos \text{Ø} \cos 90^{\circ} = \sin \text{Ø} \cot X \;\; \sin 90^{\circ} \cot h$$

$$0 = \sin \text{Ø} \cot X - \cot h$$

$$\mathbf{\tan X = (\tan h)\,(\sin \text{Ø})}$$

## Vertical sundial

*Picture 4.16: 4 directional dial*

### Definition

Vertical sundial, as the name suggests, has a dial plate which lies in a vertical plane. In other words, the shadow casts by the gnomon falls on the plate which is vertical. Vertical dials can be found on the wall of churches and other buildings. There are two types of vertical sundial, the **direct vertical dial** and the **vertical declining dial**.

### Direct vertical dials

They are oriented in such a way that they face directly toward the cardinal points of the compass. It means the wall of the building which have the vertical dial face directly to the North, South, East or West. For example, vertical direct North dial is placed on vertical wall facing North. Thus there are direct vertical dials for each of the 4 directions.

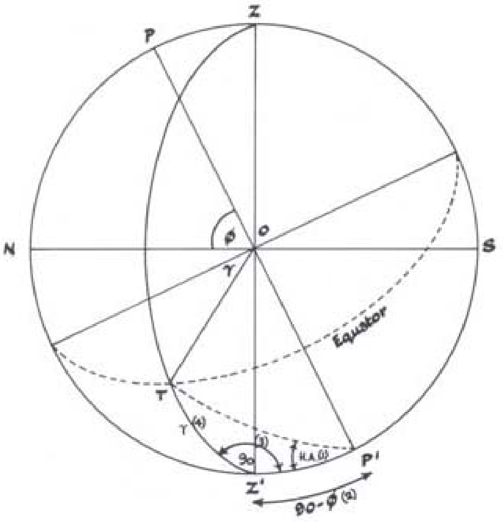


*Diagram 4.17: Drawing the Hour Line of a South Dial*

Among the 4 directions, the more commonly used ones would be a vertical north or south dial. The gnomon of these dials will be tilted at an angle equal to the co-latitude of the place. To get co-latitude, we subtract the latitude of that location from 90°. The hour lines of a North dial are in clockwise direction while the ones for South dial are in counter clockwise direction. Spherical geometry can be used to calculate the position of each hour lines. Looking at Diagram 4.20, P is the North Celestial Pole and P' is the South Celestial Pole. Z marks the location of Zenith and Z' marks the nadir. Zenith is the point in the sky directly above us. On the other hand, Z' is the point below the horizon directly below the observer. From the direction and some spherical trigonometry equation, the following equation is formulated.

$$\cos Z'P' \cos TZ'P' = \sin Z'P' \cot Z'T - \sin P'Z'T \cot Z'P'T$$

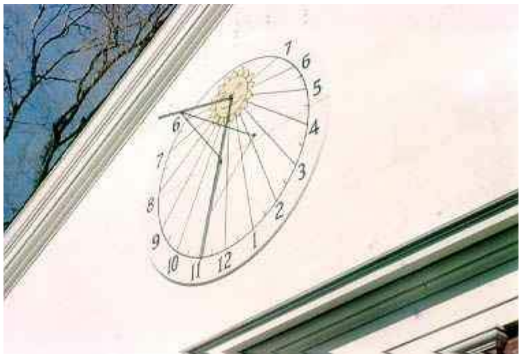
Since $TZ'P'=90^\circ$, $Z'T=\text{♈}$

$$0 = \sin (90 - \text{Ø}) \cot \text{♈} - \cot (HA) \text{ --- Equation (1)}$$

$$\tan \text{♈} = \tan (HA) \cos \text{Ø}$$

Equation 1 is similar to the equation used for horizontal dial previously. The only difference is that, in equation 1, it is sin(90- Ø) instead of sin Ø in the equation for horizontal dial.

*Picture 4.18: North dial*



*Picture 4.19: South dial*

As compared to the East and West dial, North and South dial prove to be more useful. This is because the latter can be used throughout the day whereas each of the East and West dial can only measure half the time of a day. However, the North and South dial has their own limitation. In the Northern hemisphere, a South dial would be more useful since the Sun would rise in the East, go to the South and set in the West. A North dial in the Northern hemisphere would be redundant since the Sun can only cast a shadow on it in the early morning or late evening. It is the opposite for the Southern hemisphere. At the equator, either of the dials would be useful half of the year only. The reason is that the Sun would go to the North after rising between vernal equinox and autumnal equinox. The Sun would go to the South on the other half of the year.

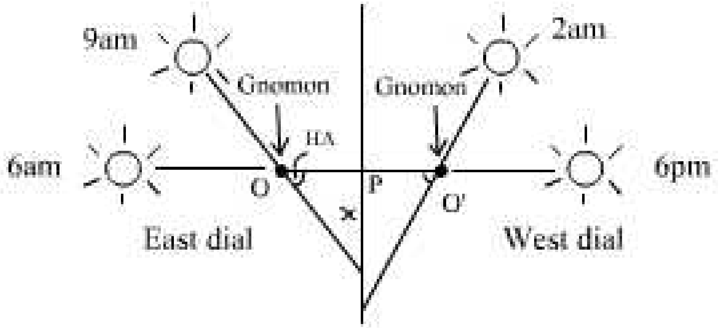


*Diagram 4.20: The hour lines of an east and west dial*

The East dial can only be used before noon. At any time in the afternoon, the Sun would be in the West. Likewise for the West dial, it can only be used in the afternoon. Due to this, the hour lines on the East dial is from 6 o'clock in the morning to 12 noon and from 12 noon to 6 o'clock in the evening. The people living further away from the equator will have a larger range as they experience a longer day time at some parts of the year. The hour lines on the plate are straight horizontal lines, one below the other instead of the usual circular pattern. This is illustrated in Diagram 4.20. When the plate is facing East, the plate would lie on the north-south plane. Therefore, the gnomon is parallel to the plate since it has to be parallel to the Earth’s axis. It also lies directly on the 6 o’clock marking since the sunlight will be straight towards the sundial at that time. To calculate and draw the hour lines, we refer to Diagram 4.20. x is the distance from the shadow to the centre, OP is the height of the gnomon and HA is the hour angle of the Sun. For each hour after 6am, 15º is added to HA. Hence for 9am, HA=45º. Once all the values are known, x can be calculated by the formula $x = OP \tan HA$

Declining vertical dials.

These is another category of vertical dial which do not face directly North, South, East or West. Instead they face toward the intermediate compass points. The four major types are the Northeast decliners, Northwest decliners, Southeast decliners and Southwest decliners. The gnomon will be at a lesser angle than co-latitude and is twisted out of the vertical in order for it to be parallel to the Earth’s axis. Since it is not possible to reorient the building to face the cardinal points, this type of dial is used.

*Picture 4.21: Declining vertical*

Looking at the diagram, the extreme upper and lower lines mark the two solstices. During the winter solstice, the shadow will fall on the upper line. The shadow will be at the lower line during summer solstice. In case of equinox, the shadow will fall onto the straight line across the centre. To determine the time, one can observe which line the shadow falls on and read the corresponding number to that line.

