Mathematical Aspects of Irish Elections

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1   Introduction

In this paper we will describe the Irish voting system by means of examples from the 2002 general election. We will also discuss the random element of the current counting procedure, the policy of only considering the last received sub-parcel when transferring, and the fact that STV is non-monotonic.

Ireland's electoral system is proportional representation by means of a single transferable vote (STV) in multi-seat constituencies. Voters are asked to rank the candidates in order of preference on the ballot paper. Proportional representation, in some shape or form, is the preferred electoral system throughout most of the democratic world. STV is also used in Australia, New Zealand, Malta and Northern Ireland. However, the preferred choice in the UK is the single member plurality or "first-past-the-post" system where only one winning candidate represents the constituency. This system can dramatically distort election results. For example, in the 1997 UK general election, the Labour party won 63% of the seats with a 43% share of the vote. In the 2002 general election in Ireland, Fianna Fáil won 48% of the seats with 41.5% of the first preference votes, and the outgoing Fianna Fáil/Progressive Democrats government together won 53% of the seats with 45.4% of the first preference votes. These two figures are often close in an STV election. Because of this, some argue that STV is more representative of the wishes of the people, and this is probably the main reason for using STV.

In 1959 and 1968 the Fianna Fáil government held a referendum trying to switch to the "straight vote" system as in the UK, but they were defeated. During a 1959 Dáil debate before the referendum Sean Lemass said: "There are not half the deputies in this house, much less half the electorate, who can give an intelligent explanation of what happens to a number three preference on a ballot paper." We hope this article may help readers in this regard.

All constituencies have three, four or five seats. Small three-seater constituencies favour the larger parties, and it is generally accepted that the optimal size for STV constituencies is at least five ([18]). In the 1977 election there were no less than 26 three-seaters, 10 four-seaters and only 6 five-seaters. Moreover, all the Dublin constituencies were three-seaters. There are reasonable arguments for three-seaters in rural areas, but not in a congested city. However, an independent boundary revision before the 1981 election removed these anomalies ([4]).

In Ireland, the candidates are listed alphabetically on the ballot paper, but they could also be listed by party or randomly. Before 1965 party affiliations were not given on the ballot paper. This change increased party loyalty among the voters. For a detailed discussion of transfer patterns in Irish elections see the book by Sinnott ([17]).

Irish political parties have long experience in vote management techniques like spreading the first preference vote among the party candidates. While this favours the big parties, it can also lead to intra-party factionalism, and Irish political culture is personified by a high degree of localism and a large number of independent candidates ([3, 4, 17]).

In order to simplify the counting process, only the last received sub-parcel is considered when transferring surpluses. We will give examples of how this policy can lead to paradoxical results in Section 8, and discuss whether this affected the result in Dublin Central in 2002. With the introduction of electronic voting, it would be possible to no longer just consider the last received sub-parcel. There are arguments for and against the introduction of electronic voting. These issues are widely discussed elsewhere, and we will not go into them further.

There is a vast literature on the mathematics of voting systems. One important objection against STV is that it is "non-monotonic". This means that increasing your first preference votes can actually prevent you from being elected! We will give a simple example of this in Section 9.

We will not go into the finer details of cases of ties, when lots need to be drawn. For further details we refer the interested reader to the Electoral Act, 1992, which is available online ([19]). While writing this paper, we consulted a number of web sites explaining the rules for the counting. We discovered that several of them did not state the rules properly. The reader may enjoy looking at some of these sites and comparing them with this paper and the Electoral Act of 1992. A good source for additional information is the web site Irish Election Database ([16]), and of

Candidate	Party	%	C 1	C 2	C 3	C 4	C 5	C 6	C 7
Ahern, Bertie*	FF	32.0	10882	10882	10882	10882	10882	10882	10882
Costello, Joe	Lab	12.2	4136	4499	4539	5046	7870	7870	7870
Fitzpatrick, Dermot	FF	7.6	2590	4855	4968	5043	5560	6264	6418
Gregory, Tony*	Ind	16.7	5664	6477	6625	7242	7242	7242	7242
Kehoe, Nicky	SF	14.6	4972	5300	5343	5487	5818	6177	6339
Mitchell, Jim*	FG	11.1	3769	3976	4063	4268	-	-	-
O'Donnell, Patrick Noel	Ind	0.3	89	95	-	-	-	-	-
O'Loughlin, Paul Thomas	CSP	1.1	366	377	-	-	-	-	-
Prenderville, Tom	Ind	0.3	97	103	-	-	-	-	-
Simpson, Tom	GP	4.3	1469	1545	1652	-	-	-	-

Table 1: Dublin Central, 4 seats, quota 6807.

course, the excellent "How Ireland voted" series of books ([8, 9, 10, 12, 13]).

An interesting classroom exercise would be to present the counting procedure as an algorithm. This is relevant because of computerized voting. For details, see the information paper on electronic voting ([20]). We attach a summary flowchart as an appendix. More detailed flowcharts can be found in ([2]). An asterisk in the tables denotes a sitting TD.

