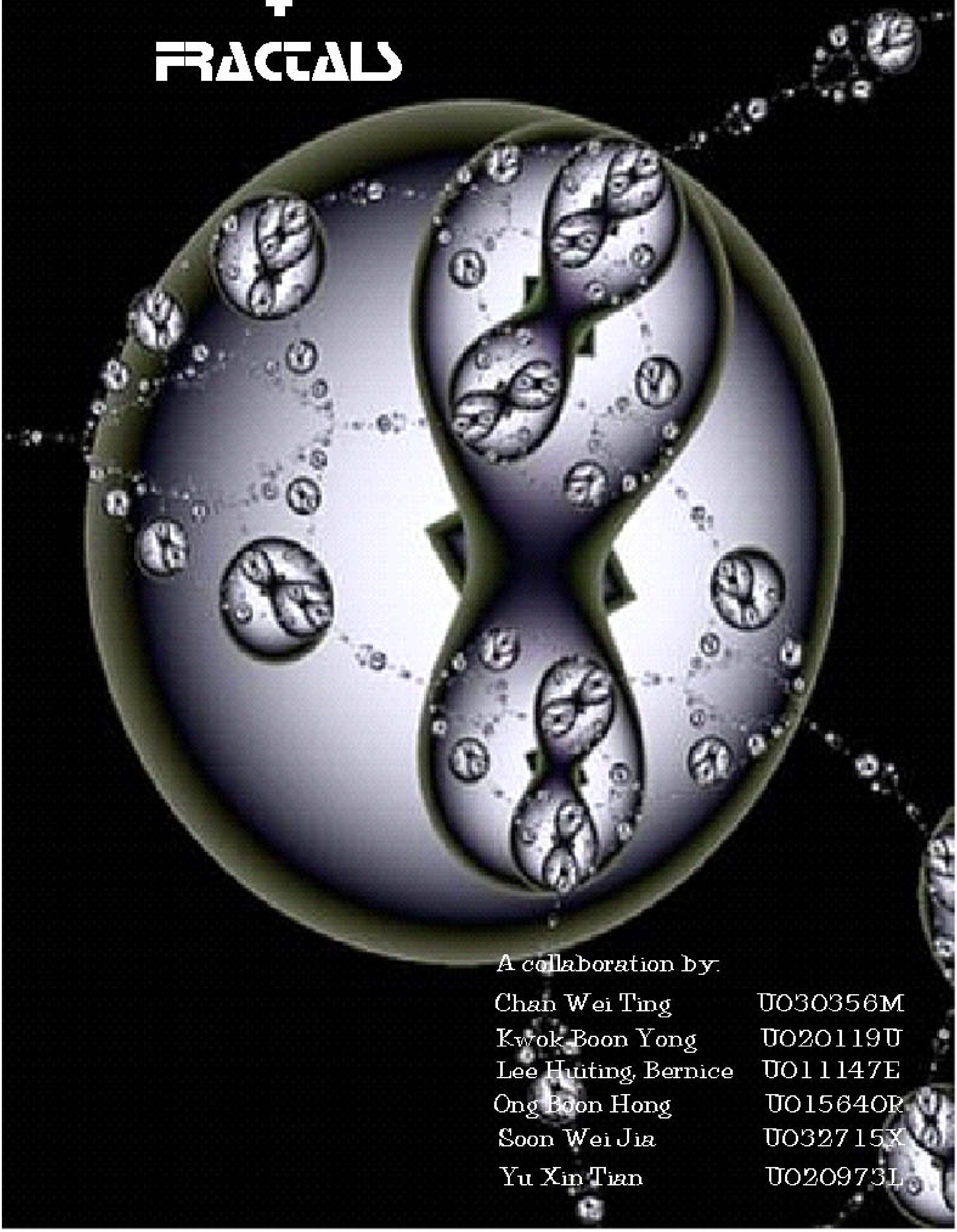
MOIRE PATTERNS
+
FRACTALS



A collaboration by:

Chan Wei Ting U030356M
Kwok Boon Yong U020119U
Lee Huiting, Bernice U011147E
Ong Boon Hong U015640R
Soon Wei Jia U032715X
Yu Xin Tian U020973L

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Introduction

Things in life hold many interpretations for different people across the globe, but some things transcend geographical and cultural boundaries. From the times of yore to the current techno-age of globalisation, such things like Arts and Mathematics have delimited the knowledge of people. These two entities seem diverse and separate; however, historic figures like Leonardo da Vinci had been able to stride the two shores, reconciling their differences and uniting their similarities.

It is the purpose of this project to attempt a similar feat, albeit with lesser expertise. We have chosen to uncover the fractal and moiré occurrences inevitably present in our daily life. These two incidents are seemingly unrelated as much as Arts and Mathematics are, but we will be testing if there is any relation between the two. In achieving our purpose, we will explain these phenomena using simple, layman terms.

> " Who has not noticed, on one occasion or another, those intriguing geometric patterns which appear at the intersection of repetitive structures such as two far picket fences on a hill, the railings on both sides of a bridge, superposed layers of fabric, or folds of a nylon curtain…the moiré effect, has found useful applications in several fields of science and technology, such as metrology, strain analysis or even document authentication and anti-counterfeiting."
>
> - Amidror [1]

