

GEM 1518K :
Mathematics in Art and Architecture
Semester II 2002/2003

Omoshiroi

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Origami

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Project Group 01 :

Ong Shin Ring

U021387J

Tay Hwee Ling

U020939W

Thng Xue Qi, Letitia
Toh Yi Ling

U021385C
U021413M

List of Contents

• Introduction and History	Pg 1
• 1) Whirl Cube	Pg 2
• 2) Ray Cube	Pg 3
• 3) Sonobe	Pg 4
• Observations of the 3 models	Pg 5
• 4) Stelleted Icosahedron	Pg 6
• 5) Cube ⁴	Pg 7
• 6) WXYZ-model	Pg 8 - 9
• 7) Cuboctahedron	Pg 10
• 8) Square Pyramid	Pg 11
• Practical Uses of Origami	Pg 12 - 13
• List of References	Pg 14

Introduction

Origami is the art of paper-folding into works of sculpture preferably without the aid of scissors or glue. The methods of folding paper into pretty forms and images made origami a highly creative art. Nowadays with the modern advancements, origami reveals a rich geometric structure that give us the infinite possibilities of exploring paper-folding.

Traditional origami is done without cutting or pasting. However, in the recent creative origami, scissors and even paste are used to help form more complicated designs. At a lower level, origami is just regarded as a hobby that can bring enjoyment. At a higher level, it actually involves a series of mathematical concepts like origami geometry and symmetry. From the animals to polygons, origami displays an immeasurable creation of art.

The beauty of origami lies in the result of a totally attractive piece of mathematical artwork which is created from a simple, flat sheet of square paper by using almost entirely folds of corners, creases, and edges.

In this report, we did case studies on 8 mathematical models created by origami. It is through this process of practical hands-on experiments that we changed our opinions of origami as being child's play. And the outcome of the

respective models displays familiar mathematical concepts which will be explored in the following pages.

Background information about Origami

The name "origami" is originated from Japanese word, "ori" which means folding and "kami" which means paper. When combined together, it formed the word origami.

The history of origami started both in the east and west. The earliest form of paper-folding is in China where people use paper that is covered with foil, to make paper house or coins to be burned in funeral rites. The origami is known to the Japanese only later in the seventh century. At that time, origami is not any kind of art that is affordable to the people, due to the fact that paper is not easily available and thus only the rich has the ability to possess it. To the Japanese, whether a completed origami is considered a success or not depends a lot on the maker's eye for its form, structure, and proportion.

In the East, paper folding is one of the key to creativity. It is widely practiced by the schools on the children to help develop their motor skills. It not only challenges the mind to create, it also helps in memorizing sequences in the process of folding for children.

1)



Name of model: Whirl Cube

Pattern: Rosette pattern; C_4

Details:

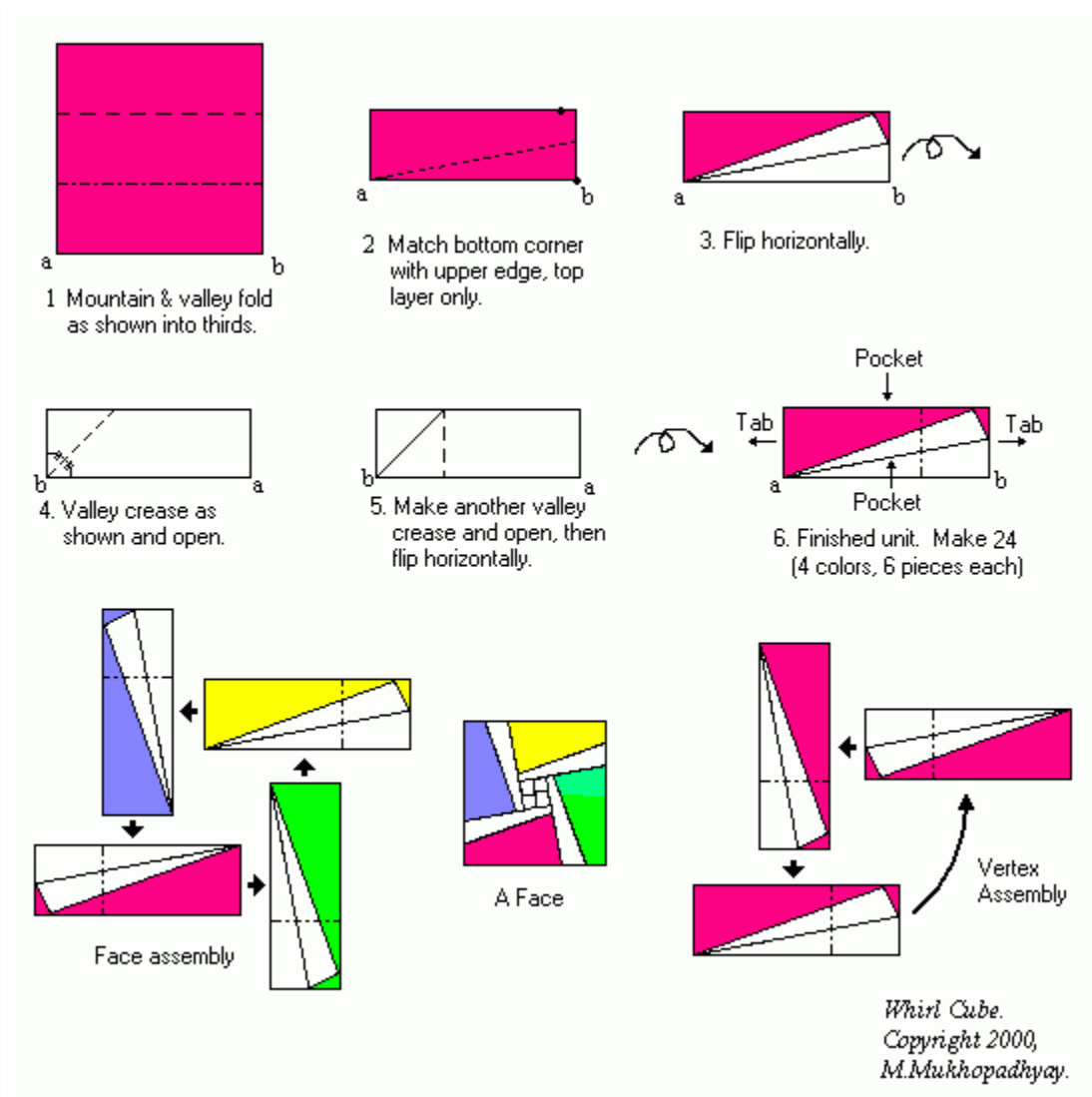
No. of faces: 6

Shape of faces: Square

No. of vertices: 8

No. of edges: 12

No. of faces at each vertex: 3



2)



Name: Ray Cube

Pattern: Rosette pattern; C_4

Details:

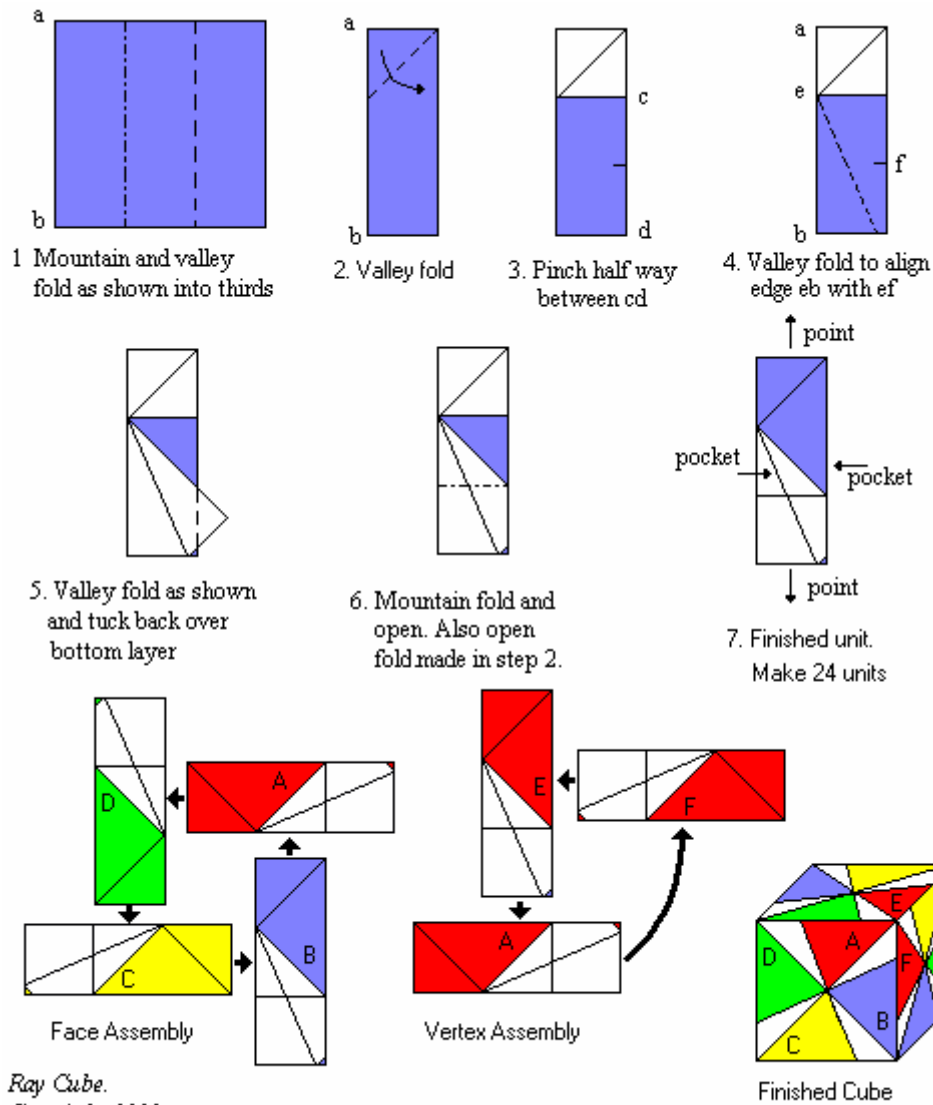
No. of faces: 6

Shape of faces: Square

No. of vertices: 8

No. of edges: 12

No. of faces at each vertex: 3



3)



Name of model: Sonobe

Pattern: Rosette pattern; C_5

Details:

No. of faces: 12

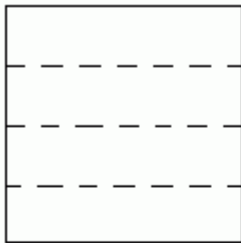
Shape of faces: Pentagon

No. of vertices: 20

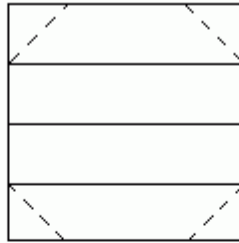
No. of edges: 30

No. of faces at each vertex: 3

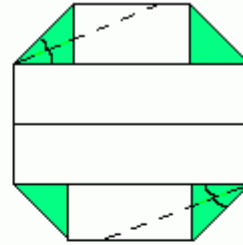
Basic Procedures:



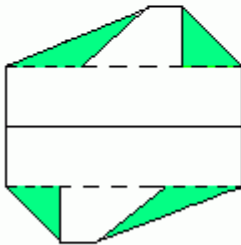
1. Valley fold with white side of paper facing you and open up.



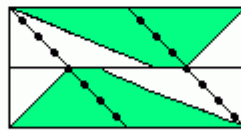
2. Valley fold corners



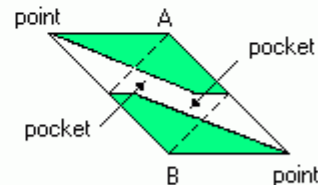
3. Valley fold to bisect the angles shown



4. Valley fold along lines shown



5. Mountain fold along beaded lines



6. Valley fold as shown. Valley or mountain fold or do not fold AB depending on kind of assembly.

© 1999 by M. Mukhopadhyay

Observations made from the Whirl cube and Ray cube

1. Both cubes are made based on the same principles of slotting each folded piece into one another to form a cube with each face that has 4-fold rotational symmetry.
2. Though all the vertices are identical (3 faces meet at a vertex), we did not manage to obtain the 3 different colours for all the vertices, in the case of

the whirl cube.

Definition of Sonobe:

A sonobe is a unit that has two points or inserting tabs, two pockets where points go in and two units can lock into each other to form a flat module. In this case, it is a stellated dodecahedron with 30 units.

Observations of the Sonobe:

The basic shape of this model is a dodecahedron. From a 3-D point of view, the base of the sonobe model is formed by five tetrahedrons that are interlocked together. These five tetrahedrons will form one pentagon.

