Sun Homework

GEK 1506 Heavenly Mathematics: Cultural Astronomy

Group 32
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Objective:

The objective of this homework is to observe the rising or setting position of the sun on a day in August, around the autumnal equinox (around 23rd September) and a day in October. We are to i) determine the date of the autumnal equinox and the zenith passage from the sunrise/sunset positions, ii) study the shape of the shadow of a ring-shaped object in the East-West plane, and iii) study the shape of the shadow path of a vertical sundial in the course of a day.

Definition of Terms

Equinox:
Equinox is an imaginary point in the sky where the ecliptic coincides with celestial equator. When the sun is at this point, it is the day of the equinox. On the day of the Vernal and Autumnal equinoxes, the sun rises due East, and sets due West, when viewed from all parts of the Earth. All locations on earth will experience equal lengths of days and nights (12 hours each) and the Sun's ray falls perpendicular to the Earth's equator. Autumnal equinox is the moment when the sun appears to cross the celestial equator, heading southward.

Declination of the Sun:
The sun's apparent yearly motion around the Earth is along the ecliptic. During its yearly motion there is certain part of the year when the sun is above the celestial equator and vice versa. Declination of an object is the perpendicular distance from that object to celestial equator. It is measured in degrees.
Declination of the sun, will be a maximum when its motion along the ecliptic is parallel to the equator at the solstices. It will be a minimum at the equinoxes when the Sun's motion along the Ecliptic is perpendicular to the equator.
Declination of the sun can be calculated from the angular difference from due East (sunrise) or due West (sunset) and the location of the observer.

Measuring Azimuth:

In order to estimate how many degrees the Sun's rising/setting point is from due East/West, we have to determine the Sun's azimuth.

Taking North as 0° and East, South, West as 90°, 180° and 270° respectively, the Sun's position in terms of the number of degrees along the horizon from North is measured by using the compass shown. The compass which we are using has the azimuth marking so an additional protractor would not be necessary.

We place the compass in such a way that the sun is visible through the needle part of the compass. The azimuth direction is then read off from the compass.

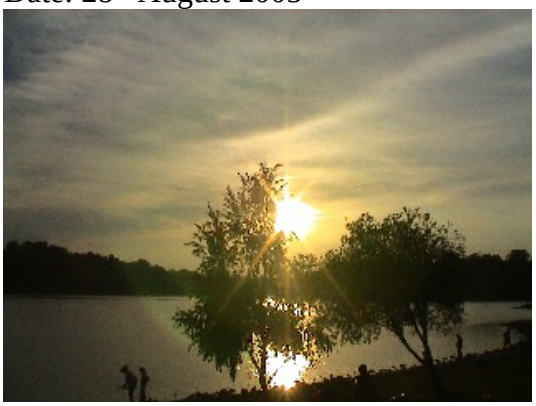
Observing the Setting Position of the Sun

Procedures:

The location chosen for this exercise was Peirce reservoir where the horizon can be better viewed without much obstruction. We chose to observe the setting position of the sun. A fixed location (beside a signboard) was chosen in order to compare the 3 observations.

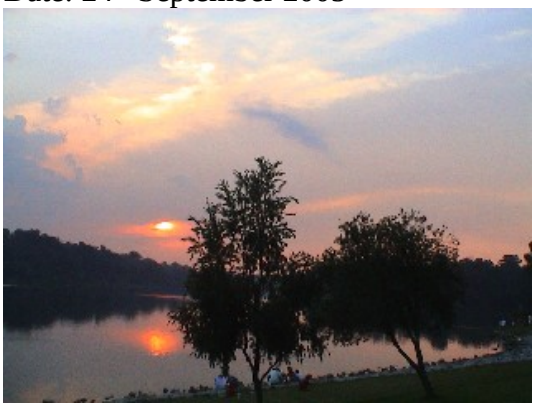
Observations:

Date: 28th August 2005



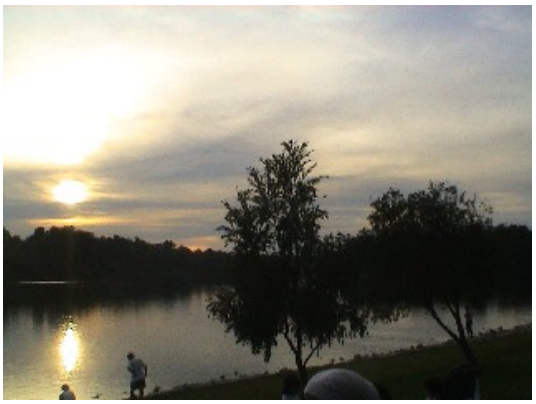
Sunset at 18:31hr
Approximate setting position of the Sun: 10º North of due West. (Azimuth: 280º) Notice that the sun is between the two trees in the foreground of the photograph.