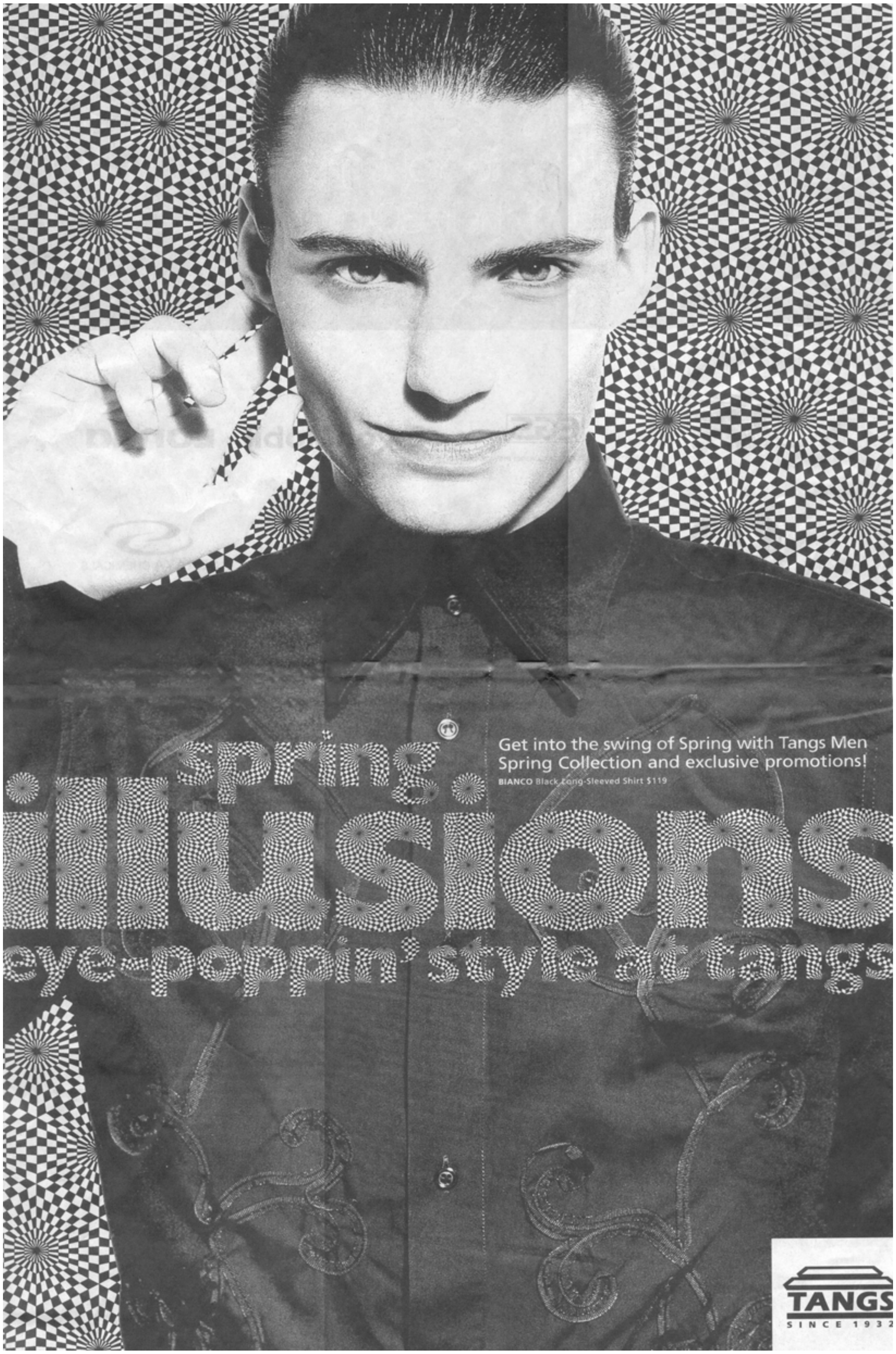
According to Amidror, the name of such an effect originated from the French and it does not refer to a French physicist who studied it (and hence should not be written with a capital letter), but a certain type of material. Specifically, this material is watered silk obtained by pressing two layers together. This glossy cloth has wavy patterns which will change as the wearer moves. Moiré patterns are like illusionary movements.

Fractals, on the other hand, are "growing structures". They regenerate layers after layers and eventually reproducing patterns that are akin to kaleidoscopic animation. Computer modelling of irregular patterns also uses fractal effect as a tool. When such patterns "grow", the movement may be outwardly similar to the moiré effect. Both are also hypnotically effective to some people!

Under separate sections, we will be reviewing the concepts (mathematical and artistic) of these two phenomena and how they work, their applications and examples in real life, and lastly a personalised moiré kit.

To affiliate, we must first unveil their forms…

[1] From Amidror's "Preface" in *The Theroy of the Moire Phenomenon*.



Above: A recent CK Tang advertisement, an example of moiré patterns in use in daily life.

Moiré patterns

Brief Description

When two similar, repetitive, grid-like patterns of lines, circles, or arrays of dots overlap with imperfect alignment on a screen, a pattern of light and dark lines appears. This is called a moiré pattern. In some cases, moiré patterns can also be created when two or more different geometrically regular patterns are superimposed. It can be composed of multiple colours and be either continually changing or still. It is, however not a physical pattern etched in the original structures but rather an optical illusion created in the vision of the viewer.

The essential quality of a moiré pattern is that it is a new pattern that emerges from two existing ones. Often, this pattern seems to resonate or create a visual depth not seen in the two original patterns individually. When the two patterns are moved relative to each other, the moiré pattern changes shape.

This is a result of the geometric distribution of dark and bright areas in the superposition: areas where dark elements of the original structures fall on top of each other appear brighter than areas in which dark elements fall between each other and fill the spaces better.

A classic moiré pattern is composed of two sets of parallel lines that are at a slight angle. (pattern B and transparency 7 in Moiré Patterns Kit).

Origins

> Forms: 16 **moyre**, 17- **moire**. < French *moire* (1639) < English MOHAIR *n*. Cf. (< French) Spanish *muer* (1734), *muaré*, Italian *moerro* (*a*1777 as *muerro*), German †*Mohr*.
>
> Definitions:
>
> - A type of fabric (originally mohair, now usually silk) that has been subjected to heat and pressure rollers after weaving to give it a rippled appearance.
>
> - Designating a wavy or geometrical pattern of light and dark fringes (stripes) observed when one pattern of lines, dots, etc., is visually superimposed on another similar pattern, or on an identical one that is slightly out of alignment with the first. Chiefly in ***moiré fringe***, ***pattern***.
>
> (Extracted from *Oxford English Dictionary Online* at http://dictionary.oed.com/)

Initially used by weavers, the word "moiré" comes from the word *mohair*, a kind of cloth made from the fine hair of an Angora goat. Later on, it underwent a process of generalisation and its meaning widened to encompass that of the shimmering quality of the French watered silk. As can be seen from the last paragraph of the citation above, it now refers also to our topic of discussion—patterns.

How It Works

When two similar line patterns overlap, dark areas are created where the black lines on the front pattern obscure the clear gaps on the rear pattern. Where the black lines on the front pattern align

with black lines on the rear, the neighbouring areas are clear and this creates a light region. It is these light and dark regions which form the moiré pattern.