## Analemmatic Sundial

The analemmatic sundial is a specific type of sundial that measures the shadow cast by the vertical gnomon which crosses the hour markings of a horizontal sundial in the shape of an ellipse. Unlike the other traditional sundials, the analemmatic sundial has a shadow-casting object that moves depending on the declination of the Sun throughout the year. Therefore, a table of dates has to be calculated to adjust the gnomon according to the date of the year. This will mean placing the gnomon at different locations, running in a North-South direction (meridian), marking out the minor (shorter) axis of the ellipse. However, this vertical pin or rod acting as the gnomon requires daily setting, to be moved about from place to place according to the Sun's declination.

*Diagram 4.22: An analemmatic dial which consists of 2 horizontal dials*

The dial, when constructed large enough, allows the person to act as the gnomon, rather than using the usual vertical gnomon. The person has to stand on a point along the minor axis of the analemmatic sundial, according to the month of the year. The analemmatic sundial is sometimes used as a piece of large mathematical structure placed in parks and gardens. This is considered unique compared to the common and traditional sundials like the equatorial sundial.

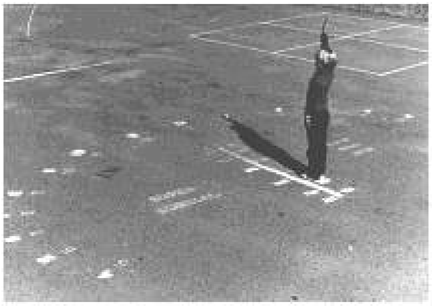


*Diagram 4.23: A human gnomon*

At around the 17th century, the analemmatic sundial was made portable as long as they are accompanied by the standard horizontal dial. However, a compass is required to orientate the sundial to enable the user to tell the actual time of the day. Permanent sundials are designed to function only at specific latitudes while portable horizontal dials can be calibrated for use at different locations, with a compass for alignment to the North. As these combined dials are large and cumbersome they were not commonly used.

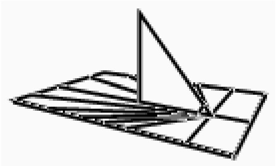
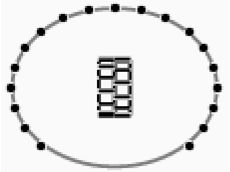
| | Traditional Dial | Analemmatic Dial |
|---|---|---|
| |  |  |
| Shape | Hour lines radiate from a central point. | Ellipse of hour point |
| Shadow-casting object | Fixed, parallel to Earth's axis. | Changes daily, vertical. |

*Diagram 4.24: Differences between a traditional dial and an analemmatic dial*

## How the Sundial works

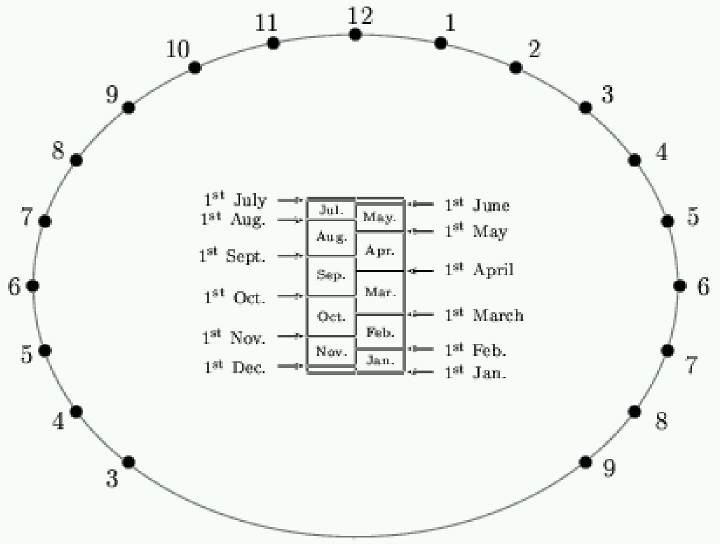


*Diagram 4.25: The elliptical dial face of an Analemmatic Sundial*

The hour band is elliptical instead of circular. Each hour will be indicated by a point along the circumference of the horizontal elliptical dial face, rather than an hour line. The dial will be able to measure time accurately throughout the year, if all the adjustments were made accordingly.

The shadow cast by the Sun on the horizontal surface moves in a clockwise manner around the pointer. However, the length of the shadow cast varies according to the time of the day and so the endpoint of the shadow may fall outside of the circle, making it quite difficult to read. Therefore, the hour point is the point of intersection between the ellipse and the circular path cast by the gnomon shadow. In order to tell time, this hour point can be calculated using equations.
The gnomon will be placed on the North-South direction on the meridian, with a distance Z from the centre of the ellipse. This requires a number of steps.

Let $M$ = Length of Major axis
$m$ = Length of minor axis
$ø$ = the latitude of the location of the dial
$d = 23.45 * \sin [360 / 365 * (284 + N)]$

First, we need to find out the declination of the Sun.
Where: $d$ = declination,
$N$ = day number away from January 1 = day 1

To calculate the length of the minor axis of the ellipse, we use the equation:

$$m = M * \sin (ø)$$

The hour angle $\theta$ of the Sun relative to the Noon (a reference point) is taken to be $0^o$ where x (in hours) is the time past Noon. Since the time taken for the apparent Sun to travel 360º across the sky in a day is roughly 24hrs, the hour angle for each hour past Noon will be 15º. Therefore, the hour angle, $\theta$ is given by $\theta = 15^{\circ}x$.

Let z be the azimuth, the angle which the Sun makes with respect to the y-axis of the ellipse. z is calculated by the equation:

$$\cot(z) = \sin(ø)$$

The distance of the point from O (center of the ellipse) along the major axis, which runs West-East is given by:

$$x = M \sin(\theta)$$

The distance of the point from O (center of the ellipse) along minor axis, which runs North-South is given by:

$$y = m \cos(t)$$
$$y = M \sin(ø) \cos(\theta)$$

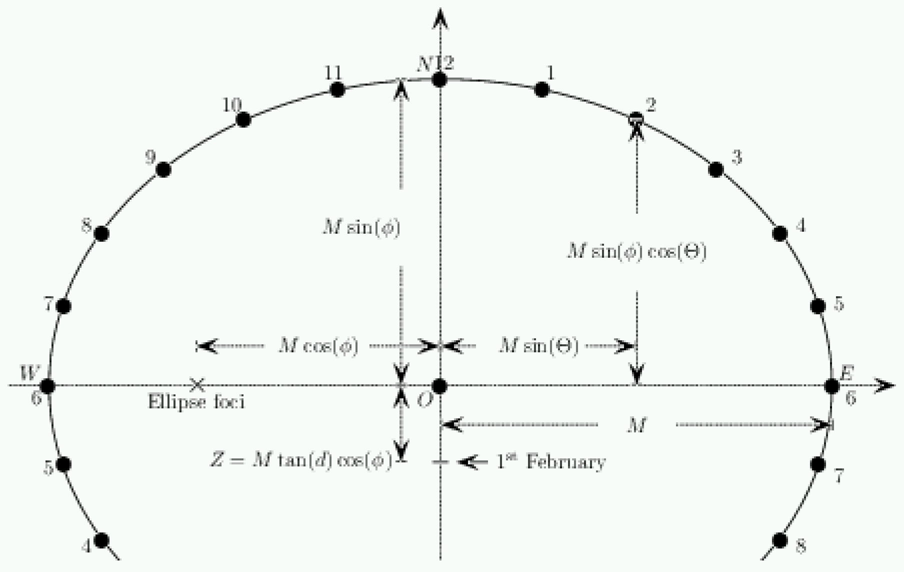


*Diagram 4.26: Calculations and hour point of the sundial.*

Therefore, where the position of the gnomon, indicating the time is Z, the intersection of the ellipse and the shadow cast by the gnomon with co-ordinate (x,y) will then be at the hour point calculated by the equation:

Z = M * tan (declination of the Sun) * cos (the latitude of the location of the dial)

**$Z = M * \tan(d) * \cos(ø)$**

# CHAPTER5: TELLING TIME AT NIGHT

We have seen how the different types of sundial in the world aid people in the past in time-keeping. Useful as it is, the sundial has a vital limitation: The necessity of sunlight for readings. During the night, the sundial is practically useless. Thus the need to search for other means of telling time at night, before the invention of mechanical clock.

## Moon Dial

As mentioned earlier, sundial requires sunlight to tell the time. Some people attempted to use moonlight to achieve the same effect. This is not totally impossible but we would need a very good weather to get the moonlight. The Moon definitely would be brightest at full Moon. It is also at full Moon that we can read off the time directly from the sundial except that it is night instead of day. If we happened to get Moon light on other days, it is also possible to use the sundial to get the time. However, we have to bear in mind that the speed of the Moon compared to Sun, when view from the Earth is slower. This means that if the Moon is at its highest altitude when it is 12 midnight today, it will only reach highest altitude after 12 midnight on the next day. This difference is 48 minutes. It can be easily derived from the equation below.

$$\frac{24\times 60\,\mathrm{min}}{360^{o}} = 4\,\mathrm{min/deg} \text{ --- Equation 1}$$

$$\frac{360^{o}}{30day} = 12\,\mathrm{deg}/\,day \text{ ---Equation 2}$$

$$4\times 12 = 48\,\mathrm{min}/\,day \text{ --- Equation 3}$$

The Sun appears to move one round along the celestial equator in a day. Since one round is 360º, we can see from the first equation that each degree corresponds to 4 minutes in time. The number of degrees that the Moon will lag behind the Sun each day is calculated in the second equation. Since it takes one month for the Moon to be in phase with the Sun again, we would get an answer of 12 degrees per day. Here, we are assuming that each month is 30 days to make the calculation simpler. If each degree is 4 minutes and as the Moon lags behind by 12 degrees each day, we can easily calculate that everyday, the Moon will be slower than the Sun by 48 minutes (Equation 3).

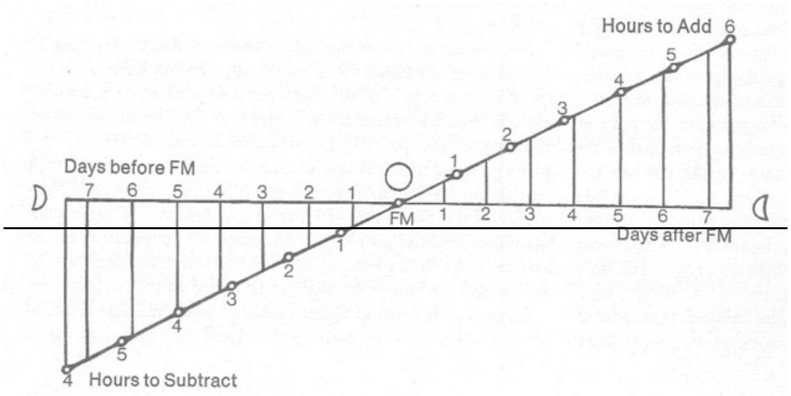


*Diagram 5.1: Moondial chart*

Therefore, for each day away from the full Moon, we need to adjust the time read from the Moon dial by 48 minutes. When it is before the full Moon, 48 minutes is to be added for each day. As for the days after the new Moon, we will have to subtract instead. The following is an example to illustrate this. Suppose that it is the 13th day of the lunar month. This is 2 days before the full Moon, so we need to subtract 1 hour and 36 minutes from the time read from the Moon dial. Therefore, if the light from the Moon casts a shadow on the Moon dial at 11 o'clock, it is actually 9:24pm. It is the same calculation for the days after the full Moon, except that addition is made to the time we get from the Moon dial instead of subtraction.

## The Big Dipper

*Diagram 5.2 : The big dipper*

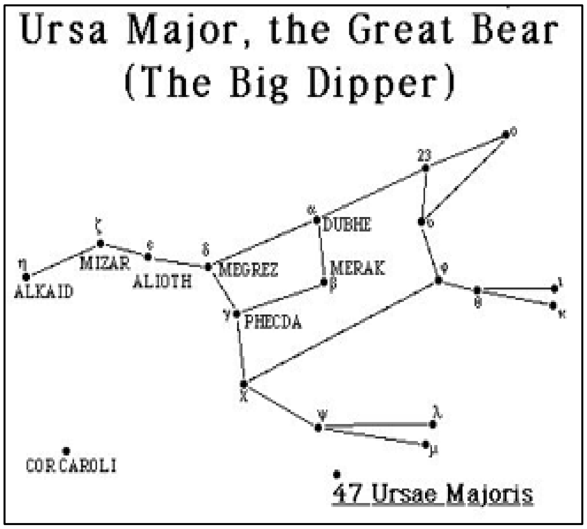


*Diagram 5.3 : Ursa Major*

The most common and popular method of telling time at night is to use the constellation, '**Big Dipper'**. It consists of seven stars: three stars form the handle and four stars form the bowl. The big dipper is not a constellation by itself. It is actually part of the constellation "**Ursa Major**", which means big bear or the greater bear. As can be seen from the 2 diagrams on the right, the big dipper is actually the tail of the Ursa Major. The big dipper has another interesting characteristic. If one draws a line from Merak to Dubhe, this line will eventually point to where Polaris is. This is actually very useful for navigation as can be seen from the story "Underground Railroad" where the slaves escape from their captivity in Southern United States by using this constellation to flee North to Canada.

Big dipper is a circumpolar constellation, which rotates around the Polaris. Depending on the latitude of the observer, it may be above the horizon for 24hrs, 12 hrs or even non-visible at all times. Since it rotates around the Polaris, it would be easier to spot if the observer is at the Northern part of the Earth. Therefore this method of telling time may not work well for people living close to the equator and definitely will not work for people living in the Southern hemisphere since it is not possible to see Polaris from their position.

