KALEIDOSCOPE

Contents

Introduction

Definition

Kaleidoscopes are devices that produce an overall pattern from the original design and the reflected images of it in the mirrors.

Characteristics

The mirror system is the crucial element of a kaleidoscope. Different types of kaleidoscopes depend on the mirror shape, the number of mirrors used, and the angle at which the mirrors are joined. The variation in these 3 factors will produce a diverse range of kaleidoscopes.

There are four primary elements found in any design:

→ Eye piece: with variations in the size and shape, and the lens used (plastic recommended for safety reasons, although glass provides a clearer view.)

→ Body: the external housing, with optional stand, rotating gears, etc.

→ Mirror system: The 2- and 3- mirror system being the most popular; configured as a triangular cylinder.

→ Object chamber: with different materials used for viewing. E.g. Paperclips, small colored stones and flowers.

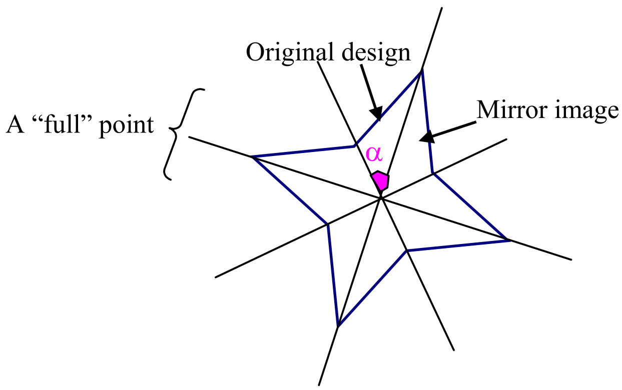
The best choice for mirrors is called the first- or front- surface mirrors, which produce sharp and clear reflections. This type of mirrors has the silvering on the front of the glass, unlike normal household mirrors. However, it is quite difficult to find in the local context.

Good kaleidoscope angle

A good kaleidoscope angle is one that produces a 'star' as an overall pattern. The star has *n* "full" points, each consisting of a copy of the original design plus its mirror image. The *n* points fit together at the point where two mirrors meet, without overlap, to form the overall pattern.

If the star has *n* points, and the points fit around the vertex without overlapping, then $n \times 2\alpha = 360°$, $\alpha = 180°/n$.

α is the angle of the original design. A 'full' point is of angle 2α. Explained in the diagram below.



This 2 mirror image, "mandala", can be extended into a 3 mirror image by adding a third piece of mirror. The image is repeatedly reflected, such that a faceted pattern is formed.

These will be further elaborated in the specific sections.

The Two-Mirror System

The two-mirror system has 3 sides. 2 of the sides are made up of mirror, shaped like a 'V' and the 3rd side, which is blackened, is put across the top. The pattern produces a *mandala,* which is a single, symmetrical, circular 'snowflake' seen at the other end of the kaleidoscope. The pattern of replication is determined by the angle between the 2 mirrors.

For example, when the angle is 45°, it will produce 360°/45°=8 reflections or 4- point star (two reflections create one point). See fig 1.

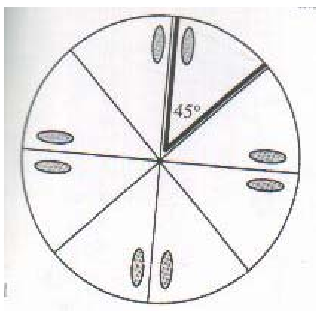


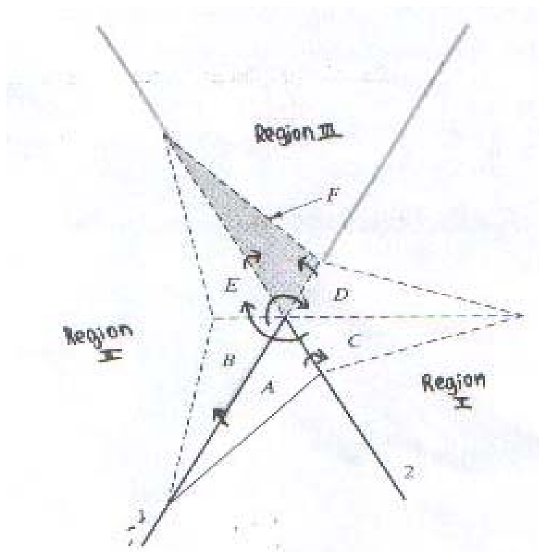
Figure 1

The following is a small list of other possible situation.