We can see how the light and dark regions form by looking at two overlapping patterns of concentric circles (pattern A and transparency 1 in Moiré Patterns Kit). The resulting moiré pattern consists of radiating dark and light lines. From this example, we can see that the dark regions are like the nodal lines of a two-source interference pattern, that is, a moiré pattern which radiates from two sources. Along such nodal lines, no light is detected by the eye because the peaks of the light waves from the clear sections of one pattern and the valleys of the light waves coming from the dark sections of the other pattern overlap and cancel each other. The differential amounts of light hitting the eye creates the illusive dark and light regions thus resulting in a moiré pattern.

Moiré patterns are magnifications of the differences between two repetitive patterns. Thus, when two patterns are aligned accordingly, no moiré pattern appears. However, the slightest misalignment of the two patterns creates a large-scale, easily visible moiré pattern. As the misalignment increases, the lines of the moiré pattern appear thinner and closer together. These can be observed from the many examples in the Moiré Patterns Kit included as part of the project.

Do note that not all moiré works, even if the base and top patterns meet the requirements. The angle of incidence must not be too wide, and the patterns should not be too diverse. For example, concentric circular patterns do not work well with radial lines.

Math in moiré patterns

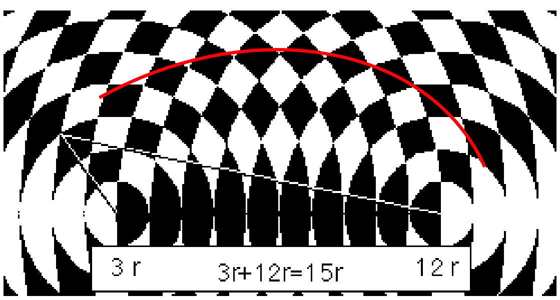


Diagram. 1.

Take for example Diagram 1, a close-up of two identical overlapping concentric circular patterns. The radius of the smallest circle in each pattern is *r*. From the diagram, we can see that the 3rd concentric circle from the centre of the left pattern intersects with the 12th concentric circle radiating from the centre of the pattern on the right. One of the bands of the moiré pattern that results moves through the intersections of the 3rd and 12th circles, 4th and 11th circles, 5th and 10th circles and so on. The moiré pattern made up of these bands is an ellipse with a radius of 15r. This radius is constant and is the sum of the radii of the intersecting circles.

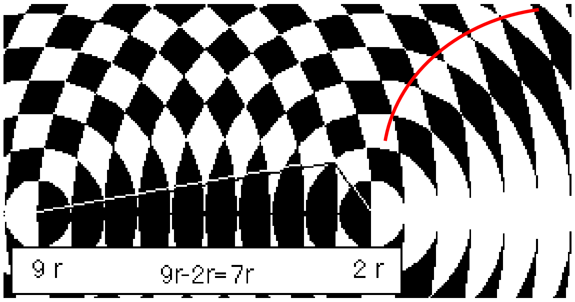


Diagram 2

Diagram 2 is of the same concentric circular patterns as in Diagram 1. However, in this diagram, we look at the other path through which the bands of dark and light regions of the moiré pattern moves. This path is a hyperbolic one which goes through the intersection between the 9th circle from the centre of the left pattern and the 2nd circle from the centre of the right pattern. The radius of this movement is constantly 7r even as the hyperbolic path moves from the intersection of the 9th and 2nd circle to that of the 10th and 3rd, 11th and 4th, 12th and 5th circles and so on. The radius, as indicated in the diagram, is the difference between the radii of the intersecting circles.

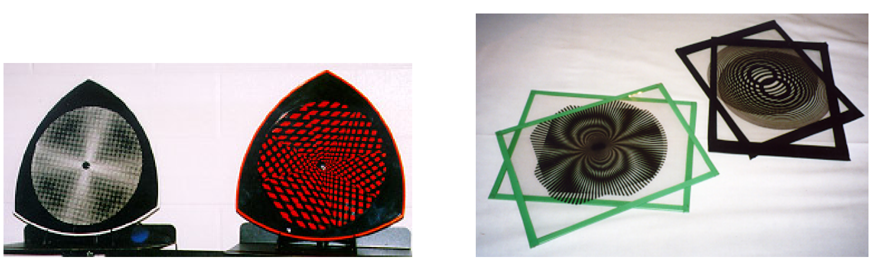
D.I.Y: Ways to create your own moiré patterns

a) You can experience moiré patterns using just one wire-mesh screen. Have a friend hold a sheet of white cardboard behind the screen, and shine a single bright light onto the screen. (On a sunny day, sunshine can serve as your light source.) Start with the cardboard touching the screen; then move it away, tilting the cardboard a little as you go. The screen will form a moiré pattern with its own shadow. Replace the cardboard with flexible white paper and bend the paper. Notice how the moiré pattern changes.

b) Use a photocopy machine to make two transparencies with similar, repetitive, grid-like patterns of lines, circles, or arrays of dots. Look through these two patterns as you move them apart and then together or as you rotate or shift the top pattern over the bottom pattern. The changing pattern of dark and light regions is the moiré pattern.

c) Go to http://eluzions.com/Illusions/Moire/pattern.shtml and use the idiot-proof moiré pattern generator on the web page.

d) The simplest way of all is to use the Moiré Patterns Kit included as part of this project.



Above: Examples of some applications of moiré patterns in art.

Moiré patterns in daily life

After understanding what a moiré pattern is, one just has to be more observant and find that such a pattern can be found in our everyday life. Look through a thin, finely woven fabric, such as a white handkerchief or some pantyhose material. Now fold the fabric over and look again through two layers. You will see moiré patterns. By sliding the fabric around, you can see the patterns dance and change (this is especially visible with *chiffon* fabric). While walking or driving past two chain-link or picket fences, moiré patterns unfold before your eyes too. You can also see similar patterns by holding two identical combs such that one is directly in front of the other and they are about a finger-width apart. Look through the teeth and you will notice the distinct moiré pattern of light and dark regions appearing. Slide the combs sideways to watch the moiré pattern move. By rotating one comb relative to another, the pattern changes in a different way.