Problems met:

To make the Sonobe model, thirty identical units must be folded. It is important to do all the folds the same way. Otherwise it will not fit with the others. It is important to note that the units are at right angles to each other. It can be quite tedious at first because the pentagons formed are interlocked. It is not a good idea to form a few pentagons and try to join them up either. Hence, it is better to start off by making a pentagon as a base and starts connecting the rest of the units together.

4)



Name: Stelleted Icosahedron

Basic Procedure:

1. Cut 6 strips of papers using 6 different colours in measurements 2cm x 24cm.
2. Using each strip, fold a witches ladder.
3. Undo the folds and using one folded strip of paper, plait the papers so that they form a nice regular shape with 5 pyramids meeting at 1 vertex.
4. Tuck in the ending point of each strip neatly. The end point always meet with the starting end!

Details:

Shape of faces: Triangle

No. of vertices: 20

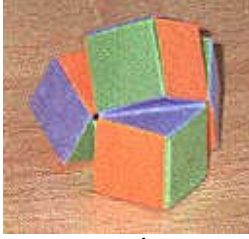
No. of edges: 90

No. of faces at each vertex: 3

Observations of this model:

1. This solid takes the basic shape of an icosahedron, but with a triangular pyramid built on top of each face of the icosahedron.
2. When all the vertices are joint up, we'll get a dodecahedron.
3. We can see that the colour combination of this solid is very interesting. If we see the solid as a dodecahedron, the 5 colours meeting at the centre of the pentagon do not repeat. The top bowl and the bottom bowl which consist of 6 pentagons each have different type of colour combination, because of the combination we get by using 6 different colours.
4. Along the equator of this solid, we can see that the colours appear in alternate, with a main colour in between the 5 different colours.

5)



Name: Cube⁴

Basic procedure:

1. Cut 3 strips of papers from 3 different colours in the measurement 4cm x 56cm.
2. Fold each strips into 14 squares, and make diagonal folds on the 3rd, 6th, 9th and 12th squares from either ends.
3. Plait the 3 strips of papers together.

Details:

Shapes of faces: Triangle and Square.

No. of vertices: 20

No. of edges: 36

Observations of this model:

1. This solid takes the basic shape of 4 cubes, but with each cube joining to 3 other cubes at 3 of the 6 faces of each cube.
2. The colour combinations is very interesting also. As we can see, at any plan view with 2 cubes place on top of one another, the centre folded square has different colour from the top and bottom faces of the cubes.
3. The top and bottom cubes showing 2 faces to me has matching colours diagonally.

6)



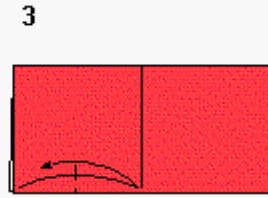
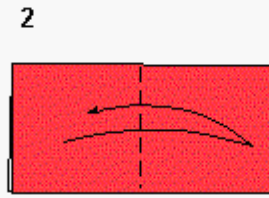
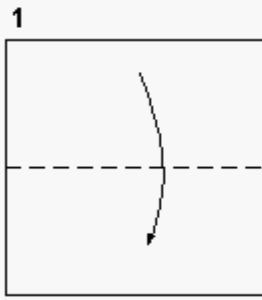
Name: WXYZ-model

Observations of this model:

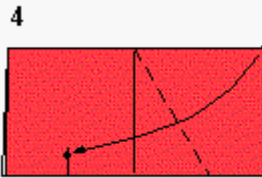
1. This model is made up of 4 differently coloured triangles intersecting together in the centre.
2. There's actually 6 points where all 4 colours intersect n face one another. The other points are where only 3 triangles face one another, and there are different combinations of 3 colours. In all, there are 14 points of intersections, and 2 of each 3-colors combination.
3. Given the folding proportions, all the triangles intersect and cut along one another at exactly $\frac{2}{3}$ of the centre of each line of symmetry.
4. There are 4 colours and therefore, given the shape as an equilateral triangle, 3 different colours cut along the 3 lines of symmetry separately.
5. The order of the colours is reversed on either side of any three-colour combination because of the intersecting at the centre.
6. (Based on top view) Along any plane on the triangles through the centre, there are 6 points of the edges outlining the shape of a hexagon, 3 by a coloured triangle, and one from one of each remaining triangles.