The method to use big dipper to find time is as follows: Imagine Polaris as the centre of the clock face. Draw a line from Polaris passing through the two pointer stars, Dubhe and Merak. This line forms the hand of the clock. This will be the imaginary sky clock (Diagram 5.4). Note that the clock is a 24 hour clock and not 12 hours since the stars take approximately one day to go one round around the sky. Each hour on the imaginary clock is 360° ÷ 24 =15°. It is important to note that this clock turns counter clockwise, which is different from the normal clock.

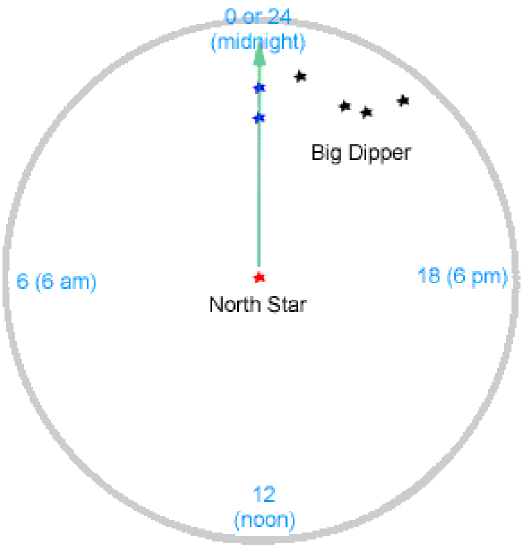


*Diagram 5.4 : Midnight on 7th of March*

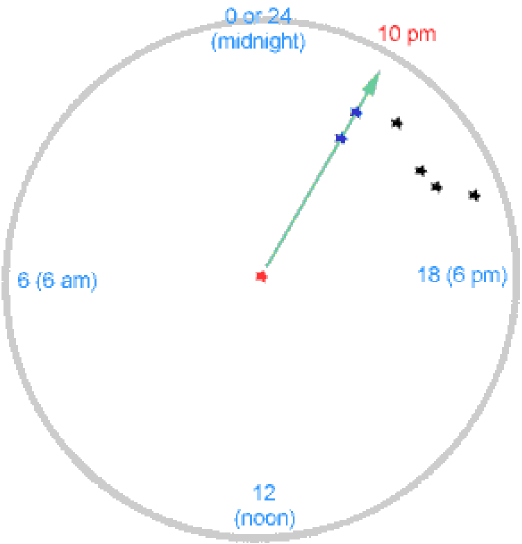


*Diagram 5.5 : 10pm on 7th of March*

The two pictures above are what an observer will see on March 7, standing in Mexico with longitude 117ºW in longitude. In the diagram, 12 midnight is directly on top. The position where we normally see as 3 o'clock is now 6pm. The 6 o'clock position is now 12 noon and 9 o'clock is 6am. One thing to note is that it is not possible to tell the time in the day when the Sun is out since the sunlight would prevent us from seeing any of the stars.

The clock hand on in Diagram 5.5 is pointing at what we usually see as 1 o'clock. However, the clock hand here moves in a counter clockwise direction. Furthermore, when the clock hand moves by 1 hour, the actual time actually moves 2 hours. Therefore when the pointer stars are at 1 o'clock position, it is actually 10pm.

7th of March is the day where the pointer stars are directly on top of Polaris at 12 midnight. On other days, the 12 midnight marker would shift as the stars are moving faster by 4 minutes each day.

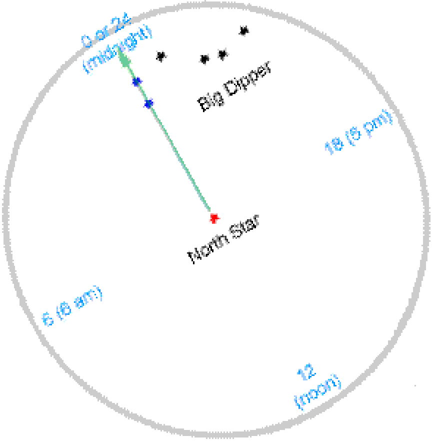


*Diagram 5.6 : Midnight on 6th of April*

$$\frac{24 \times 60\,\mathrm{min}}{360^{\circ}} = 4\,\mathrm{min/deg}$$

The stars take 1 year for to be in phase with the Sun again. Therefore each day, it will be faster by approximately 1 degree and lead by 4 minutes. After 30 days, the stars will lead by 2hours.

$$\begin{aligned} &30 days \times 4\,\mathrm{min} = 120\,\mathrm{min} \\ &= 2 hrs \end{aligned}$$

Therefore, at 12 midnight, the pointer stars will no longer be directly on top of Polaris. Instead, it is at 2am (of Diagram 5.4) of the imaginary clock. 12 midnight position of the pointer stars will be as shown in Diagram 5.6. This means that we always have to remember where the 12midnight marker is in order to tell the time.

Alternatively, we can just read the time from to the imaginary clock and minus the number of hours that we are ahead of, according to the days from 7th March. Ignoring the fact that the midnight marker moves (meaning the midnight marker is as shown in 5.4), we will read Diagram 5.6 as 2am on 6th of April. As we are 1 month after 7th of March, the stars will be ahead of the Sun by 2hrs. So we make a calculation:

$$2am - 2hrs = 12am$$

Thus the time is actually 12 am. To be exact, we can actually minus 4 minutes for each day after 7th of March. For example, if I am reading the clock during 10th of March, I will need to minus 12 minutes from the reading I get.

## The Nocturnal

*Diagram5.7: The nocturnal*

It has been proven a difficult task to get an accurate value for the time at night. The methods described above will only give us an approximation of the actual time. The degree of error is about ±30minutes. In order to get a more precise time, human created an apparatus, nocturnal to help them. People then created an apparatus, the nocturnal, in order to find time more precisely. Nocturnal, also known as star clock, also uses the pointer stars and Polaris to calculate the time. The principle is the same as what we had discussed previously. However, one advantage of nocturnal over the big dipper is that it makes reading the time from the imaginary sky clock easier and with less error.

A nocturnal consists of 2 circular plates riveted together. Inscribed on the back plate are the months and days of the year. Diagram 5.8 below shows a nocturnal with zodiac sign markings instead of our usual month. For instance, 20th of September will be indicated as 1st day of Libra. The other plate is called the hour plate. As the name suggest, the hour plate marks the 24 hours of a day. The marking is numbered 1 – 12 for day time, and again 1 – 12 for night time.