One common occurrence of moiré patterns is found in computer monitors and television sets. The pattern exists as an ordered wavy pattern superimposed over the screen in a series of ripples, waves, and wisps of intensity variations. An explanation for the occurrence of a moiré pattern is as such: The pixels, which are generated by the video board, cannot be perfectly aligned with the phosphor dots or stripes on the screen. For some pixels, the CRT beam hits the screen phosphors directly at the centre and this produces a bright pixel (light region). As for other pixels, the beam hits off-center and this produces a dimmer pixel (dark region). The moiré pattern is actually a map of their alignment over the screen.

Such an occurrence of moiré patterns in computer monitors and television sets does not represent a defect in the monitor but it is a result from a practical limitation in display technology. In order to completely eliminate Moiré patterns, the dot or stripe pitch on the monitor would have to be significantly smaller than the size of a pixel, which is generally not possible.

These are a few ways how we can minimize moiré patterns in monitors and television sets:

1) Change the image size or shift resolutions. Moiré patterns are most noticeable when the ripples are separated by a few millimeters. By increasing or reducing the image size or the pixel size, you can literally adjust the intensity of the moiré pattern thus making it less noticeable.

2) Do not use a dim window background. The dimmer the Window Background color, the more noticeable the moiré patterns will be throughout the entire window background because the beam size is smaller at low intensities.

Another occurrence of moiré patterns often overlooked is in printing and scanning half-tone images. An example would be the scanned Tangs advertisement on page 3 of this report. If you zoom into the bottom of the picture of the man, you can see moiré patterns. Halftone printers use layers of colored dots to create an image and sometimes when the dots do not line up properly, a moiré pattern is created. Dots must be printed at a particular angle to avoid this. When a scanner samples a halftone image, interference may occur between the pattern of dots in the image and the pattern of the sampling, creating a moiré pattern. To reduce this problem, scanning and imaging software have installed a "defringe" option.

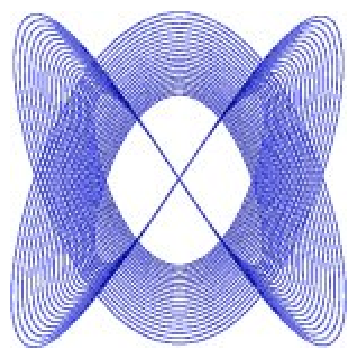
Applications of Moiré patterns

Moiré patterns are used by scientists in detecting motion and surface abnormalities such as stress in metals, the flatness of a surface, or air flow over the wing of an airplane. An example is Projection Moiré Interferometry (PMI). It is an instrument which used in the aerospace industry to acquire model deformation measurements in large wind tunnels in a highly accurate and efficient way. Moiré fringe patterns (as shown in the picture below) are obtained by using the instrument and the patterns are processed to obtain a quantitative, spatially continuous representation of the model surface shape or deformation.

These patterns are also used in illustrating the orbits of planets, microwave and holography activity. Mathematicians use them in studying properties of ellipses, parabolas and hyperbolas. Moiré patterns have also been used to measure the topography of three-dimensional objects.

Moiré in Design/ Art

Besides having several useful applications, moiré patterns are linked to design and have been pleasant to people’s eyes for many years. The Japanese and other Asian people have created fabrics and baskets with it for centuries. More recently, the French became known for their moiré textiles, particularly scarves. Ribbons, dresses, jackets, and many other items used by people all round the world are made with moiré patterns. Below is an example of Victorian style moiré pattern wallpaper. Moiré patterns were quite popular in Victorian times.



Left: Victorian wallpaper.
Right: An example of one of Vasarely’s works.

Influenced by the graphic arts and science's evolving understand of optics and visual perception, optical or op artists such as Victor Vasarely often used moiré patterns and optical illusions in the early sixties. By experimenting with color and line, op artists made vibrant shapes that seem to pulsate. In this case, Vasarely used moiré patterns to create depth and texture.

Fractals

Introduction




"Philosophy is written in this grand book - I mean universe - which stands continuously open to our gaze, but which cannot be understood unless one first learns to comprehend the language in which it is written. It is written in the language of mathematics, and its characters are triangles, circles and other geometric figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering about in a dark labyrinth."

- Galileo (1623)

Imagine a picture of the coastline of Singapore.

Measure the distance of the coastline with a kilometre-long ruler.

What if it was measured with metre-long rulers? Which measurement would give a larger measurement?

Since the coastline is jagged, it would be easier to get into the nooks and crannies with the metre-long ruler as compared to the longer kilometre-long ruler, so it would yield a greater and more accurate measurement. Now what if a centimetre-long ruler was used instead? It would be possible to measure the teeniest and tiniest of crannies there. So the measurement would be even bigger!

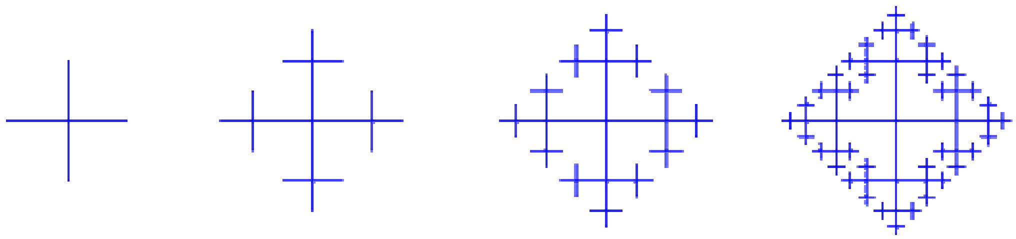
What if at every point on the coastline was jagged? Measure it with shorter and shorter rulers, and the measurement would get longer and longer, and more and more accurate. Measure it with infinitesimally short rulers, and the coastline would be infinitely long.

That's fractals!

What is a fractal?

"To define a thing is to substitute the definition for the thing itself."

-Georges Braque (1875 - 1940)



Benoit Mandelbrot defined a fractal as:

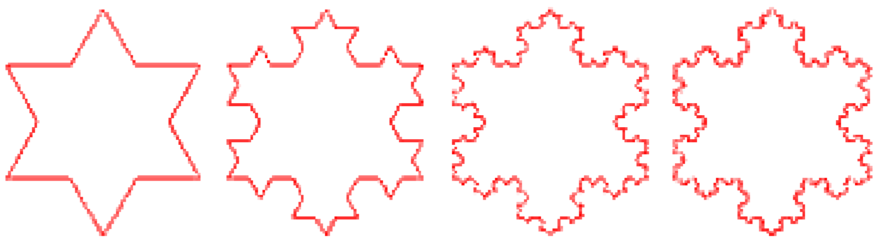
A rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced/size copy of the whole.