Basic procedure:

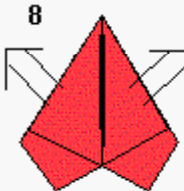
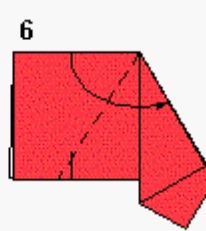
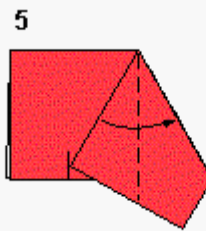
WXYZ



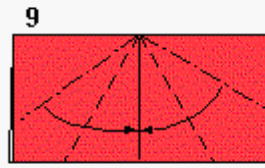
pinch



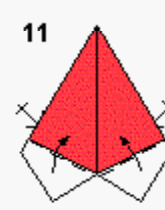
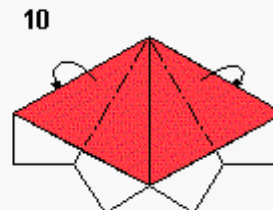
fold to pinch line



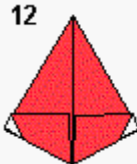
open out



squash both ends



repeat at rear

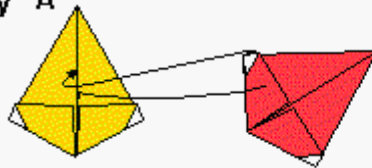


make 12 units

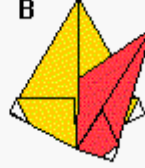
best effect is when each triangle in final model is a different colour

for this you need 3 units in each of 4 different colours

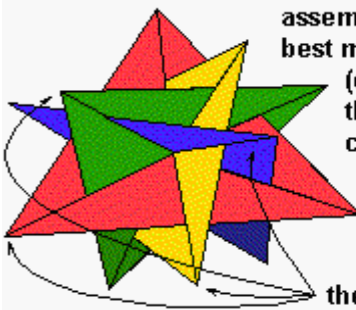
Assembly A



B

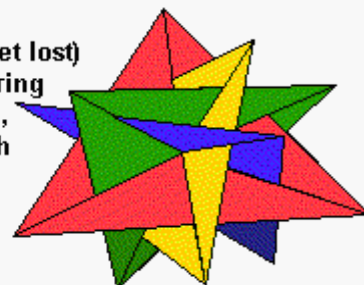


flaps in one unit go into pocket in next unit



assembly can be tricky (you can get lost)
best method is to join 4 units in a ring
(one of each of the 4 colours),
then add units to extend each
colour into a triangle

these are the initial 4 units



© model Tung Ken Lam
© diagrams D.Petty

7)



Name: Cuboctahedron

Details:

No. of faces: 14

Shape of faces: Square and Triangle

No. of vertices: 12

No. of edges: 24

No. of faces at each vertex: 4

Observations of this model:

1. This Archimedean model is made up of 6 squares and 8 triangles.
2. The length of the sides of the triangles and squares are the same.
3. Each triangle touches 3 more of its own kind at the edges and 3 of the other kind along its side. On the other hand, each square touches 4 triangles along each side.
4. From top view, the outline of the model either looks like a cube (looking from above the square) or a hexagon (from above the triangle).
5. The centre of the model is the hexagon.
6. Each shapes faces the same shape on the opposite side of the cuboctahedron, but the triangle will never face the same angle as the opposite one (one will be pointing upwards while the other, downwards).
7. The basic shape of the unit from the model actually sees a lot of symmetry in the folding process. E.g., the use of an "X" as the basic shape, of the use of 4-fold symmetry repeatedly as well as the occurrence of triangles and squares during the folding procedure.

8)



Name: Square Pyramid

(Note that in this case, the picture above does not represent the model we did, however, the same principles apply)

Observations of this model:

1. This model is made up of 2 pyramids with a common square base.
2. At any vertex, there is the "X" shape because 4 triangles meet at the point.
3. At the point of rest on any surface, it is one of the triangles at rest as the base.
4. It may look like triangles are either touching base to base (with the bottom pyramid) or side to side (in the respective pyramid), but because the sides are all equal in length, there is actually no definite "top" and "bottom" pyramid as the top and bottom are interchangeable with changes of the axis.
5. The centre of the model is a square; there are only 4 vertexes of the model along any plane.
6. The opposite faces of the model are parallel to each other.