2 Outline of the Counting Procedure

We will start with an outline of the procedure, and then discuss the finer points later on.

At the beginning of the count for a constituency, all valid ballot papers are thoroughly mixed together and then sorted according to first preferences. If we denote the total valid poll by P and let N be the number of seats, then the quota Q is calculated by the formula

$$Q = 1 + \frac{P}{N+1},$$

where we round down if this number is not an integer. This is the smallest number of votes that guarantees election.

If the number of votes for any candidates is equal to or greater than the quota they are deemed elected. For example, in Dublin Central (Table 1), $N = 4$ and $P = 34034$, so $Q = 6807.8$ by the above formula. Thus the quota is 6807 and Bertie Ahern with 10,882 votes is deemed elected on the first count.

The surplus of an elected candidate is the number of votes by which that candidate's total number of votes, original and transferred, exceeds the quota. When

a candidate is elected, the surplus ballots may be transferred to the remaining candidates according to the next preference on the ballots. The ballots are physically divided into *sub-parcels* (literally different piles of ballots). Some ballots may not have a next preference for a continuing candidate, and these ballots are called *non-transferable*. Ballots that do have a lower preference for a continuing candidate are called *transferable*.

The general idea is that a surplus is transferred if and only if at least one of the following conditions are met.

1. If transferring the surplus could elect the highest remaining candidate.

2. If transferring the surplus could bring the lowest remaining candidate equal to or above the second lowest remaining candidate.

3. If transferring the surplus could bring the lowest remaining candidate up to one quarter of the quota (candidates whose total reaches that figure qualify for reimbursement of some election expenses).

However, there are exceptions. As we will discuss in Section 4, there may be more than one surplus present. In that case we must see if the *sum* of the undistributed surpluses satisfies these conditions, so satisfying at least one of these conditions is not necessary. In addition, the surplus of another candidate may be larger, in which case that surplus is transferred first, so satisfying at least one of these conditions is not sufficient.

3   When is a Surplus Transferred?

A common misunderstanding is that the surplus is transferred if it could change the order of any of the lower-placed candidates or it could result in any candidate reaching one quarter of the quota. This is not true! We are only required to consider whether it could change the position of the *lowest*-placed candidate or result in the *lowest*-placed candidate reaching one quarter of the quota. For example, in Dublin Central (Table 1) after count 4, Gregory's surplus of 435 is not enough to move Mitchell up, so Mitchell is eliminated and the surplus is not distributed. However, Costello is only 3 ahead of Fitzpatrick, so distributing the surplus could have changed the order between them. Similarly, in Galway West (Table 5) after count 8, Fahey's surplus could have changed the order between Lyons and McDonagh, but it is not transferred at that point. It is transferred after count 10 when Callanan has been eliminated.

We now give examples of where these conditions are or are not met.

Candidate	Party	%	C 1	C 2	C 3	C 4	C 5	C 6	C 7	C 8
Aylward, Liam*	FF	20.5	12489	12489	12489	12489	12489	12489	12489	12489
Browne, Fergal	FG	9.0	5468	5530	5670	5762	5894	-	-	-
Collins-Hughes, Eddie	Ind	2.7	1614	1644	-	-	-	-	-	-
Hogan, Phil*	FG	12.9	7841	7852	8074	8239	8903	12238	12238	12238
Kiernan, Tom	SF	3.4	2078	2098	2216	-	-	-	-	-
McGuinness, John*	FF	15.4	9343	9371	9657	10085	10527	10654	10878	12111
Nolan, Billy	Ind	0.6	335	-	-	-	-	-	-	-
Nolan, M.J.	FF	14.3	8711	8772	8887	9140	9204	9832	9895	11009
O'Brien, Michael	Lab	6.1	3732	3739	3946	4220	-	-	-	-
Pattison, Seamus*	Lab	0.0	-	-	-	-	-	-	-	-
Townsend, Jim	Lab	7.0	4272	4330	4415	4568	6625	7646	7653	10615
White, Mary	GP	8.2	4961	5003	5401	6055	6696	7329	7355	-

Table 2: Carlow-Kilkenny, 5 seats, quota 12169. Note that Seamus Pattison is the Ceann Comhairle and is returned unopposed.

As an example where none of the conditions are met, we consider Carlow-Kilkenny, count 1 (Table 2). Aylward is elected but his surplus of 320 is not transferred because it does not satisfy any of these conditions. Let us check: Firstly, the highest continuing candidate is more than 320 from the quota. Secondly, the lowest candidate, Billy Nolan, is more than 320 below the second-lowest candidate, Collins-Hughes. And finally, adding 320 to the lowest candidate's total would not bring Billy Nolan up to one quarter of the quota. Note that in fact Aylward's surplus is not transferred until count 7!