*Diagram5.8:Close up of nocturnal*

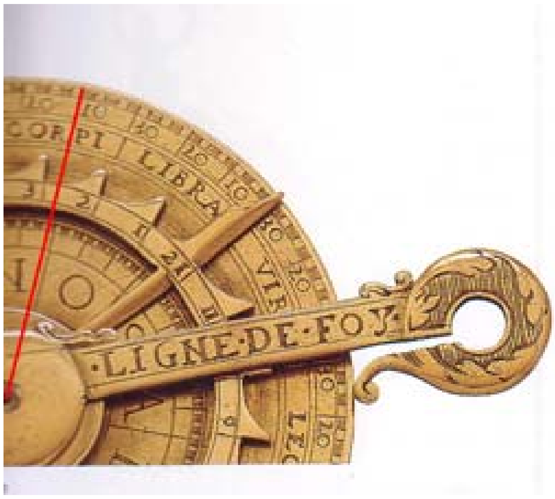


*Diagram5.9:Red line pointing to 2:40am*

To read the time, the user has to first adjust the pointer of the hour plate to point to the current date. The aim of this is to adjust the 12 o'clock marking so that it is at the correct place when we read the time. This is similar to the adjustments we make when using the big dipper, by subtracting 4 minutes for each day after 7th of March. Then the observer holds the handle of the nocturnal and looks at Polaris through the centre hole of the nocturnal. After locating the North Star, the hour plate is rotated using the movable arm such that it aligns to the 2 pointer stars of the big dipper. When it is aligned, the nocturnal arm will indicate the hour of the night. Diagram 5.8 shows the nocturnal hand pointing at

10pm. To get a more precise reading, we can use the day of the month, 1 – 30, on the outer plate. From the picture, we can see that the number 1- 30 correspond to 2 hours.

Each increment of the number on the outer plate is equal to additional 4 minutes to the hour we get, as $120\,\text{min} \div 30 = 4\,\text{min}$ To illustrate this concept, look at Diagram 5.9. If the red line is the nocturnal hand, we can see that it is now between 2am to 3am. If we look at the outer plate, we can see that the red line is pointing at 10, which is 10 divisions after 2am on the hour plate.

$$10 \times 4\,\text{min} = 40\,\text{min}$$
$$2am + 40\,\text{min} = 2:40am$$

Therefore the more accurate time would be 2:40am. Note that if we draw a line from the 2 on the hour plate to the outer plate, the line correspond to the 0 on the outer plate. The previous is the ideal case where we can just read the 10 on the outer plate and multiply it by 4. But this will not always happen. If the line from the 2 on the inner plate coincides with 10 on the outer plate and the red line is at 14, then we would have to take 14 minus 10, which equals to 4 and use 4 in the multiplication. (Not use 14)

The time obtained from the nocturnal is **true time**. It is not the actual time used in country. To get **standard mean time** which is shown on the watch, we can use the methods chapter 6.

The nocturnal is a very useful instrument for telling the time at night from the Pole Star and big dipper. With the nocturnal the time estimation is ±15 minutes. This is a large improvement in accuracy from the method describe previously. It also eliminates the hassle of calculating how many days we are from 7th of March by rotating the pointer of the hour plate.

## The Crux clock

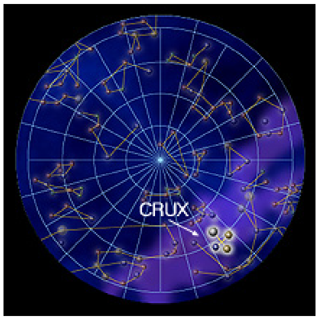


*Diagram 5.10: South Pole and Crux*

Only the people in the Northern hemisphere are able to use the big dipper to calculate the time, and not those in the Southern Hemisphere. These people can instead use the Southern Cross, also known as Crux to find out the time in the night.

The Crux is made up of 4 bright stars: Acrux, Mimosa, Gacrux and Delta Crucis. Note that in Diagram 5.10 the Crux is upside down. Diagram 5.11 shows how it looks like when it is up right.

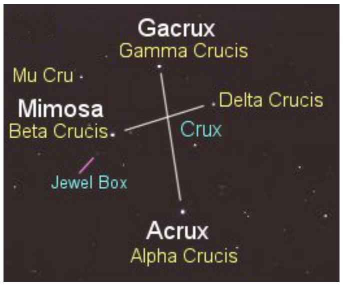


*Diagram 5.11: Crux*

As it is circumpolar, the Southern Cross will rotate around the South Pole. Similar to the method of telling the time using big dipper, the centre of the clock face is the South Pole. However, unlike North Pole, where there is Polaris, South Pole does not have a star there. It is only the black sky where we can see all the stars rotate around it. The hand of the clock is the line joining Gacrux and Acrux. The day when the 12 midnight marker is at 12 o'clock position is 29th of March. The stars here will rotate in clockwise instead of counter clockwise. This can be easily understood by rotating a pen in a counter-clockwise direction when view from the top of the pen. Not changing the direction of rotation look at it from the tip of the pen. You will see that the rotation is clockwise.

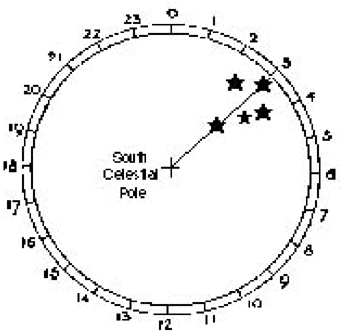


*Diagram 5.12: Crux clock*

Since the rotation, when looking at the South pole is clockwise, we can image it to be similar to a normal clock except that each hour on a normal clock will be 2 hours on the crux clock. So the 3 o'clock position is actually 6pm on the crux clock. Diagram 5.12 on the right shows the a 24 hour clock with the clock hand at 3am.

If the date is not 29th of March, we have to make adjustments. The adjustment is very similar to how it is described previously when using big dipper. For every 30 days after 29th of March, we can minus 2 hours from the reading from the clock. Using the diagram above as illustration, if what we see on the 28th of April is as shown, then we need to minus off 2 hours from 3am. The actual time is then 1am. To be more accurate, we can also minus 4 minutes for each day after 29th of March. During 31st of March, we can minus 8 minutes from the reading we get.

Similarly this method of telling time has its limitation. Since the nature of this method is similar to the big dipper method, it also shows the local true time instead of the standard mean time.

## Astrolabe

### History

The astrolabe is from the Greek word ‘*astron*’ and ‘*lambanien*’ which means ‘one who catches the heavenly bodies’. It is considered one of the oldest scientific astronomical instruments in the world, with a history of almost 2000 years. It is used to measure several astronomical events quite accurately, some which are the time of the night and day and also the position of the celestial bodies. It is very popular with astronomers throughout history as it is portable and multi-functional.

Greek astronomer, Hipparchos refined the projection theory and principle of the astrolabe, eventually its invention. It was later, in around 8th century that the astrolabe was introduced to Islamic countries through translation of Greek texts which covered contents related to astrolabes. It was widely used by the Islams, who found the astrolabe very useful mainly because it is able to help locate the direction of Mecca and also the time of the day which enabled them to know prayer times more accurately.

After the Islams conquered Spain, the astrolabe was introduced and imported to European countries. The Europeans discarded the feature of Islamic prayers on the astrolabe and included features like timekeeping methods and astrological information. Slowly in the 14th century, the astrolabe was made more complex and it became a more popular and important astronomical tool. It was considered to be prestigious and of high education to have the knowledge to use the astrolabe at that time.

The astrolabe was slowly phased out in around 1650 as a timekeeping tool due to the invention of more accurate, reliable and specialized instruments. One of them is the pendulum clock. The astrolabe is much appreciated by astronomers and is used as an educational tool till today.