DEGREE(x°)	IMAGES (360°/x°)	n-POINTED STAR
90	4	2
60	6	3
45	8	4
36	10	5
30	12	6
25.7	14	7
22.5	16	8
20	18	9
18	20	10
16.8	22	11
15	24	12

Process of reflection

The following diagram shows a 2-mirror kaleidoscope with 60 degree between the 2 mirror, producing 6 images and 3-pointed star.

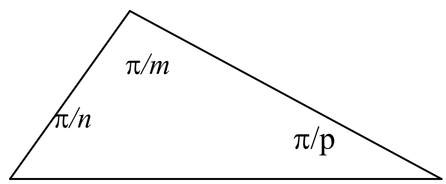


A reflects onto C in mirror 2, A reflects onto B in mirror 1, C reflects onto E in mirror 1 and B reflects onto D in mirror 2. F is made up both the left reflection of part D in mirror 1 and the right reflection of part E in mirror 2. A reflects in the clockwise direction (mirror 1) and anticlockwise direction (mirror 2). At the nth image in region III, user will get to see a combination of reflections from both the clockwise and anticlockwise direction in either a direct-direct copy manner or a reverse-reverse copy manner. The images in region III are same. For instance, B and C are reverse copy of A while E and D are direct copy of A. Images get reflected this way until the n-pointed star is completed with the last piece of image in region III. Hence, you can make any number of pointed stars you wish just by adjusting the angle between the 2 mirrors.

The Three-Mirror System

Summation of good kaleidoscope angles

A good triangular kaleidoscope must have angles which are good kaleidoscope angles. The three angles of such a kaleidoscope must have radian measures π/n, π/m and π/p for some whole numbers n, m, and p all bigger than 1.



From the diagram above, this tells us that the angle of a good kaleidoscope triangle must not be obtuse. Also, the sum of these angles must add up to π. In other words,

$$\pi/n + \pi/m + \pi/p = \pi$$

Consequently,

$$1/n + 1/m + 1/p = 1 \quad \text{------ equation (1)}$$

The solutions to equation (1) will tell us about the possible good kaleidoscope triangles. To find the solutions, we can first assume that n, m, and p increase in magnitude:

$$2 \leq n \leq m \leq p$$

Then starting with $n = 2$, we can try to fill in the following table.

n	m	p
2	?	?

If $n = 2$, then $1/m + 1/p = ½$. Since $n \leq m \leq$ p, the first possibility for m would be 2. However it doesn't work because we would be having a triangle with two right angles. So, we try $m = 3$. Therefore p would have to be 6. So our table looks like this.

n	m	p
2	3	6

With $n = 2$, the next choice for m would be 4. This forces p to be 4, and our table will looks like the following.

n	m	p
2	3	6
2	4	4

With $n = 2$, the next choice for m would be 5. This would force p to be less than 5 which is not good. The same argument would work for m bigger than 5. This would force p to be smaller than m. Thus, there are no more possibilities with $n = 2$.

Consider now that $n = 3$. The first choice for m would be 3. This forces p to be 3 also. Now our table looks like this.

n	m	p
2	3	6
2	4	4
3	3	3

If $n = 3$ and $m > 3$, then we would have $1/p \leq 1/m \leq 1/3$ and

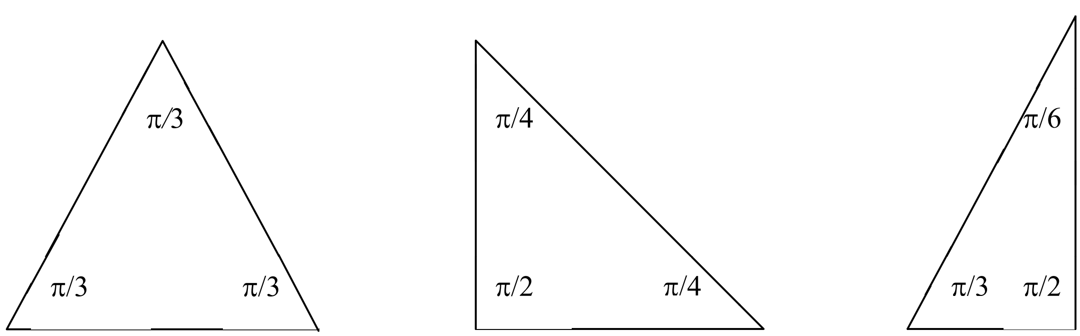
$$1/n + 1/m + 1/p = 1/3 + 1/m + 1/p < 1/3 + 1/3 + 1/3 = 1$$