Next an example of where only condition 1 holds; Cork East, count 1 (Table 3). Ned O'Keeffe is elected on count 1 with a surplus of $10574 - 9158 = 1416$. The highest remaining candidate has 8340, which is $9158 - 8340 = 818$ away from the quota. The surplus is therefore transferred on the next count because it satisfies condition 1. However, it does not satisfy conditions 2 or 3.

Next an example of where only condition 2 holds; Carlow-Kilkenny, count 7 (Table 2). Aylward's surplus of 320 from count 1 is finally transferred, but just barely! Mary White from the Green Party has 7329 votes while Jim Townsend from Labour has 7646. The difference is 317. If White had received 4 votes less, she would have been eliminated, and Carlow-Kilkenny would have finished the count without a single surplus transfer!

Finally an example of where only condition 3 holds; Kerry South, count 4 (Table 6). O'Donoghue is elected on count 1 with a surplus of $9445 - 9162 = 283$. One quarter of the quota is 2290.5 and Casey has 2216 after count 3. The surplus is not transferred on counts 2 or 3, but it meets condition 3 (and does not meet

Candidate	Party	%	C 1	C 2	C 3	C 4
Ahern, Michael*	FF	18.2	8340	9204	9204	9204
Bradford, Paul*	FG	15.4	7053	7224	7718	8231
Manning, Patrick	CSP	0.4	187	189	-	-
Mulvihill, John	Lab	10.5	4813	4838	5559	-
Murphy, June	SF	5.7	2624	2685	-	-
O'Keeffe, Martin	GP	2.5	1136	1151	-	-
O'Keeffe, Ned*	FF	23.1	10574	10574	10574	10574
Sherlock, Joe	Lab	10.5	4792	5015	6192	8660
Stanton, David*	FG	13.7	6269	6324	6978	8438

Table 3: Cork East, 4 seats, quota 9158.

Candidate	Party	%	C 1	C 2	C 3	C 4	C 5	C 6	C 7
Brolchain, Niall	GP	4.4	2193	2216	2225	2241	2308	2416	2450
Callanan, Danny	SF	3.0	1468	1485	1492	1498	1509	2166	2200
Coistealbha, Sean AC	SF	2.7	1311	1373	1379	1383	1397	-	-
Cox, Margaret	FF	6.6	3269	3697	3700	3704	3752	3801	3870
Cuiv, Eamon*	FF	20.1	9947	9947	9947	9947	9947	9947	9947
Fahey, Frank*	FF	14.6	7226	7902	7903	7904	7993	8120	8318
Grealish, Noel	PD	5.5	2735	2787	2789	2791	2858	2876	3121
Healy-Eames, Fidelma	FG	2.7	1320	1340	1343	1347	-	-	-
Higgins, Michael D.*	Lab	10.5	5213	5315	5330	5343	5510	5646	5772
Lyons, Donal	PD	4.0	1995	2017	2017	2026	2058	2068	2545
Manning, Eileen	Ind	0.2	96	97	100	-	-	-	-
McCormack, Padraic*	FG	9.6	4760	4850	4853	4859	5260	5318	5502
McDonagh, Michael	FG	4.6	2279	2300	2301	2306	2619	2632	2670
McDonnell, Declan	PD	3.0	1462	1479	1480	1484	1499	1506	-
Nulty, Joseph The Ice Man	Ind	0.2	76	77	-	-	-	-	-
Scallon, Dana Rosemary	Ind	3.4	1677	1727	1731	1740	1812	1872	1920
Walsh, Seamus	Ind	4.9	2439	2559	2573	2588	2626	2742	2774

Table 4: Galway West, 5 seats, counts 1 to 7, quota 8245.

	C 8	C 9	C 10	C 11	C 12	C 13	C 14	C 15	C 16
Brolchain, Niall	2651	3108	3111	3267	3391	3749	3775	-	-
Callanan, Danny	2306	-	-	-	-	-	-	-	-
Cox, Margaret	4082	4297	4314	4623	4782	5244	5273	5585	5906
Cuiv, Eamon*	9947	9947	9947	9947	9947	9947	9947	9947	9947
Fahey, Frank*	8318	8318	8318	8318	8318	8318	8318	8318	8318
Grealish, Noel	3233	3315	3323	4544	5098	5325	5352	5684	6215
Higgins, Michael D.*	6095	6664	6682	7171	7498	8151	8244	10580	10580
Lyons, Donal	2713	2787	2798	-	-	-	-	-	-
McCormack, Padraic*	5865	6001	6012	6285	7623	8420	8420	8420	8420
McDonagh, Michael	2748	2808	2812	2888	-	-	-	-	-
Walsh, Seamus	2944	3154	3155	3234	3328	-	-	-	-

Table 5: Galway West, 5 seats, counts 8 to 16, quota 8245.