Date: 24th September 2005



Time: 18:26hr
Approximate setting position of the Sun: 0º, due West (Azimuth: 270º). The sun in the photo is now slightly to the left of the trees in the foreground.

Date: 22nd October 2005



Time: 18:14hr
Approximate setting position of the Sun: 10º South of due West. (Azimuth: 260º)
It is obvious in the photo that the setting sun is very much to the left of the 2 trees, as compared to the previous 2 occasions.

Evaluation and Analysis:

Comparing the 3 observations and the azimuth readings, it is observed that the sun's position changes from Northwest on 28th August to due West on 24th September, and then to Southwest on 22nd October.

The declination of the sun decreases from 10º in August to 0º near the day of the autumnal equinox. The declination continues to decrease from 0º near the day of the equinox to -10º or 10º below the celestial equator on a day in October.

At the June solstice and December solstice, the sun's declination is a maximum (±23.5º) with respect to the celestial equator, due to the tilt of the Earth's axis with respect to its orbit. The declination of the sun will decrease to a minimum (0º) on the day of the Vernal equinox and Autumnal equinox. Hence, the declination will vary between ±23.5º and 0º when it is not the time of the equinox.

The day of the equinox can be determined through the azimuth of the sun when it is setting. This will be the day when the sun set directly due West on the equator and the declination will be 0º. It is also the day where the day time and the night time are equal. It is observed that this occurs around the day of our 2nd observation (where the estimated date in theory is 23rd September).

Studying the Shape of the Shadow of a Ring-shaped Object

Procedures:

The 3 observations for this exercise were conducted at almost the same time, around 5:14pm – 5:30pm. It is assumed that the shape of the shadow will not change in relation to location. A flat hoop was used as it is wide enough for better observations. This 'ring' was placed vertically on the ground in the East-West plane and

Observations:

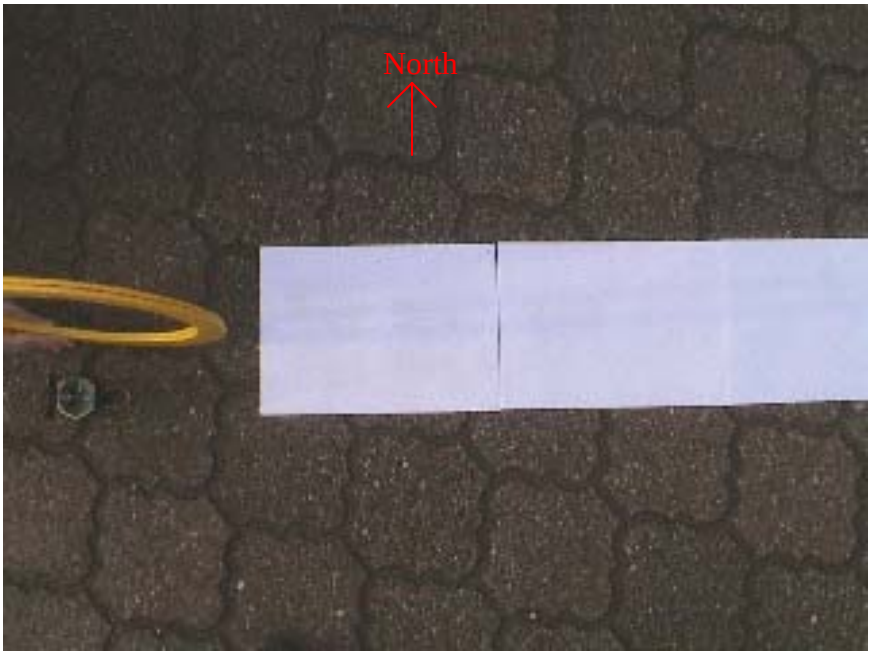
1) Date: 31st August 2005
Time: 17:14hrs

With the ring placed vertically on the ground in the East-West plane, the shadow formed is in the shape of an ellipse and fall below the east west plane. The shadow of the top part of the forms the bottom of the ellipse. The top portion of the ellipse is formed by the bottom part of the ring.

2) Date: 24th September 2005
Time: 17:26hr

On this day, the sun set almost directly due West. The shadow of the upper half of the ring falls exactly on the lower half. A thick, flat line is formed, and it is unable to separate the shadow of the upper and lower portion of the ring.