This is not good. Similarly, if $n \geq 4$, then $1/p \leq 1/m \leq 1/n \leq 1/4$ and

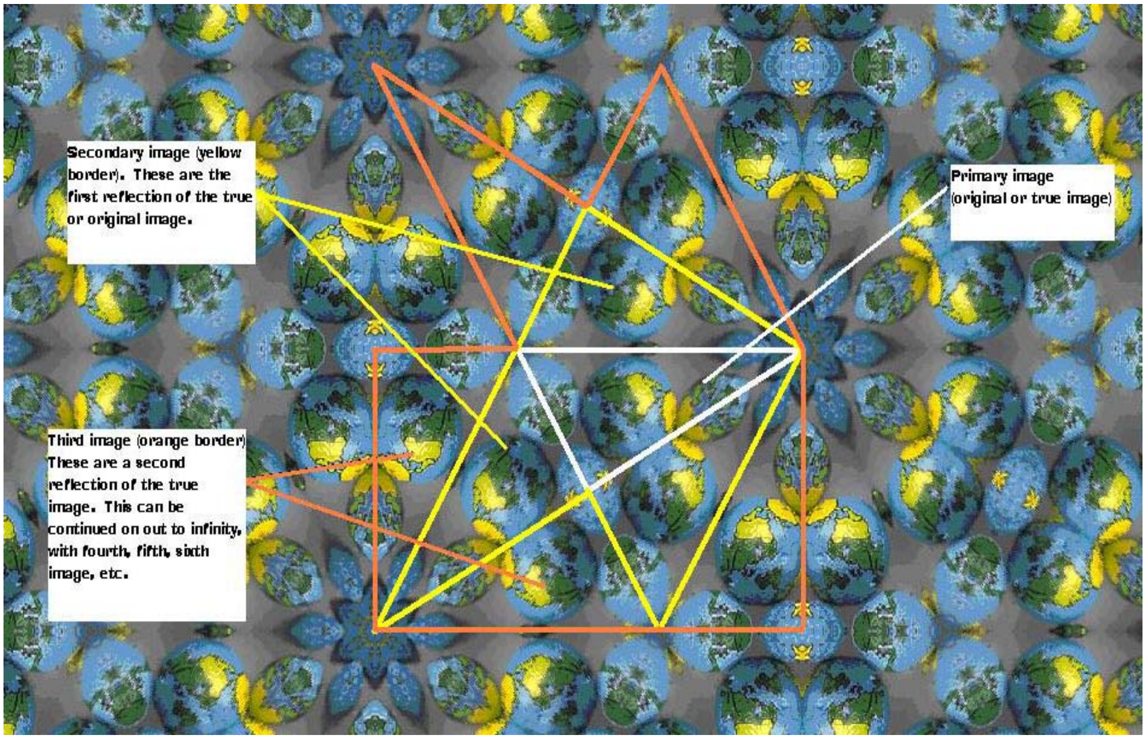
$$1/n + 1/m + 1/p \leq 1/4 + 1/4 + 1/4 = 3/4$$

Again this is no good. Therefore the table of possible good kaleidoscope triangles is complete as it is above.

In conclusion, there are only three possibilities as shown below.



Process of reflection

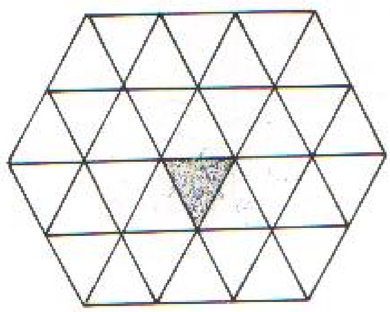


This diagram shows the order of reflection in a three mirror kaleidoscope. The original image is bordered by white. First reflections are those bordered by white of the original object and yellow. Second reflections are those that are bordered by yellow and orange and so on.

Types of symmetry

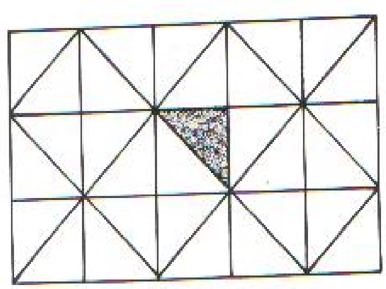
The angle of the image must be evenly divided into 360°. Also, the three angles of the mirror must add up to 180°. We shall introduce the three possible mirror configurations, which are pure symmetrical patterns. The images that are linked together are exact and real.