He coined the name "fractal" in 1975 from the Latin *fractus* or "to break".

The basic concept of fractals is that a large extent of *self similarity* is enclosed within. This means that they usually contain little copies of themselves buried deep within the original. And they also have infinite detail.

Self-similarity: Each minute fraction, when magnified, can replicate a larger portion exactly. Every magnified section is identical to any other magnified section.

- is *self-similar* and
- has *fractional (fractal) dimension*.
- formed by *iteration*



How it came about

"Every contrivance of man, every tool, every instrument, every utensil, every article designed for use, of each and every kind, evolved from very simple beginnings."

- Robert Collier

The development of fractals can generally be split into 2 broad eras, before and after the invention of computers.

Pre-computer Fractals:

Fractals have come up as an important question two times before the invention of computers. The first time was when British cartographers discovered a problem with measuring the length of the coastline of Britain. Measuring the coast on more enlarged and detailed maps, a lengthier measurement was obtained. Measurements by examining the most comprehensive maps obtained a dimension that was over double the original. The closer they examined the longer the coastline became. Little did they know that this is a property of fractals i.e. a finite area being bounded by an infinite line

The second instance of pre-computer fractals was noted by the French mathematician Gaston Julia who tried to visualise a complex polynomial function. In 1918 Julia published a beautiful paper *Mémoire sur l'itération des fonctions rationnelles, Journal de Math. Pure et Appl.* **8** (1918), 47-245, concerning the iteration of a rational function *f*. Julia gave a precise description of the set J(f) of those z in C for which the *n*th iterate $f^n(z)$ stays bounded as *n* tends to infinity. It received the Grand Prix de l'Académie des sciences.

Above: Gaston Julia

Using a function in the form of $Z^2 + c$ (where c is a complex constant with real and imaginary numbers), he performed iterations by taking the x and y coordinates of a point and substituting them

into Z in the form of x + yi (where i is the square root of negative one). The resulting pair of real and imaginary numbers were substituted back into Z and this was executed repeatedly for numerous recursions.

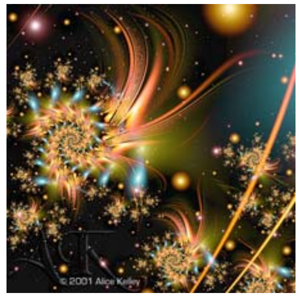
Seminars were organised in Berlin in 1925 to study his work and participants included Brauer, Hopf and Reidemeister. H Cremer produced an essay on his work which included the first visualisation of a Julia set.

Although he was famous in the 1920s, his work was essentially forgotten until B Mandelbrot brought it back to prominence in the 1970s through his fundamental computer experiments.

Post-computer Fractals

Above: Benoit Mandelbrot

Benoit Mandelbrot, an IBM employee came up with the idea of writing a program using a formula such as $Z*n^2 + c$ (where Z is a complex number and c is a complex constant with real and imaginary numbers) and running it on a computer. Mandelbrot was the first person to utilise computers to perform the countless repetitive calculations to make a fractal.

How it works

Fractals are often formed by performing repetitive iterations.

Making a fractal:

1. Take any familiar geometric figure (a triangle or line segment, for example) and modify it so that the new figure is more complicated or sophisticated in a unique way.
2. Then in a similar fashion, modify the resulting figure, and get an even more complex figure.
3. Now modify the resulting figure using the same procedure and get an even more intricate figure.
4. Repeat the above for any number of iterations desired and obtain a fractal.

Interesting things to note

Not every iterative process produces a fractal. Take a line segment and chop off the end. What is the resulting figure? Just another line segment— hardly an intricate pattern at all, and definitely not a fractal. It would be possible to continue the iterative process over and over, chopping off the end of the line segment, but it would serve no purpose other than to produce a shorter and shorter line segment.

Below is a picture of a similar iterative operation that *is* fractal. Take a line segment (see below) and remove the middle third. The resulting figure increases in its complexity. At the earliest drawings of the fractal curve, few clues to the underlying mathematical structure will be seen. Repeat the above by removing the middle third of resulting line segments. With subsequent drawings of the fractal cure, sudden changes may appear and details emerge more clearly as the fractal is redrawn.

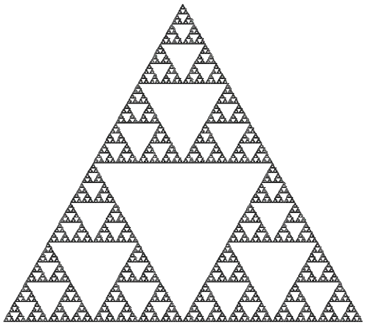
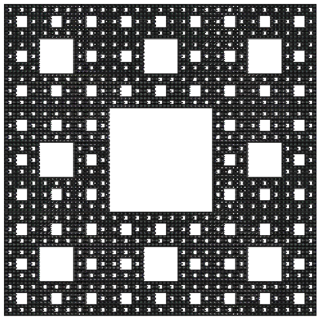
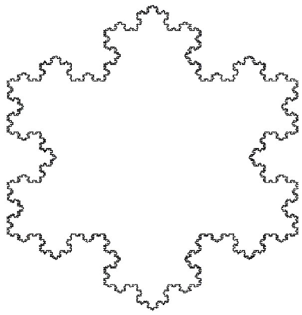
The result is a fractal called Cantor dust, named after the mathematician Georg Cantor, and is the two-dimensional version of the Cantor set.

Cantor Dust Types of Fractals

Currently three broad categories of fractals are commonly studied:

- Iterated function systems.

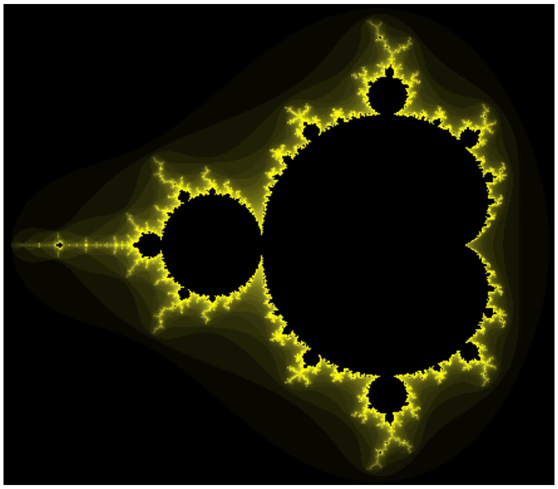
  These have a fixed geometric replacement rule (Cantor set, Sierpinski carpet, Sierpinski gasket, Peano curve, Koch snowflake).