Candidate	Party	%	C 1	C 2	C 3	C 4	C 5	C 6	C 7
Barry, Donal	Ind	2.5	934	-	-	-	-	-	-
Casey, Sheila	FG	5.3	1934	2033	2216	2220	-	-	-
Fitzgerald, Seamus	FG	12.4	4539	4732	4791	4811	6083	-	-
Fleming, Tom	FF	18.9	6912	6995	7255	7407	7593	8276	8381
Grady, Donal	Ind	3.7	1346	1449	-	-	-	-	-
Healy Rae, Jackie*	Ind	17.0	6229	6455	6819	6893	7147	8409	8584
Moynihan-Cronin, Breeda*	Lab	14.5	5307	5480	5946	5979	6432	9442	9442
O'Donoghue, John*	FF	25.8	9445	9445	9445	9445	9445	9445	9445

Table 6: Kerry South, 3 seats, quota 9162.

conditions 1 or 2) after count 3 so it is transferred on count 4.

4   Multiple Surpluses

There could be more than one surplus. For example, this would happen if an elected candidate's surplus was not transferred, but then another candidate was elected. An example of this is Dublin Central (Table 1). On count 4 Gregory was elected, but his surplus was not transferred then. Instead, Mitchell was eliminated. This resulted in Costello being elected on count 5. There were then two surpluses present.

There could also be more than one candidate reaching the quota in the first count as in Cavan-Monaghan, where two people were elected on the first count. Again there were two surpluses present.

When more than one surplus is present, the largest is checked against the three conditions. If the largest surplus on its own does not meet any of the three conditions, the largest surplus will be transferred on its own if the *sum* of the surpluses meets one or more of the conditions. Otherwise we proceed with the elimination of candidates. We consider the sum of all untransferred surpluses to make sure that we have exhausted all possibilities before a candidate is eliminated.

Note that it does not matter who reached the quota first. The only time an earlier surplus gets priority is if the two are equal.

The question of whether a surplus should be transferred is considered again after each count. For example, in Carlow-Kilkenny (Table 2), Aylward was elected on the first count but the surplus was not transferred until the 7th count.

5 Elimination

If none of the three conditions for transferring a surplus are met for any of the surpluses or the sum of the untransferred surpluses, we eliminate the candidate with the lowest number of votes. All the transferable votes of the eliminated candidate are transferred to the remaining candidates according to the next preference. The non-transferable votes are classified as "non-transferable not effective," since they did not have any effect on the election, and are set aside.

It is important to consider any untransferred surplus. Suppose the three lowest candidates are A with 2004 votes, B with 2002 votes and C with 1 vote. If there is a surplus of 2000 present, then it will not be transferred because it cannot move C ahead of B. But when eliminating C, it may seem like we could eliminate B and C together, since just eliminating C alone could not move B ahead of A. However, we must not forget the untransferred surplus. We must first eliminate C and then transfer the surplus. The transferred surplus could result in B moving ahead of A.

It is more efficient to exclude several candidates in one count. For example, in Dublin Central (Table 1) three people were eliminated at count 3 and in Westmeath four were eliminated in one count. The law says that this may be done when we have checked two conditions. Firstly, the combined vote of the candidates to be eliminated together with any untransferred surplus must be less than the vote of the candidate above them. In that case, separate elimination of some of them and transfer of any surpluses would not be enough to bring the highest of them above any of the candidates above them. Secondly, we must make sure that if any of the candidates to be eliminated are below one quarter of the quota, separate elimination of some of them and transfer of any surpluses would not bring any more of them over one quarter of the quota. In order words, nobody should be eliminated before it is clear whether they can claim reimbursement of some expenses.

In Galway West (Table 4), Nulty and Manning are eliminated separately. Nulty is eliminated after count 2 and Manning is eliminated after count 3. According to the law, they could have been eliminated together. One quarter of the quota is 2061 and it is true that Nulty's votes could have brought Lyons over that, but that is not the question. Lyons is not about to be eliminated at this stage. Notice that we could not eliminate Healy-Eames at this stage, because the votes from Nulty and Manning could have brought her ahead of McDonnell. Galway West was the longest count in 2002 with 16 counts, but that was partially unnecessary.

6   Filling of Last Seats

As the count proceeds, the non-transferable ballots are being set aside. In effect, the total poll is decreasing and the quota is being reduced. It can thus happen at the end that candidates are elected without reaching the official quota. After the final count, the candidate or candidates who have the highest votes among those remaining will be deemed elected, even if they have failed to secure a quota. Carlow-Kilkenny (Table 2) is a 5-seater, but since Pattison was the Ceann Comhairle and automatically elected, only four seats were up for election. Only two of the remaining five candidates had reached the quota in the end, so two were elected without making the quota.

After all the seats have been filled, if a candidate could possibly reach one quarter of the quota then surpluses should still be transferred. In Donegal South-West in 1982, three candidates were elected on the first count. However, the counting continued until the fourth count, with the surplus of all three elected candidates being transferred, in order to determine whether the Workers Party candidate could claim reimbursement of some expenses, which he did ([16]).