- The easiest and simplest combination is the 60°-60° – 60° equilateral triangle. It divides the circle into 6 symmetrical folds patterns of continuous triangles as shown in the figure below.



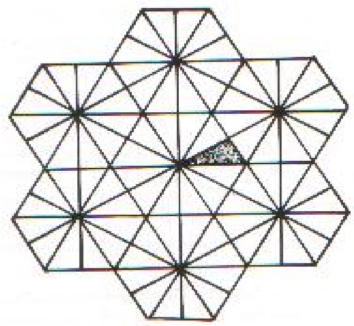
60° - 60° - 60°

- The second combination of angle is the 45°-45°-90° isosceles triangle. The image has repeated square patterns with the square actually being produced by two triangles. It produces eight symmetrical fold patters at 45° and 4 symmetrical fold patterns at 90° as shown in the figure below.



45° - 45° - 90°

- The last combination of angle is the 30°-60°-90° with three different angles producing three different symmetrical fold patterns. The three types of symmetry are 90° that produce 4 folds, 60° that produces 6 folds and 30° that produces 12 folds, joined together in a honey comb effect as shown in the figure below.



30° - 60° - 90°

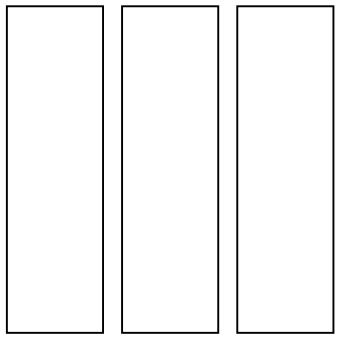
Odd angles

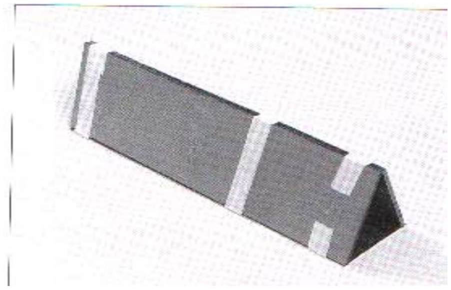
The previous examples have angles that divide 360° evenly producing pure symmetrical results. However, odd angles divide the 360° fractionally. The odd angles produce continuing designs but only showing fractional parts of the actual images. It produces geometric patterns but continuing is disturbed. Direct scene is not nicely produced by the back sector, the one furthest from the direct view although the odd divider angles of 360° create equal size sectors.

If the eye piece hole is large enough for the viewer to change his angle of view, then he will be able to see that the back sector content that only shows part of the actual image. The arrangement has one spoiling feature and it is not clear. However, if the eye piece hole is small, viewer will not be able to see the flaw of the symmetry.

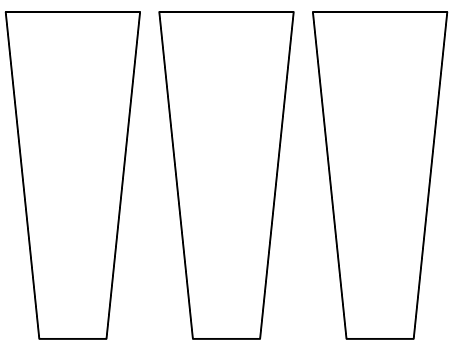
2 D and 3 D images

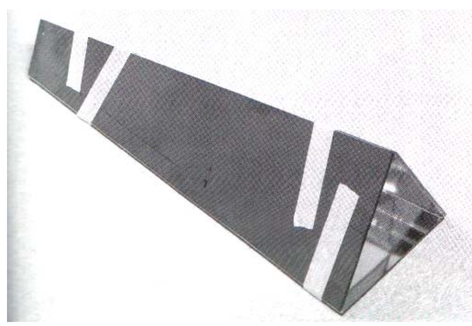
2 dimensional image can be seen using three rectangular mirrors joined together. The shape of the three mirrors will look as shown below:



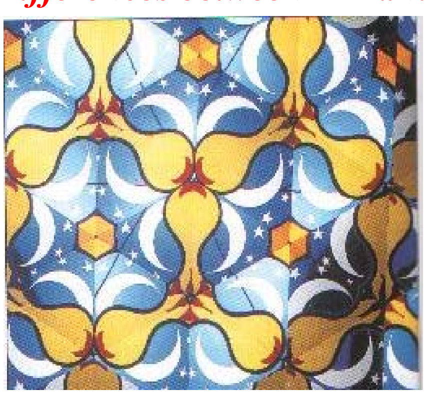


Tapered mirror system provides a spherical 3 dimensional image. When the viewer looks through the eyepiece hole, the image looks like a globe. The large opening shows a spherical 3 dimensional appearance while the small opening shows an exact and real image which are enlarged, so that viewer will be able to see and examine the objects. This will increase the brightness because it allows more light to enter the system. This system has three trapezoid mirrors as shown below.