Sierpinsk's Triangle ***Sierpinski's Carpet*** ***Koch Snowflake***

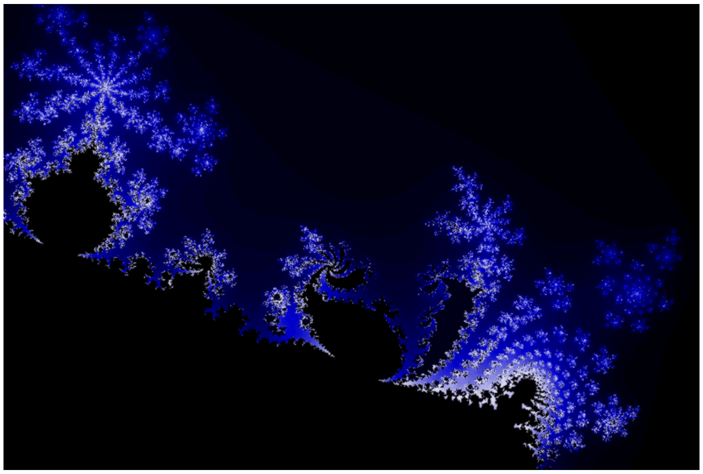
- Recurrence Relations

  Fractals defined by a recurrence relation at each point in a space such as the complex plane. Recursion is a way of specifying a process by means of itself. More precisely, "complicated" instances of the process are defined in terms of "simpler" instances, and the "simplest" instances are given explicitly. Functions whose domains can be recursively defined can be given recursive definitions patterned after the recursive definition of their domain. These fractals are also called *escape-time fractals* (Mandelbrot set and the Lyapunov fractal).



Mandelbrot

- Random fractals, generated by stochastic rather than deterministic processes (Fractal landscapes , Lévy flights).



Fractal Landscape - Coastline

Of all of these, only Iterated function systems usually display the well-known "self-similarity" property--meaning that their complexity is invariant under scaling transforms. Fractals such as the Mandelbrot set are more loosely self-similar: they contain small copies of the entire fractal in distorted and degenerate forms.

Fractal Dimensions

One of the unique things about fractals is that they have non-integer dimensions. That means that from the 3rd dimension, looking at this on a flat screen (an approximation of the 2nd dimension), fractals are in between the dimensions. Fractals can have a dimension of 1.8, or 4.12.

Although fractals have non-integer dimensions, they always have a smaller dimension than what they are on. That means that if a fractal was drawn using lines, that fractal cannot have a dimension greater than the surface it was drawn on.

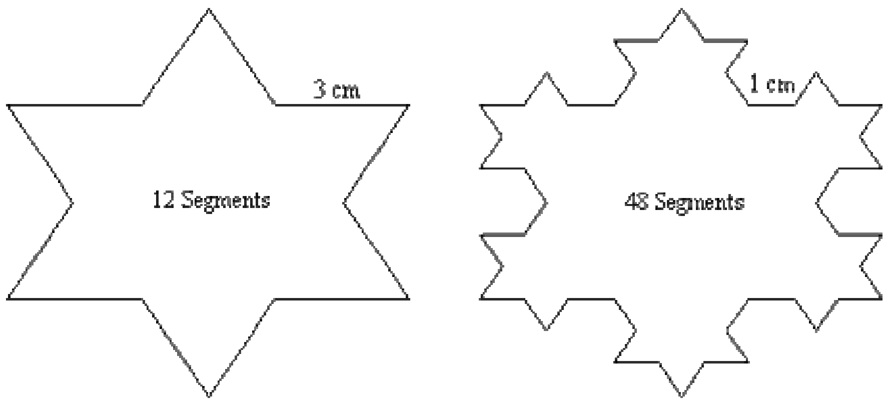
And how exactly does one calculate how many dimensions a fractal has? In math, dimension means much more than a statement of an object's shape or size. Dimension has been simplified for the public to provide a layman understanding.

Calculations can be simplified with logarithms. If a cube was used and its edge length was multiplied by 2, then 8 of the old cubes could be fitted into the new cube. Taking these two numbers, you can find that log 8 / log 2 equals three. So, a cube has a dimension of 3, which we already knew. 8 is also 2 raised to the 3rd power. Not a coincidence.
It can be assumed that for any fractal object (of size P, made up of smaller units of size p), the number of units (N) that fits into the larger object is equal to the size ratio (P/p) raised to the power of d, which is called the Hausdorff dimension.

$$N=\left(\frac{P}{p}\right)^d \quad \text{-or-} \quad d=\frac{\log N}{\log (P/p)}$$

Using Koch's Curve as an example and using only line segments that are 3 centimetres long (P), a simple Koch's Curve with 12 segments, 3 centimetres per segment can be constructed. This figure is also known as the Star of David. Taking that to the next level and using line segments which are 1

centimetre long (p), 48 line segments are formed. By cutting the length of the line segments by one third (P = 3, p = 1, P/p = 3), the number of line segments used (N) goes up four times (48 segments for p divided by 12 segments for P equals 4). That means N = 4, P/p = 3, so d = log 4 / log 3. Calculations show that Koch's Curve has a dimension of 1.2618595071429!



Fractal Geometry

Where classical geometry deals with objects of integer dimensions, fractal geometry describes non-integer dimensions. The common understanding of most people is that the world is made up of zero dimensional points, one dimensional lines and curves, two dimensional plane figures like squares and circles, and three dimensional solids such as cubes and spheres.