### Components of Astrolabe

*Picture 5.13: Various parts of an astrolabe*

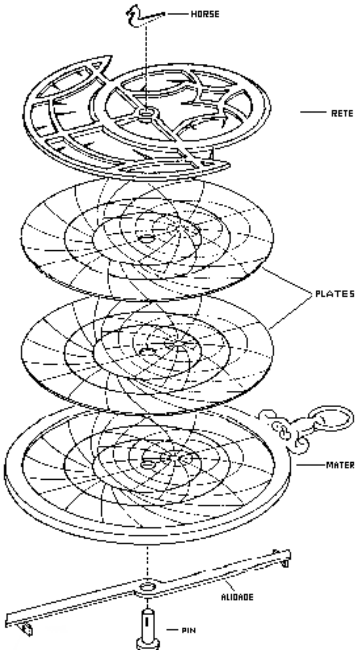


*Diagram 5.14: Parts of an astrolabe*

An astrolabe is made up of a front part and back part. The front part consists of fixed and rotating part which includes mater, plates and rete. At the back of it, there is an **alidade** which is used for looking at celestial objects.

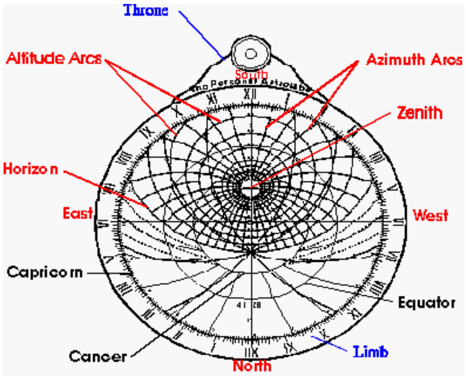
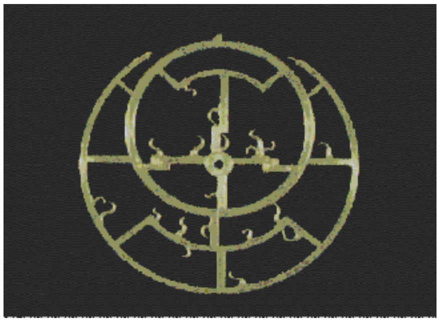


*Diagram 5.15: Detailed plates*

*Picture 5.16: Plate of an astrolabe*

These form the major parts of the astrolabe. The mater is usually made of brass disk and is about 0.6cm thick with a radius of about 7.5cm. It hollowed out in the centre to hold a set of thin brass plates placed on top of it. Markings are made on the edge of the disk which is called the limb. The scale used in astrolabe includes degree and 24 hours. The plate, which is also called climate or tympanum, has altitude and azimuth marking on it. Observing celestial objects movement using the horizon celestial coordinate system depends on the observer's latitude and hence if the astrolabe is used in different latitudes, the plate can be changed for the closest user latitude.



*Picture 5.17: Rete*

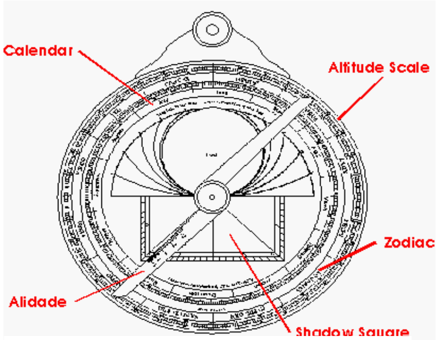


*Diagram 5.18: Back of an astrolabe*

The **rete** which is made of brass has cutaway parts to reveal the readings of the plate beneath it. It is a skeletal brass disc that is fitted above the plate. On the rete, there are labeled pointers which represent the more prominent stars in the sky. The rete also consists of a ring which is divided into 12 sections of 30 degrees to represent the sign of the zodiac. The **ring** is also called ecliptic as it tracks the movement of the Sun in the course of a year. The rete is made to rotate to simulate the daily movement of the celestial objects in the sky. A clock type hand called rule is fitted on top and held in position by a pin passing through the instrument.

The back of the astrolabe has a rotating alidade to measure the altitude stars and Sun or other celestial objects. Alidade usually can be used for observation. It has different scales depending on the location and the time when the astrolabe are made. Islamic instruments often included scales used to find the direction of Mecca and determining the prayer times.

*Picture 5.19: An astrolabe*

### Telling time with astrolabe

To use an astrolabe as a clock, an observer holds the astrolabe on the ring. A bright star is identified and observed through the alidade. The rete is then rotated such that the pointers of that star are above the altitude of the plates corresponding to the altitude of that star in the sky. Now the clock has been set and the Sun altitude can be determined. With that, we can tell the true local time.

# CHAPTER6: EQUATION OF TIME

The effect for the discrepancy of the measurement of time is due to two reasons which cause the analemma: The Earth's tilt of the equatorial plane from the ecliptic plane and the eccentricity of the Earth's orbit.

When the Earth rotates on its axis, the Sun appears to move across the sky. The Sun's path through the sky, marked by the gnomon's shadow, changes every day due to the 23.5 º tilt of the Earth's axis from the perpendicular to the ecliptic plane. Since the Earth's axis is inclined to its orbital plane, the plane of the Earth's equator is also inclined to the plane of the Earth's orbit around the Sun. This is called **obliquity**. The North Pole is tilted toward the Sun half of the year, and away from the Sun on the other half of the year. Other than the apparent eastward drift amongst the stars, the Sun moves along the ecliptic Northwards or Southwards with respect to the celestial equator. The Sun that we see everyday moves along the celestial equator. That is only the daily motion. As the true Sun moves towards or away from us, it therefore lags behind or moves ahead of the mean (fictitious) Sun at different times of the year, resulting in uneven speed.

Other than the tilt of the Earth, the change of the Earth's speed around the Sun also causes the varying lengths of the day. This is due to the fact that the Earth's orbit around the Sun is an ellipse and so its distance from the Sun varies throughout the year. As the Earth's orbit around the Sun in an elliptical manner, the distance between the Earth and the Sun is minimum (at the **perihelion**) around Dec 31 and is maximum (**aphelion**) around 1 July. The great German astronomer, Johann Kepler discovered that the line joining the Earth and the Sun sweeps over equal areas in equal times in its yearly revolution around the Sun. Kepler's 2nd Law, states that "The radius vector sweeps equal areas in equal times". Therefore, in order for the Earth to cover the same area of the ellipse in a given interval of time, its speed changes in the orbit. The Earth moves faster (apparent longitude changes faster ), where the force of gravity is higher, when it is closer to the Sun and slower when it is further from the Sun. Specifically, the solar time indicated by the Sun will be slower relative to clock time between perihelion and aphelion and faster relative to clock time between aphelion and perihelion.

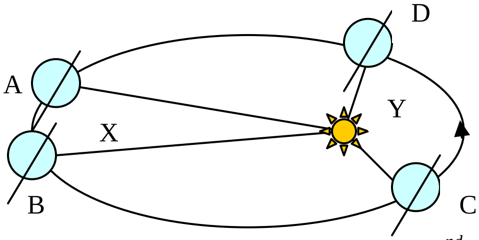


*Diagram 6.1: Illustration of Kepler's 2nd law. To cover equal areas (X and Y) in equal times, the Earth has to travel over a larger distance, faster along CD than along AB*

Together, the tilt of the Earth's axis and the Earth's elliptical revolution round the Sun results in varying speeds of the apparent Sun's motion, leading to variations of the length of a day. Thus, these contribute to the difference in the true local time and standard mean time, known as the 'Equation of Time'. The following graph illustrates the combination of the two effects and enables correction of the time measured on the sundial in order to find the standard mean time.