3) Date: 22nd October 2005
Time: 17:20hr



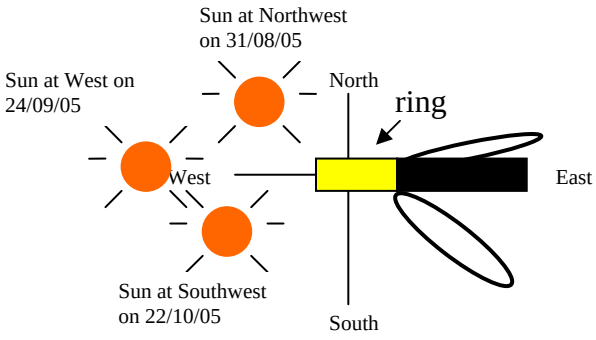
We took a photo of the projection of the ring's shadow on pieces of white paper as it was too dark for the shadow to be visible. Here, the shadow formed is again in the shape of an ellipse. The shadow is above the east west plane. The top part of the ring casts a shadow showing the top part of the ellipse. The bottom portion of the ellipse is formed by the lower part of the ring.

Evaluation and Analysis:

The shadows formed in the observations on 31st August and 22nd October were in the shape of an ellipse. The shadow formed on 24th September produces nearly a straight line, and this situation is similar to the shadow path of the vertical sundial. This method can also be used to determine the day of the equinox, using the same principle of declination. However, only locations near the equator will see the sun rising straight up from East and set direct west during the equinox.

On 31st August, the sun sets in the Northwest. This will cast a shadow below the east west plane as seen in the diagram on the right..

On 22nd October, the sun sets in the Southwest. The shadow formed will be above the east-west plane.



Studying the Shape of the Shadow Path of a Vertical Sundial

Procedures:

The exercise was conducted at a platform near Bishan Park for all 3 occasions. A stick was placed on the ground (with blu-tack) and its shadow marked throughout the course of the day with a chalk. At least 5 markings are to be drawn for each observation. A curve was drawn to join up the markings, indicating the shadow path of this vertical sundial. The photographs are mostly taken from the same position, facing the North.

Observations:

Date: 31st August 2005


Taken at 10:29hr


Taken at 11:45hr


Taken at 12:51hr


Taken at 14:27hr


Taken at 15:24hr


Taken at 16:23hr


Taken at 17:08hr


Shadow path, facing East


Shadow path, facing West

The shadow of the stick is formed stretching from Southwest to Southeast, forming a curve. When viewed facing North, the shadow path can be said to be downward curved, lying South of the vertical stick.

Date: 23rd September 2005

Taken at 9:51hr

Taken at 10:50hr

Taken at 11:50hr

Taken at 12:55hr

Taken at 13:51hr

Taken at 14:58hr

The shadow path, facing North

Shadow path, forming a straight line almost.

Another photo of the shadow path, taken from directly the top of the stick.

The shadow of the stick cannot be seen in the photograph taken at 12:55hr. This is not because of the absence of sunlight. The photographer's shadow is deliberately put in to show that there is sunlight but the shadow of the stick falls directly on the position of the stick. As the sky turned cloudy after 3pm, we had to abort the observation. A curve was drawn nonetheless, as we have obtained more than 5 readings. The shadow of the stick formed almost a straight, stretching from direct West to direct East.

Date: 22nd October 2005


Taken at 10:00hr


Taken at 10:59hr


Taken at 11:49hr

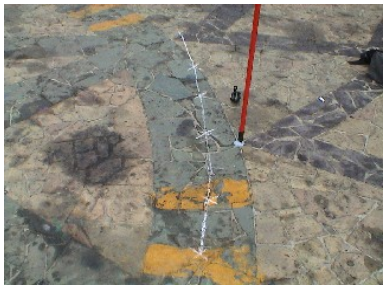

Taken at 12:56hr


Taken at 14:32hr


Taken at 15:36hr


Taken at 16:43hr


Shadow path, facing North


Shadow path, facing East

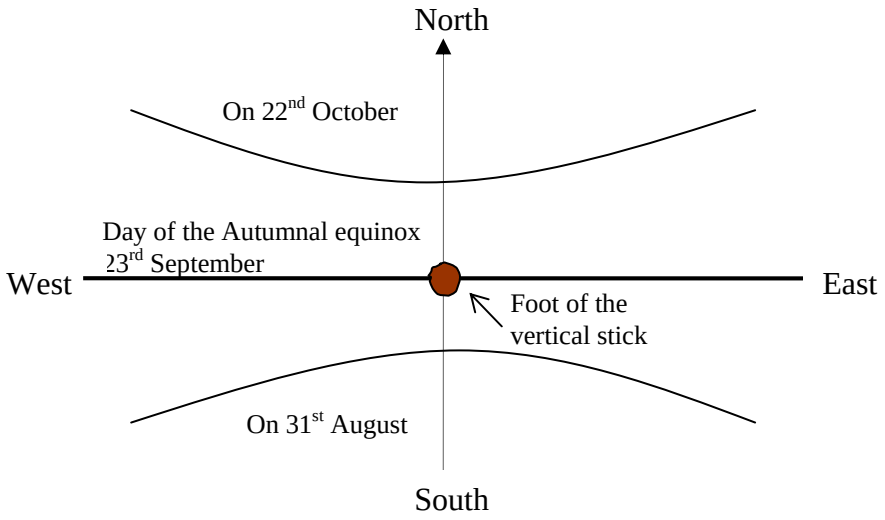
After several attempts in the rainy month of October, we finally managed to conduct this exercise on a sunny day. The shadow of the stick is formed stretching from Northwest to Northeast. When viewed facing North, the shadow path can be said to be an upward curve, lying North of the vertical stick.