However, many natural phenomena are better described with a dimension part way between two whole numbers, as discovered by British cartographers. Although a straight line has a dimension of one, a fractal curve will have a dimension between one and two depending on how much space taken up. The more that flat fractal fills a plane, the closer it approaches two dimensions. Likewise, a undulating fractal landscape will reach a dimension somewhere between two and three. So a fractal landscape made up of a large hill covered with tiny bumps would be close to the second dimension, while a rough surface composed of many medium-sized hills would be close to the third dimension

Major differences between fractal and Euclidean geometry

Fractal	Euclidean
Modern invention	Traditional
No specific size/scale	Based on Characteristic size/scale
Appropriate for geometry in nature	Appropriate for man-made objects
Described by algorithms	Described by formulas

Firstly, fractals are a relatively new discovery. The field of fractals has only been formally studied in the last decade compared to Euclidean geometry which dates back over 2000 years ago.

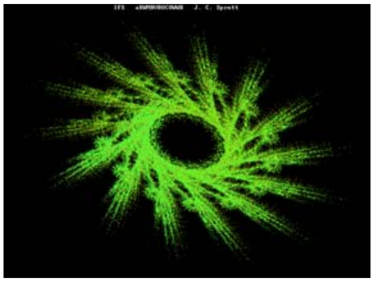
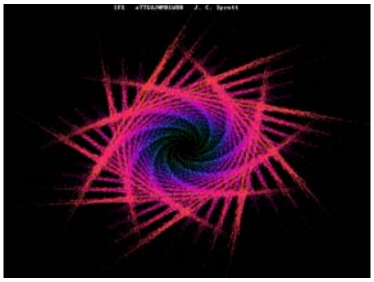
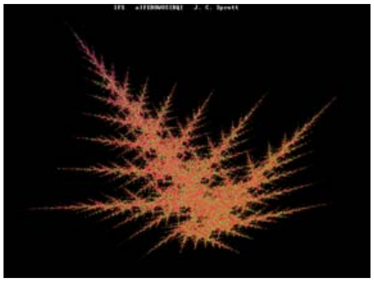
Secondly, Euclidean shapes have a few characteristic sizes or length scales (e.g. the radius of a circle or the length of a side of a cube), whereas fractals have no characteristic sizes. Fractal shapes are self-similar and independent of size or scaling. .

Third, Euclidean geometry provides a good description of man-made objects whereas fractals are required for a representation of naturally occurring geometry. It is likely that this limitation of our traditional language of shape is responsible for the striking difference between mass produced objects and natural shapes.

Finally, Euclidean geometry are defined by algebraic formulae, for example

$$x^2 + y^2 + z^2 = r^2$$

defines a sphere. Fractals normally result from an iterative or recursive construction or algorithm.

What they are used for

" Rarely do great beauty and great virtue dwell together. "

- Petrarch (1304 – 1374)

What possible use could mathematical pictures that have non-integer dimensions have? Besides looking pretty, that is.

Fractal dimensions can measure the texture and complexity of everything from coastlines to mountains to storm clouds.

Nature is full of fractals. Twigs on trees look like the branches which they grow on, which look like the tree itself. Fern leaves are another good example. Ever wondered about why art appeals to the individual’s aesthetic sense? The reason paintings are so appealing is that their fractal dimensions are similar and mimic those found in nature. That is the reason why paintings look natural, even if they're just splashes of paint on canvas.

Self-similarity can be found in every other object in the world, which is why fractals can make such outstanding reproductions of nature. Artists have created very realistic looking landscapes composed of just a few fractal equations.

"The most useful fractals involve chance ... both their regularities and their irregularities are statistical."

- Benoit B. Mandelbrot.

Fractals win prizes at graphics shows and appear on t-shirts and calendars. Their chaotic patterns appear in many branches of science. Physicists find them on their plotters. Strange attractors with fractal turbulence appear in celestial mechanics. Biologists diagnose dynamical diseases.

Random fractals have the greatest practical use because they can be used to describe many highly irregular real-world objects. Examples include clouds, mountains, turbulence, coastlines and trees. Fractal techniques have also been employed in image compression, as well as a variety of scientific disciplines.

Fractals offer an alternative method of observing and modelling complex phenomena that may be superior to Euclidean Geometry or the Calculus developed by Leibnitz and Newton. A rising cross disciplinary science of complexity coupled with the power of desktop computers brings new tools and techniques for studying real world systems and understanding the universe.

Fractals also have **technological applications**. Antennas have always been a problematic area that has forced many engineers to resort to using trial and error because of the complexity of electromagnetism. Antenna arrays consist of thousands of small antennas which are either placed randomly or regularly spaced. Fractals offer the ideal combination of uncertainty and order using fewer components. Parts of fractals have the disorder, while the fractal as a whole provides the order. By bending wires into the shape of Koch's Curve, more wire can be fit into less space, and the jagged shape also generates electrical capacitance and inductance. This eliminates the need for external components to tune the antenna or to broaden its range of frequencies by increasing the efficiency of the antenna. Motorola has started using fractal antennas in many of its cellular phones, and reports that they're 25% more efficient than the traditional piece of wire. They have lower manufacturing costs and are capable of operating on multiple bands. The journal *Fractals* has found that for an antenna to perform consistently at all frequencies it has to be symmetrical around a point and it must be self-similar, which explains the high efficiency of fractal-shaped antenna.

Fractals are being used in the control of **fluid dynamics**. Engineered fluid containing fractals are used for process intensification by controlling the scaling and distribution of fluids. Fractals allow fluid properties like eddy size or concentration distributions to be adjusted in a highly controlled manner, and can bring about benefits such as reduction of energy costs, process use and device design pressure.

Fractals are now used to store photographic quality images in a tiny fraction of the space previously required. **Fractal compression** is a lossy compression method used to compress images using fractals. A lossy data compression method is one in which a file is compressed and then decompressed upon to retrieval. The retrieved file may well be different to the original, but is "close enough" to be useful in some way. The method is best suited for photographs of natural scenes. The

fractal compression technique relies on the fact that in certain images, parts of the image resemble other parts of the same image.

It has been known that fractals can be used to resemble real-life objects. The development of fractal compression came as a result of research into applications of fractals. As more research was carried out on this field, one question arose: would it be possible to turn an image into a fractal by approximation?

With the faster computer nowadays, the goal of compressing real images into fractal images was achieved. The advantage of using fractal compression is that it is possible to enlarge fractals without getting a blocky picture because the fractal "smoothes out the details", making it much better for pictures that require enlargement. This smoothening effect is due to infinitely detailed nature of fractals.