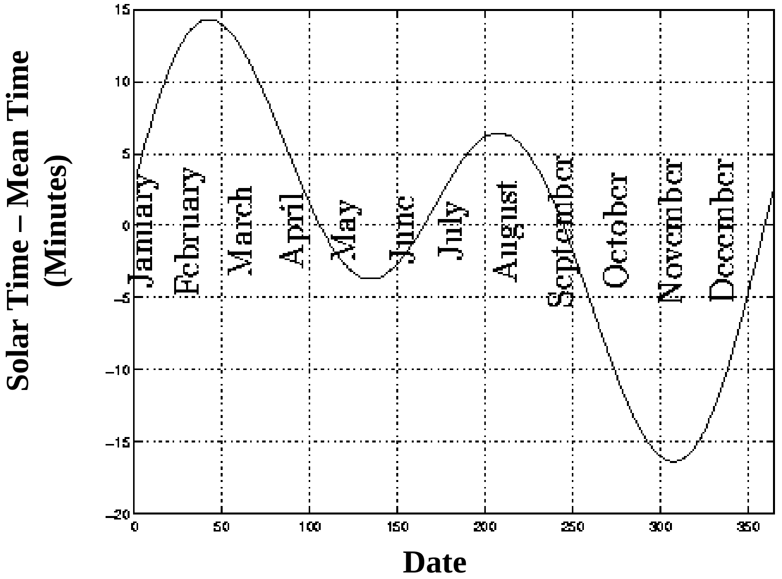


*Diagram 6.2 : The Equation of Time, a combination of the 2 effects.*

In February, the greatest difference between clock-time (standard mean time) and sundial time (true local time) will be positive, and the greatest negative difference will occur in November. This difference can vary as much as 12 ½ minutes to 16 ½ minutes in a year. It is zero on 16 April, 15 June, 1 September and 25 December and has maxima and minima near 12 February, 15 May, 27 July and 4 November.

## Declination

As discussed in the previous section, one of the causes of the Equation of time is due to the obliquity or tilt of the Earth's axis with respect to its orbit. When the path of the Sun is traced throughout the course of a day, it will show a distinct line. Therefore the sundial serves to tell the time of the day, depending on the declination of the Sun. The Sun's yearly apparent motion around the Earth on the ecliptic determines its shift of direction path, North and South of the equator over the course of the day, and this shows the Sun's declination. Declination of the Sun, which is the projection of the Sun's motion onto the equator, will be a maximum when its motion along the Ecliptic is parallel to the equator at the solstices and will be a minimum at the equinoxes. At the solstices and equinoxes, the Sun will reach the meridian at noon and so the equation of time due to obliquity will be zero.

## Solstices and Equinoxes

During summer in the Northern hemisphere, the Sun's path in the sky will be higher and stays above the horizon longer. The Sun rises near the East, moves North of the equator and sets near the West. In winter, the path is lower in the sky. The days are shorter and the Sun rises near the East, moves South of the equator and sets near the West. On the days of the solstices, the declination of the Sun from the ecliptic is the greatest. The Summer solstice (usually 21st June) marks the day when the Sun reaches its furthest northern position, and its furthest southern position is marked at the Winter Solstice (usually 21st December). On the day of the Vernal and Autumnal equinox, all locations on Earth experience equal lengths of days and nights (12 hours each). The path of the gnomon on a sundial will then trace out a straight line as the declination of the Sun is 0°. On any other dates, the shadow of the gnomon will gradually change between the Summer and Winter Solstice lines.

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http://www.math.sfu.ca/histmath/Special/Introduction.htm

# LIST OF ILLUSTRATIONS

Diagram 2.1: Obelisk used in early Egypt. The ancient Egyptian shadow clock on top measures the hours from sunrise till noon and is reversed for measurement until sunset.
http://physics.nist.gov/GenInt/Time/early.html

Diagram 2.2 : Early water clock
http://physics.nist.gov/GenInt/Time/early.htm

Diagram2.3: Su Sung water clock tower
http://physics.nist.gov/GenInt/Time/early.html

Picture 3.1 : Sundial
*Clock a History.* (Floyd)Ernie Cummings,(February 1997)
*http://users.commspeed.net/k6xf/clock.htm#ONE*

Picture 3.2 : Ti Timespace – built in 2000 at Gosport
*New Sundials. Internetworks Ltd of Epsom, England*, (7 March 2005)
*http://www.sundials.co.uk/newdials.htm*

Diagram 3.3 : Water clocks from Egypt date back to 1400 BC.
*A Brief History of Clocks: From Thales to Ptolemy*. Jesse Weissman, (unknown)
http://www.perseus.tufts.edu/GreekScience/Students/Jesse/CLOCK1A.html

*Picture 3.4 : Candle clock used as alarm clock*
*The History of Timekeeping.* Elytra Design, Cambridge MA, (unknown)
*http://www.elytradesign.com/ari/index.htm*

*Picture 3.5 : Various types of hourglass*
*The History of Timekeeping.* Elytra Design, Cambridge MA, (unknown)
*http://www.elytradesign.com/ari/index.htm*

Picture 3.6 : Replica of an early 17th-century foliot-and-verge clock
*The Engines of Our Ingenuity*. John H. Lienhard, (2000)
http://www.uh.edu/engines/epi1506.htm

Picture 3.7 : Spring driven clock
*The History of Timekeeping.* Elytra Design, Cambridge MA, (unknown)
*http://www.elytradesign.com/ari/index.htm*

Diagram 3.8 : Parts in pendulum clock
*Clock a History.* (Floyd) Ernie Cummings,(February 1997)
*http://users.commspeed.net/k6xf/clock.htm#ONE*

Picture 3.9: Pendulum Clock
*The History of Timekeeping.* Elytra Design, Cambridge MA, (unknown)
*http://www.elytradesign.com/ari/index.htm*

Picture 3.10 : Quartz clock
*The History of Timekeeping.* Elytra Design, Cambridge MA, (unknown)
*http://www.elytradesign.com/ari/index.htm*

Picture 3.11 : Laboratory cesium frequency standard
*A Walk Through Time*. National Institute of Standards and Technology (NIST) Physics Laboratory, (May 1995):
*http://physics.nist.gov/GenInt/Time/early.html*

Picture 4.1 : An early brass sundial from 18th Century, Gemany. The small mechanism allows correction for the Equation of Time.
unknown

Diagram 4.2: A typical Sundial
*Sundial*. Britannica.com Inc. and Encyclopædia Britannica, Inc. ,(2000)
http://www.britannica.com/clockworks/sundial.html

Diagram 4.3: Apparent motion of the sun
Self drawn

Diagram 4.4: The shadow of a gnomon
Rohr, René R. J, *Sundials : history, theory, and practice* (New York: Dover Publications, 1996)