7   How is a Surplus Transferred?

The surplus transfer is where the counting rules get more complicated. When a candidate is elected on the first count, their transferable papers are divided into sub-parcels (literally, piles of ballots) according to the second preference. These sub-parcels are distributed as we explain below. On later counts, when the votes credited to an elected candidate consist of original and transferred votes, the *last received sub-parcel* consists of those ballots received in the count that resulted in the candidate reaching the quota. In this case, the returning officer examines the papers contained in the last received sub-parcel by the candidate who has just been deemed to be elected, and arranges the transferable papers in this sub-parcel in further sub-parcels according to the next available preferences for continuing

candidates recorded on those papers. A new sub-parcel is also made for the non-transferable papers.

If candidate A is elected on the first count, then the last sub-parcel consists of all A's first preferences. However, if A is elected on a later count, the last sub-parcel will *not* involve the ballots that have A as a first preference, so the transfers do not reflect the preferences of the voters who put A as their first preference. See section 8 for further discussion of the last sub-parcel.

Note that if the number of transferable papers is less than the surplus, some of the non-transferable votes of that elected candidate are used to make up the remainder of the surplus. These ballots are called *"non-transferable not effective"* and are set aside. The remainder of the non-transferable votes are retained as part of the candidate's quota and placed with that candidate's papers. These are called *"non-transferable effective"* votes. (For example, if A is 50 votes below the quota and receives a surplus distribution of 100 votes, then A's surplus is 50. Suppose that the 100 votes received have 40 transferable and 60 non-transferable. The surplus of 50 consists of 40 transferable and 10 non-transferable not effective ballots.) This terminology is not entirely self-explanatory! You may think of it as follows. If a non-transferable vote is included in the quota, it was effective and is called "non-transferable effective". But if it has no further preferences for continuing candidates and was not used in bringing the last candidate over the quota, then it was not effective in any way and is called "non-transferable not effective".

Unfortunately, it gets even more confusing. Some detailed voting tables gives the number of "non-transferable votes". However, that is not really the number of non-transferable votes! It is the number of non-transferable not effective votes. To understand this, it is necessary to understand why the number of "non-transferable votes" is listed. If you add the number of votes after the last count is usually quite a lot smaller than the total number of votes cast. The difference is the number of non-transferable not effective votes. Listing the number of "non-transferable votes" shows that no votes have disappeared.

So in particular, if the table says that there were zero "non-transferable votes", that does *not* necessarily mean that there were no non-transferable papers! It just means that the were no non-transferable not effective papers. In other words, the surplus was smaller than the number of transferable papers. There could still be non-transferable effective papers that were used to make up the quota. So there is no need to account for any reduction in the total number of votes. However, when studying transfer patterns ([17]), the actual number of non-transferable (effective or not effective) papers is of interest.

This sounds complicated, but let us consider some simple examples. There are two cases.

Case 1: Number of transferable papers $\leq$ surplus.   In this case all transferable papers are transferred. Consider Dublin Central, count 7 (Table 1). Gregory is elected on count 4 with a surplus of $7242 - 6807 = 435$ votes. His surplus is distributed on count 7, and 154 votes are transferred to Fitzpatrick and 162 to Kehoe. His last received sub-parcel , which took him over the quota, has 617 votes. The number of non-transferable effective votes can be calculated by subtracting the surplus from the number of ballots in the last received sub-parcel. In this case we have $617 - 435 = 182$ non-transferable effective votes. The number of non-transferable not effective votes can be calculated by subtracting the number of papers transferred from the surplus. In this case we have $435 - (154 + 162) = 119$ non-transferable not effective votes. Thus, in the last received sub-parcel of 617 votes, there are in fact a total of 301 non-transferable votes. Of these, 182 are effective (retained as part of the quota) and 119 are not effective (used as part of the surplus). This leaves $617 - 301 = 316$ votes to be transferred.

Case 2: Number of transferable papers $>$ surplus.   In this case transferable papers are transferred in proportion, from the top. The number of papers to be transferred from each sub-parcel (which is X, say) bears the same proportion to the number of papers in the sub-parcel Y as the surplus S bears to the total number of transferable papers T. In other words,

$$\frac{X}{Y} = \frac{S}{T}$$

and we use this formula to calculate X. This is what generally happens if someone is elected on the first count, but theoretically a candidate whose voters do not give any other preferences but who makes the quota on the first count could be a counter example. In that case the number of transferable votes would be 0 and hence less than the surplus. (We remark that a surplus can be 0, as happened with O'Flynn in Cork North-Central in 2002.)