Basic Procedures (for both Cuboctahedron and Square Pyramid):

1. Refer to <http://hverrill.net/origami/quilts/quilt1-0.html> for the making of the basic unit.
 - Basic Unit of both models:



2. The completed look of the 2 models are done by ourselves based on the basic unit.

Uses of Origami

Origami has been appreciated as a beautiful form of art since ancient times. Folding is only a part of this interesting art form. Origami folders have the opportunity to make creative designs and figures. Besides that, people have found many ways to use origami in several areas over the years. For instance, teachers have discovered that it is a wonderful teaching tool with many educational benefits. In addition, paper folding is being used in physical therapy, in mental health programmes, commercial use and as a source of entertainment and enjoyment. Origami has become a significant activity that has many advantages.

Schools have begun to realise the educational value of origami. It is a great hands-on activity and a wonderful resource that teaches students how to follow directions, encourages cooperation among students, improves motor skills, and it helps develop multi-cultural awareness. Math teachers have found that they can use origami to teach geometry, fractions, and problem solving. Language teachers also make use of origami to introduce chapters in literature, poetry, and creative writing. Science and social studies teachers use origami to introduce lessons as well. Hence, origami has become a very useful teaching tool in education.

Many physical therapists use origami as a fun and relaxing way to exercise hand muscles of the patients who are trying to rehabilitate. People who suffer from arthritis or other physical problems also benefit from folding origami as their hand muscles are strengthened. Psychologists use origami in their medical field too. They have found that the coordinated work of both hands when folding origami helps with the development of motor skills,

attention, memory, and imagination processes. The folding of origami can also be helpful to some people because it can act as a tool to relieve stress.

Origami can be applied to commercial use. Robert Lang, an engineer at JDS Uniphase, in Santa Clara, California, is one of the leading origami designers in the world. He uses engineering principles in designing origami. He has developed a computer algorithm, called Tree Maker, for the design of complicated origami objects, and such as multi legged insects or an antlered moose. During talks by Rainer Hoffmann of EASi Engineering Alzenau, Germany, and Tomoko Fuse, an origami designer in Nagano, Japan, more applications of origami are introduced. One example will be the automotive industry. EASi does computer-assisted engineering for the industry. One of its specialties is software for simulating the deployment of airbags. Although basic origami is not directly applicable, origami design techniques have been used in complex three-dimensional models for the folding patterns of airbags, which need to fit compactly in steering wheels, dashboards, and the curved roof structure of cars.

Origami is one good source of entertainment. People who enjoy origami find it fun and fascinating. Some of them like to share ideas with other people who enjoy folding origami models. There are several origami clubs and organisations where members share ideas and interest. There are also large origami conventions where people from all over the world attend.

Origami is not just about folding papers. There is a wide range of origami applications that are useful to the society. Some of the main ones are discussed in the above. This goes to show that origami does have its contributions in many areas.

The Heart As Origami by Jane Hirschfield

*Each one has its shape. For love, two sleeping ducks.
For selfless courage, the war horse.
For fear of death, the daylily's one-day flower.
More and more creased each year, worn paper thin,
and still it longs for them all.
Not one of the lives of this world the heart does not choose*

List of References:

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- <http://www.mizushobai.freemove.co.uk/patterns.htm>
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- <http://www.math.ksu.edu/main/events/special/openhouse/oh01origami.htm>
- Origami Mathematics Page
- <http://web.merrimack.edu/hullt/OrigamiMath.html>
- Origami Tessellations
- <http://hverrill.net/pages~helena/origami/tessellations/>
- Orihouse Knotology, the art of creating spheres from strips of paper
- <http://home.wanadoo.nl/gerard.paula/origami/knotology.html>
- Fancy Folding
- <http://home.att.net/~mathtrek/muse0301.htm>
- A Fun Guide to Origami
- <http://www.abc.lv/thinkquest/tqjunior-entries/5402/index.html>

- In The Fold: Origami Meets Mathematics
- www.siam.org/siamnews/10-01/origami.pdf
<<http://www.siam.org/siamnews/10-01/origami.pdf>>