Diagram 4.5: A sundial plate representing the hour lines.
*How do I read the sundial?* Robert C. Berrington, (October 14, 1995)
http://physics.uwyo.edu/~rberring/sundial/node4.html

Diagram 4.6: The analemma, part of the meridian of mean time
Rohr, René R. J, *Sundials : history, theory, and practice* (New York: Dover Publications, 1996)

Diagram 4.7: Conical dial
*Greek and Roman Sundial*. Gibb(unknown)
http://www.math.sfu.ca/histmath/Special/methods%20of%20analysis.htm

Picture 4.8 : An outdoor equatorial sundial
*Equatorial.* The North American Sundial Society, Inc. (unknown)
http://www.sundials.org/faq/

Diagram 4.9 : Fixing an equatorial sundial.
*Analemmatic sundials: How to build one and why they work*, Chris Sangwin, Chris Budd
http://plus.maths.org/issue11/features/sundials/

Picture 4.10 : An equatorial sundial used in the 19th Century, also known as the solar clock.
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Diagram 4.11: Setting the dial with a latitude triangle
Self drawn

Picture 4.12 : A garden-variety horizontal dial
*Horizontal Dials*. The North American Sundial Society, Inc.(unknown)
http://www.sundials.org/faq/

Diagram 4.13: Side profile of a horizontal sundial
*The Mathematics Of Sundials*. Liew Huay Ling, Lim Siew Yee,(2000/2001)
http://www.math.nus.edu.sg/aslaksen/projects/sundials/

Diagram 4.14 : Constructing a horizontal sundial
unknown

Diagram 4.15: Deriving the hour angle
Self drawn

Picture 4.16 : 4 directional dial
*Vertical Dials and Reclining Dials.* The North American Sundial Society, Inc. (unknown)
http://sundials.org/faq/types/vertical.htm

Diagram 4.17: Drawing the hour line of a south dial
*The mathematics of Sundials*. Liew Huay Ling, Lim Siew Yee(2001)
http://www.math.nus.edu.sg/aslaksen/projects/sundials/diff_vertical_direct.html

Picture 4.18 : North dial
*Vertical Dials and Reclining Dials* The North American Sundial Society, Inc. (unknown)
http://sundials.org/faq/types/vertical.htm

Picture 4.19 : South dial
*Vertical Dials and Reclining Dials.* The North American Sundial Society, Inc. (unknown)
http://sundials.org/faq/types/vertical.htm

Diagram 4.20: The hour lines of an East and West dial
Self drawn

Picture 4.21: Declining vertical dial
Vertical Sundial (unknown)
*http://ourworld.compuserve.co.uk/__121b_fv0pIQVY/leoZ5wgBvfOt2eQiq2DkoOCypB++9QZOaVqYsTVWMGjDQ==*

Picture 4.22:
Stephenson, Bruce, *The universe unveiled : instruments and images through history* (New York : Cambridge University Press, 2000)

Picture 4.23:
The North American Sundial Society. (unknown)
http://www.sundials.org/faq/

Diagram 4.24: Difference between a traditional dial and an analemmatic dial
*Plus magazine*. C.J Budd and C.J. Sangwin (June 2000)
http://pass.maths.org/issue11/features/sundials/

Diagram 4.25: The elliptical dial face of an analemmatic dial
Rohr, René R. J, *Sundials : history, theory, and practice* (New York: Dover Publications, 1996)

Diagram 4.26: Calculation and hour point of the sundial
*Plus magazine*. C.J Budd and C.J. Sangwin (June 2000)
http://pass.maths.org/issue11/features/sundials/

Diagram 5.1: Moondial chart
Rohr, René R. J, *Sundials : history, theory, and practice* (New York: Dover Publications, 1996)

Diagram 5.2: The big dipper
*Big Dipper*. Jerry Lodriguss, (1974-2005)
*http://www.astropix.com/HTML/C_SPRING/BIGDIP.HTM*

Diagram 5.3: Ursa Major
*Ursa Major*. (unknown)
http://www.astro.wisc.edu/~dolan/constellations/constellations/Ursa_Major.html

Diagram 5.4 : Midnight on 7th of March
Telling Time with the Stars. Troop 326 (unknown)
http://community.palouse.net/troop326/startime.html

Diagram 5.5 : 10pm on 7th of March
Telling Time with the Stars. Troop 326 (unknown)
http://community.palouse.net/troop326/startime.html

Diagram 5.6: Midnight on 6th of April (edited by us)
Telling Time with the Stars. Troop 326 (unknown)
http://community.palouse.net/troop326/startime.html

Picture 5.7: The nocturnal
Stephenson, Bruce, *The universe unveiled : instruments and images through history*
(New York : Cambridge University Press, 2000)

Picture 5.8: The close up of nocturnal
Stephenson, Bruce, *The universe unveiled : instruments and images through history*
(New York : Cambridge University Press, 2000)

Picture 5.9: Red line pointing to 2am
Stephenson, Bruce, *The universe unveiled : instruments and images through history*
(New York : Cambridge University Press, 2000)

Diagram 5.10 : South Pole and Crux
*Crux - The Southern Cross.* Jennifer Bergman. (November 7, 2003)
http://www.windows.ucar.edu/the_universe/crux.html

Diagram 5.11: Crux
*Crux And Southern Centaurus.* Jim Kaler, (unknown)
http://www.astro.uiuc.edu/~kaler/sow/cru-t.html

Diagram 5.12: Crux clock
*Determining Time From the Southern Cross*. Michael Smith (1999)
http://users.hunterlink.net.au/~madms/cruxstuff.html

Picture 5.13: Various parts of an astrolabe
Humboldt State University, 1 Harpst Street, Arcata, California
http://www.humboldt.edu/%7Erap1/EarlySciInstSite/Instruments/PlanAstrolabe/ESI.PlanAstExp.jpg

Diagram 5.14: Parts of an astrolabe
*The Astrolabes*. James E. Morrison, Janus
http://astrolabes.org/parts.htm

Diagram 5.15: Detailed plates
*The Astrolabes*. James E. Morrison, Janus
http://astrolabes.org/parts.htm

Picture 5.16: Plate of an astrolabe
Humboldt State University, 1 Harpst Street, Arcata, California
http://www.humboldt.edu/%7Erap1/EarlySciInstSite/Instruments/PlanAstrolabe/ESI.PlanAstExp.jpg

Picture 5.17: Rete
*The Astrolabes*. James E. Morrison, Janus
http://astrolabes.org/rete.htm

Diagram 5.18: Back of an astrolabe
*The Astrolabes*. James E. Morrison, Janus
http://astrolabes.org/back.htm

Picture 5.19:
Stephenson, Bruce, *The universe unveiled : instruments and images through history*
(New York : Cambridge University Press, 2000)

Diagram 6.1 : Illustration of Kepler's 2nd law.
Self drawn

Diagram 6.2 : The Equation of Time, a combination of the 2 effects.
*Analemmatic sundials: How to build one and why they work*. Chris Sangwin, Chris Budd
http://plus.maths.org/issue11/features/sundials/