Let us look at Dublin Central, count 1 (Table 1). Bertie Ahern gets 10882 first preference votes and is elected on the first count. His surplus is $10882 - 6807 = 4075$. All Ahern's papers are put into sub-parcels according to the second preferences, and the non-transferable papers are also put into a sub-parcel. Since these are first preference votes and no candidates have been eliminated, there probably are not any non-transferable papers, so we will assume that all his papers are transferable. How many are there in each sub-parcel? We do not know. All we know is the ratio in which the surplus is transferred. For the sake of illustration, let us guess that Fitzpatrick's sub-parcel has say 6050 papers and Kehoe's sub-parcel has say 878 papers. How did we guess these figures? We know the numbers of transferred ballots. The ratio S/T, where T is the total transferable vote for Ahern

and S is his surplus is

$$\frac{S}{T} = \frac{4075}{10882} = 0.374471\ldots$$

If Fitzpatrick's sub-parcel contains 6050 papers, then we should therefore transfer

$$(6050) \times (0.374471) = 2265.55\ldots$$

papers to him, and transfer

$$(878) \times (0.374471) = 328.786\ldots$$

papers for Kehoe.

[Note: How do we deal with the fractions? We compute the corresponding numbers for all the other candidates and round up the answers that have the largest fractional part, and we round up as many of these as are needed in order to make the total number of transferred ballots equal to the surplus (for further details see [19]). It is precisely at this point that a mistake was discovered in the program used for counting the votes electronically in 2002, see [2]. The program rounded down all these numbers, whereas some should be rounded up.]

After calculating the proportions for each candidate, we transfer 2265 papers from Fitzpatrick's sub-parcel to the top of his pile of first preference votes. A key point here is that we transfer the *top* 2265 papers as a *sub-parcel*, from the Fitzpatrick pile of Ahern's first preferences to the *top* of the Fitzpatrick pile of first preferences. It turns out here that we also round up the figure for Kehoe, so we transfer a sub-parcel of the top 329 papers to his pile.

If we are in Case 2 on a count later than the first, the surplus is transferred in proportion as in the above example, but the transferred papers are only taken from the top of the last received sub-parcel of votes that brought the elected candidate over the quota. The papers for inclusion in the surplus are taken from the top of each candidate's sub-parcel of next preferences contained in the sub-parcel of papers last received by the elected candidate. See Section 8 for further discussion of this.

To illustrate, let us look at Dublin Central, count 6 (Table 1). Costello is elected on count 5 with a surplus of $7870 - 6807 = 1063$. His last received sub-parcel of votes, which took him over the quota, contains 2824 votes. On the next count, his surplus is transferred. 704 goes to Fitzpatrick and 359 to Kehoe. So we see that the whole surplus is transferred and there are no non-transferable non-effective papers. Were there any non-transferable effective votes in the 2824? We do not know. If there were, they were put in a separate sub-parcel and the remaining votes were arranged according to the next preferences for the remaining candidates. After the proportions are calculated, the required numbers of papers were taken from the top of these piles and transferred to Fitzpatrick and Kehoe.

Let us sum up the three examples we have considered. For Ahern and Costello the number of transferable papers is greater than or equal to the surplus. Therefore we need to guess the number of non-transferable papers and compute ratios. For Gregory, the number of transferable papers is less than the surplus, so we know that all the ones that were used to make up the quota were also non-transferable, and we do not need to worry about ratios.

What is the difference between Ahern, Costello and Gregory in this respect? First of all, it depends on which count we are at. All of Ahern's papers are first preference votes, and although some people may only vote for him, it would be extremely unlikely if we were in Case 1 on the first count. However, once the papers have been transferred around a few times, we are more likely to run out of further preferences. There are at least two reasons for this. Firstly, we are further down on the list of preferences – for Ahern we are looking at second preferences, while for Costello and Gregory it is at least third preference. Secondly, the candidates listed as preferences may already have been eliminated.

Another factor is the candidate's party. Gregory runs as an independent. Independent voters do not necessarily follow vote management, and may have fewer preferences, or they may have preferences for other independent or small-party candidates who have been eliminated in earlier counts. Costello's (Labour) surplus is the result of a Fine Gael to Labour transfer, so they are likely to be "well-managed" votes with lots of preferences. Gregory's surplus comes from the Green Party candidate Simpson, and only about half of them has a vote for Fitzpatrick (FF) and Kehoe (SF). However, voter transfers can be surprising. 34% of the 1063 papers in the Mitchell to Costello parcel that are transferred have a Fine Gael, Labour, Sinn Féin transfer.

Traditionally, the transfer system has worked against Fianna Fáil ([10]), as supporters of virtually all other parties used their preference votes against them and Fianna Fáil candidates who had done well on the first count were overhauled by candidates from other parties.

8   The Last Received Sub-parcel Problem

There are alternative methods of distributing the surplus to the method used in the Republic of Ireland. Each of these methods has its own advantages and disadvantages. We discuss this issue in this section.

In order to save time, only the last received sub-parcel is considered when transferring surpluses in an Irish general election, as we discussed in the previous section. Unfortunately, this could cause fundamental distortion. There is a discussion of this in the book by Sinnott ([17]), see also [6].