Fractal geometry and chaos theory are also responsible for a new perspective to view the world. For centuries, the line was used as a basic building block to understand the objects around us. Chaos science uses a different geometry called fractal geometry. Fractal geometry is a new language used to describe, model and analyze complex forms found in nature.

Things that fractals can model include:

- plants
- weather
- fluid flow
- geologic activity
- planetary orbits
- human body rhythms
- animal group behavior
- socioeconomic patterns
- music

and more…

The Link between Moiré and Fractal Patterns

We started out this project trying to establish or to even find a relation between the two seemingly different phenomena. We came across many websites as well as references in books, but there is apparently no definite way to explain their relationship mathematically.

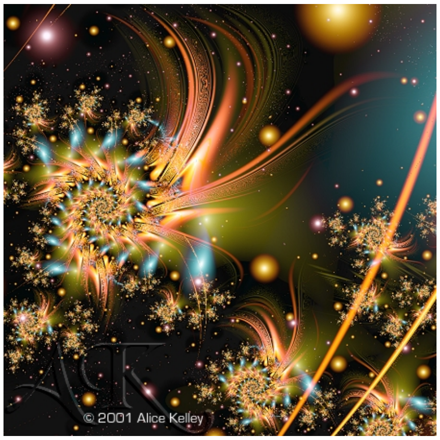
If the base layer of the moiré pattern has no angle of incidence with the top layer, there will not be any moiré effect. Based on the principle that there must be an angle of interception (incidence), or misalignments of the elements, there is none here and hence the only "boring" patterns you would expect to see white strips appearing or disappearing from sight.

Since we can pair a basic moiré pattern with any random pattern with similar elements as long as they coincide at some angle, we can pair it with fractals—a "live" pattern that is consistently mutating; perhaps up till the point where the fractals regenerate to infinity and it appears to be an array of parallel lines to the base moiré.

There are sites online that boast of the beauty of fractal-moire hybrid (see http://www.sente.co.uk/ss/gridcitymain.htm for such proof!) However, they do not include in any FAQ a section which tries to explain why and how they work. This could perhaps be a trade-secret.

Knotplot was another application we found that can actually be seen as a reconciliatory effort (http://www.pims.math.ca/knotplot/KnotPlot.html) You can randomly or draw lines in this program, and with a few clicks of the mouse, you can see the pattern repeat, move and mutate, much like how fractals and moiré might work together.

We will end this discussion on the relationship between the two effects in our Conclusion to follow.



Conclusion

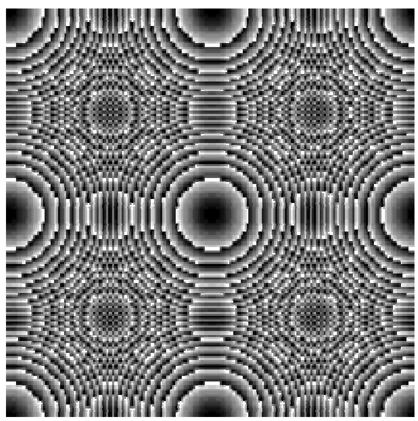
"In mathematics you don't understand things. You just get used to them. "

- Johann von Neumann (1903 – 1957)

We have tried to explain the moiré and fractal effects in simple terms and a connection between them seems to be missing, from the mathematical perspective. It could be due to our inadequacies in the deeper knowledge of these two fields that prevented us from exploring it further through the equations that mathematicians have presented on these two phenomena.

For instance, we were unable to understand one such equation that was used explain a circles-and-squares fractal moiré pattern when we searched for such a linkage online

$$Z_{n+1} = Z_n^{\,2} \mid m|$$



Above: a circle-and-square fractal moiré pattern

In any case, we did manage to find applications that merge them both. Things like screensavers (though we feel that they are rather unpleasant looking) have been created which boast of the successful hybridisation of the two art-math genres.

We wait to see if such combinatory possibilities would be explored and new (perhaps more beautiful) applications created for the advancement of these two topics. Who knows? Perhaps the pseudo-scientific world might see fractal-moirés as a means for hypnotism.

References:

Amidror, Isaac. "Introduction." *The Theory of the Moire Phenomenon.* Dordrecht: Kluwer Academic Publishers; 2000.

Binde, Melissa D. "Fractals as Art." http://www.fractality.com/

Bogomolny, Alexender. "Modular Art." http://ccins.camosun.bc.ca/~jbritton/modart/jbmodart.htm

Conors, Mary Ann. "Exploring Fractals." http://www.math.umass.edu/~mconnors/fractal/fractal.html

Davidson, Michael W. and Kirill I. Tchourioukanov. "Moire Patterns." http://micro.magnet.fsu.edu/primer/java/scienceopticsu/moire/moire.html

Harrington, Doug. "Amazing Seattle Fractals." http://www.fractalarts.com/ASF/

Kearney, Mike. ""Process Intensification using Engineered Fluid Transporting Fractals." http://www.arifractal.com/process%20intensification%20paper.pdf .

Kelley, Alice. "Seeking the Spiral." http://www.fractalus.com/cheshirecat/

Lanius, Cynthia. "Why Study Fractals." http://math.rice.edu/~lanius/fractals/WHY/

Permadi F. "Moire Patterns." http://www.permadi.com/java/moire/pattern250.html

WiseHuman Software. "Fractal Fantasy." http://www.bykeyword.com/pages/detail5/download-5570.html

ArtbyMath Gallery. http://www.artbymath.com/

Hop's Gallery. "Ellipses and Hyperbolas." http://www.clowder.net/hop/Elpshypr.html

Mathematik.com. http://www.mathematik.com/Moire/

SandlotScience. http://www.sandlotscience.com/

"Fractals: an Introduction." http://ejad.best.vwh.net/java/fractals/intro.shtml

"Moire Patterns." http://www.exploratorium.edu/snacks/moire_patterns.html

"Moiré Patterns that follow Parabolas." http://math.unice.fr/~irem/ciiim/moire/demparab/parabola.html

"Moire Phenomenon." http://lite.bu.edu/lite1/moire/index.html

"Mind Boggling Fractals." http://www.mbfractals.com/

"Sprott'sFractal Gallery." http://sprott.physics.wisc.edu/fractals.htm

"The Fractal Microscope." http://archive.ncsa.uiuc.edu/Edu/Fractal/Fractal_Home.html