Evaluation and Analysis:

As the Sun moves across the sky from east to west each day, the shadow which is cast on the ground by the tip of the stick, traces a path from west to east.

Only on the days of the equinox will the shadow path formed by the tip of the stick placed on a horizontal surface on the equator trace out a straight line, as the declination of the sun is 0º. On any other dates, the shadow will gradually change between the Summer and Winter solstice lines. When the Sun is above the celestial equator (In this case August, in the period between the Summer solstice and Autumnal equinox), this path is concave toward the vertical stick (downward curve). Conversely, when the Sun is South of the equator (In this case, October, in the period between the Autumnal equinox till Winter solstice), the shadow path is concave away (downward curve) from the vertical

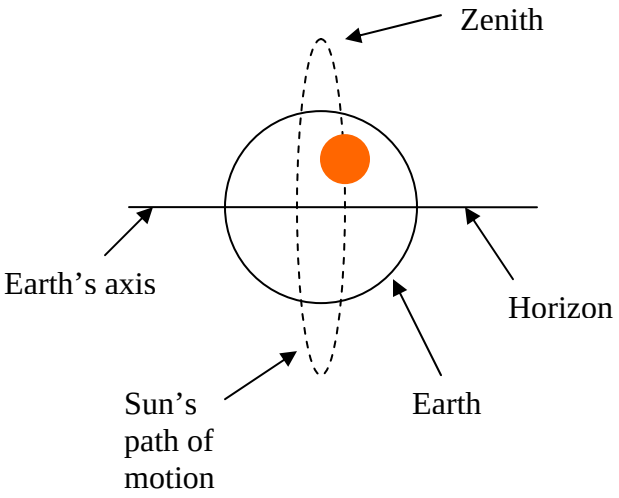
stick. If we combine the shadow paths formed on the 3 days of observations, the result will be as follows:



Shadow paths traced by the tip of the vertical stick (gnomon of vertical sundial) in Singapore (near the equator), for the 3 days of observations.

Depending on the declination of the Sun, there will be a different shadow path on other days of the year. During August, the sun will rise in the east goes to the north and set in the West. This can be seen in the diagram above. The sun moves above the east-west thus gives rise to the downward curved shadow. At October, the sun travel south of the east west plane and cast the shadow that looks like an upward curve.

At the equator, the sun will pass through zenith on the day of the equinox. This can be illustrated by the diagram on the right. During equinox, the apparent daily motion of the sun is on the celestial equator. For an observer on the equator, the celestial equator is perpendicular to the horizon as shown on the right. It can be seen that the zenith position coincides with the path of the sun at noon.



Sundial with vertical gnomon is not as useful as compared to others. It can be seen that we need to know the date before we can know which line to read to get the time. This is different from the sundial with pyramid gnomon where it can be used throughout the year.

Limitations:

It is possible to have experimental error in our observations and measurements as we are not located directly on equator (Singapore is located on latitude 1º North). There may also be human error due to instability and a possibility of parallax error. Therefore, the readings we obtained will have ± 1° of error. Our ring may not be aligned directly in the East/West plane and our markings may not be accurate. By keeping our hands steady and taking multiple readings, we tried to reduce the amount of error. Another assumption is that the magnetic declination is negligible in Singapore, which may lead to lower validity of the results.

The shadow of a vertical sundial, the shadow of a ring and the setting position of the sun can be quite useful to enable us to determine the day of the equinox, but not a convenient one. There has to be sufficient sunlight for the above 3 observations and recordings to be made, as seen from the difficulties we have encountered on cloudy or rainy days.

Another consideration is the location where the observations are made. Determining the day of the equinox with a straight shadow line is only applicable in locations at or near the equator. At other latitudes the shadow would be a curve during equinox. A simple way of knowing if it is equinox is to consider the azimuth of the sun and the observer's location. If the observer is at latitude 20º in the Northern hemisphere, then on the day of the equinox, the sun will rise at 20º south from east. (i.e. azimuth 110º)As for people living in the Southern hemisphere of 20º, the sun would rise at 20º north from east (i.e. azimuth 70º)

References:

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http://freepages.pavilion.net/users/aghelyar/sundat.htm