In order to understand how this policy can distort the results, we have con-

	C 1	C 2	C 3
A	12000	12000	12000
B	9001	11001	11001
C	9000	9000	10001
D	9995	9995	9995

Table 7: County X, 3 seats, quota 10000.

structed some hypothetical examples. Suppose County X is a three-seater with 39996 votes and hence a quota of 10000, and that the first preference vote is given as in Table 7. Assume that voters for candidate A all give their second preference to candidate B and their third preference to candidate C. The surplus from A will then first push B over the quota, and is then transferred to C who also is elected.

There are two things to notice. First of all, the same sub-parcel of votes in involved in electing all three candidates. In a sense, the votes cast for A become "transferable super-effective"! Secondly, this happens regardless of the preferences of B. Even if all first preference votes for B have D as their second preference, there will be zero transfers to D, since we only consider the sub-parcel transferred from A. However, if we look at all of B's votes, as much as 82% of them could have a next preference for D, so D could have received 819 votes instead of 0 if all of B's votes were taken into account! Obviously one can argue that this outcome distorts the will of the public.

We will now consider a variation of this. Suppose that there is a fifth candidate E, and that the first count figures are as in Table 8. Suppose that all of the (first preference) votes for E have A as their second preference, B as their third preference, and C as their fourth preference. When E is eliminated, the transfers will then first elect A, then B and finally C. Again, this happens regardless of the preferences of A, B and C. Suppose that D has wide cross-party appeal, and that all the voters for A, B and C have D as their second preference. The only problem is that the 5% who vote for E do not like D. So D is the first choice of (just under) 25% and the second choice of 70% and still fails to get elected! Notice how the current system totally ignores the fact that 70% have D as their second preference, while the ballots with E as first preference are "transferable super-effective"! In fact, in this example the only ballots where further preferences are considered are the votes for E. This election result would not be very representative of the public will, and some argue against STV because things like this can happen. However there is a solution, as we will discuss below.

Let us consider first a real life example where the transfer patterns are probably complex, although we cannot know for sure. After count 5 in Dublin Central (Table 1) Kehoe (Sinn Féin) was ahead of Fitzpatrick (Fianna Fáil) before the

	C 1	C 2	C 3	C 4
A	9994	12000	12000	12000
B	9001	9001	11001	11001
C	9000	9000	9000	10001
D	9995	9995	9995	9995
E	2006	-	-	

Table 8: County Y, 3 seats, quota 10000.

transfer of the surplus from Costello (Labour). However, the papers that brought Costello over the quota on count 5 were all from the elimination of Mitchell (Fine Gael). Now consider the transfer of Costello's surplus. The ballots in this surplus would mostly have Mitchell (FG) as the number 1 preference. The result was that 704 ballots were transferred to Fitzpatrick (FF) and 359 to Kehoe (SF). These Fine Gael voters preferred FF to SF in the ratio 2 to 1. What would have happened if all of Costello's votes had been considered? Perhaps a majority of these Labour voters would have ranked Kehoe (SF) above Fitzpatrick (FF), and the final outcome would have been different.

We remark that this was the only example in the 2002 election when transfers from Fine Gael or Labour helped elect a Fianna Fáil candidate.

With electronic voting there is no longer any reason for considering the last received subparcel only. All the votes with all their further preferences could be taken into consideration by transferring all the papers, but at a fraction of their value. This is called the Gregory method, named after J.B. Gregory who proposed it in 1880. It is used in Northern Ireland for local and European elections, but not for Westminster elections ([4]). In Ireland it is only used for the Seanad panel elections, where the number of votes is about 900. It is being proposed for use in local elections in Scotland, and appears to have been passed by parliament there. To use this method here (in a Republic of Ireland general election) would require a change in legislation.

The exact details of how to implement the Gregory method turn out to be quite subtle, and there are many different approaches, including the Meek method ([6, 14, 15]). Some methods involve changing the rules on transfers of votes to allow transfers to candidates who have already been elected. Of course, additional surpluses are then created and these, in turn, may be transferred. Some methods also recalculate the quota in the course of the count.

Due to limitations of space we will just give some pointers and comments to relevant papers. The article by Farrell and McAllister ([5]) discusses the changes in the law in Australia in 1983, and introduces the terminology *inclusive Gregory method* and *weighted inclusive Gregory method*. We feel that using the term inclu-

sive Gregory method to describe the method introduced in the 1983 law is somewhat unfortunate. The legislators probably intended to implement the *weighted* inclusive Gregory method, but did not phrase the law correctly. It is easy to see that from a mathematical point of view, the inclusive Gregory method is just a mistake, and does not deserve serious attention. However, it is curious to see that it was actually implemented, and the paper by Farrell and McAllister is important reading.

The briefing note by James Gilmour ([6]) for the Scottish local government and transport committee compares different methods of counting. His distinction between the philosophies of the "inclusive" and "exclusive" approaches is valuable and seems not to be mentioned elsewhere. It would be interesting to know if such considerations were previously brought up by Gregory or others. In the past, it seems likely that the practical limitations of the manual vote count has dictated the choice of methods, rather than intentional and fundamental difference of philosophy. However, with electronic voting it is possible to implement a wider range of methods, and it makes sense to consider the distinction.