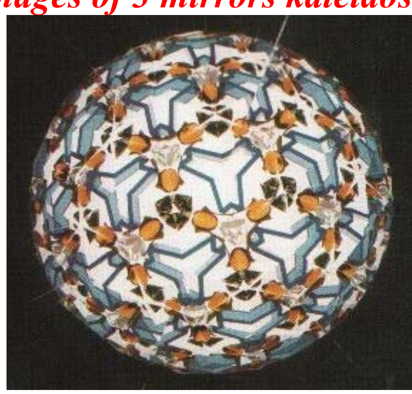




Differences between 2 D and 3 D images of 3 mirrors kaleidoscopes



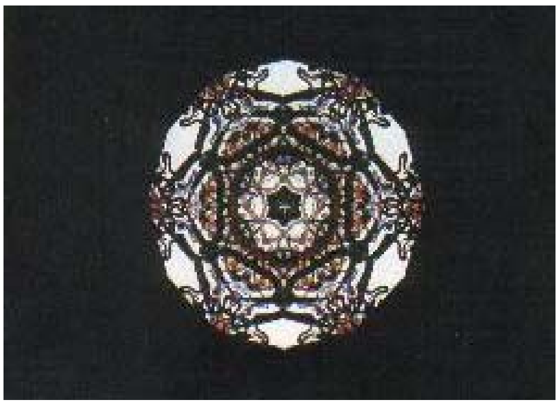
2 D image



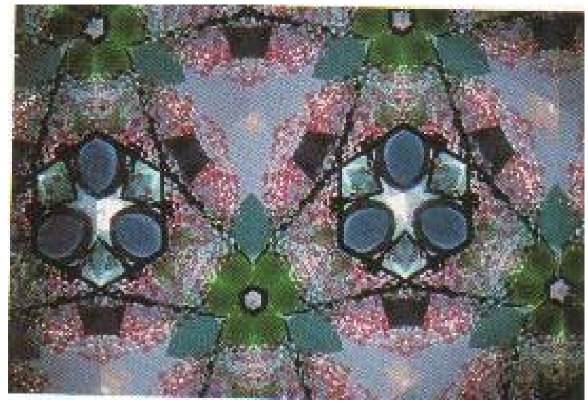
3 D image

Differences between 2 and 3 mirror kaleidoscopes

Firstly for two-mirror kaleidoscope, the image produced is a circular pattern surrounded by a black field, which is known as mandala-like image. Mandala is a single, symmetrical, circular “snow flake” at the end of the tube. Whereas for three-mirror kaleidoscope, it creates an image that repeats one triangular pattern continuously, hence producing a faceted pattern that fills the entire field of view.



Two-mirror image, by Charles Karadimos



Three-mirror image, by Carolyn Bennett

Next, for two-mirror kaleidoscope, the angle between the two mirror planes can of any degree that is between 0° to 90°, as long as it divides equally into 360°. However for three-mirror kaleidoscope, beside that each of the three angles cannot be obtuse and must be evenly divided into the 360° of the circle, the sum of the three angles must be 180°.

Steps in making a kaleidoscope

The steps below will explain how to make a 3 mirror tapered kaleidoscope that gives a 3 dimensional image.

Materials needed:

3 mirrors
Clear triangle glass
Viewing object
Transparency paper
Compressed foams
Masking tape
Double-sided tape
Blue tack
Glue

Steps:

1) Firstly, clean the 3 mirrors. Then arrange them into a triangular shape with 60° at each corner, using masking tapes.

2) Compressed foams are attached behind each piece of mirrors to support them and hold them in the right angle.

3) White cardboards are wrapped around the kaleidoscopes with beautiful wrapping papers.

4) The viewing object is made from placing various beads placed on the transparency using glue. Then it is attached to the bottom of the kaleidoscope using double- sided tape.

5) After cleaning the clear triangle glass. It is fit onto the top of the kaleidoscope using blue tack.

6) The kaleidoscope is now really for viewing when placed vertically on the table.

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