Since the ballots are thoroughly mixed at the start of the counting process, the transfers should represent a random sample of the total number of papers which have the same preferences up to that candidate. Even with the preliminary mixing, there is still this random element in the counting procedure. This is discussed by Gallagher and Unwin ([7]). They constructed a probability model and studied past election results. The only relevant cases are when the final difference in votes between the last elected candidate and the runner-up is less than or equal to twice the number of randomly selected surplus votes that have reached these two candidates via other candidates. Their conclusion is that about one in five of all results could theoretically have been affected. However, the probability of a result being affected is usually very small. They only found 5 cases where the probability was greater than 10%, but see also [12, p. 176].

As we have said, with the introduction of electronic voting the need to take a random sample could be removed because a computer could count all the preferences on all the ballots instead of taking a random selection. In the 2002 election, some constituencies did have electronic voting. However, in those constituencies the "random sample" legislation had to be obeyed. This involved the computer ordering all the votes in order to choose some at random. According to the government information paper ([20]), the computer was programmed to randomly order the papers by means of a random number generator called the Lehmer algorithm ([11]). This is a simple procedure involving modular arithmetic.

We remark that the last two paragraphs have been concerned with two important areas of mathematics, probability theory and number theory!

X1	X2	X3	X1	X2	X3
7997	4002	4001	7999	4000	4001
9998	6002		7999		8001

Table 9: County Z, quota 8000, original count on the left, recount on the right.

9 STV is Non-monotonic

One odd aspect of STV is that it is non-monotonic. This means that getting more votes may prevent you from being elected. This is discussed extensively in the literature on voting ([1, 4]). In this section we will give a simple hypothetical example that shows that STV is non-monotonic and briefly mention possible real-life examples.

Three candidates from party X are fighting for the last seat in county Z. When X3 is eliminated, the transfers are evenly split among X1 and X2 and X1 is elected. But X2 does not like X1, and demands a recount. The recount shows that two X1 ballots had been placed among the X2 ballots, so at the second to last count X1 actually has 7999 and X2 4000. But when X2 is eliminated, it turns out that all the votes have X3 as their next preference, and X3 is elected!

Proponents of STV say that these occurrences are rare. It is hard to tell when this actually happens in Ireland, but Irish elections are often close and mathematically surprising things do happen frequently. According to "How Ireland Voted" ([9, 12, 13]) there were possible instances in 1987, 1992 and 1997. In 1989 in Dublin North-East, after the ninth count, Sean Haughey (Fianna Fáil) had 5,356 votes, Pat McCartan (Workers Party) had 4,995 and Neil Holman (Progressive Democrats 4,971. Holman was eliminated, and his transfers took McCartan ahead of Haughey. If 25 of Haughey's votes had been cast instead for Holman, McCartan would have been eliminated instead of Holman and McCartan's transfers might have elected Haughey ahead of Holman ([12, p. 87]). In 1992 in Dublin North-East, Cosgrave might have been elected if 25 of his votes had been cast instead for McCartan ([9, p. 78]). In 1997 in Limerick East, Jackman might have been elected instead of Kemmy if 150 of his votes had been cast instead for Ryan leading to the elimination of Kemmy ([13, p. 145]).

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Appendix: Flowchart

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                          ┌──────────────────┐
                          │   Find Quota     │
                          └──────────────────┘
                                   │
                                   ▼
                          ┌──────────────────┐
                          │ Find Number of   │
                          │ First Preferences│
                          │ For Candidates   │
                          └──────────────────┘
                                   │
                                   ▼
 ┌──────────────┐    Yes   ┌──────────────────┐
 │ Deem         │◄─────────│ Did Any          │◄──────────────────────────────┐
 │ Candidate    │          │ Candidate        │                               │
 │ Elected      │          │ Reach Quota?     │                               │
 └──────────────┘          └──────────────────┘                               │
        │                           │ No                                      │
        │                           ▼                                         │
 ┌──────────────┐    No    ┌──────────────────┐   Yes   ┌──────────────┐      │
 │ Do We        │─────────►│ Do We            │────────►│ Decide which │──────┤
 │ Fill the     │          │ Distribute       │         │ One and      │      │
 │ Last Seats?  │          │ A Surplus?       │         │ Distribute   │      │
 └──────────────┘          └──────────────────┘         └──────────────┘      │
        │ Yes                       │ No                                      │
        ▼                           ▼                                         │
 ┌──────────────┐          ┌──────────────────┐                              │
 │              │          │ Eliminate a      │                              │
 │ Finish       │          │ Candidate and    │                              │
 │              │          │ Distribute Votes │                              │
 └──────────────┘          └──────────────────┘                              │
                                   │                                         │
                                   └─────────────────────────────────────────┘
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