Undergraduate Research Opportunities Programme in Science
(UROPS)

INDIAN CALENDARS:

COMPARING THE SURYA SIDDHANTA AND THE ASTRONOMICAL EPHEMERIS

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Introduction

The history of calendars in India is a remarkably complex subject, owing to the continuity of Indian civilization and to the diversity of cultural influences. In the mid-1950s when the Calendar Reform Committee made its survey, there were about 30 calendars in use for setting religious festivals. Up till today, numerous calendars are still used in India for different purposes: The Gregorian calendar is used by the government for civil matters; the Islamic calendar is used by the Muslims; the Hindus use both the solar and the lunisolar calendars for both civil and religious purposes.

The aim of this report is to briefly describe the workings of the Indian solar and lunisolar calendars, and to highlight the differences between the two methods of measuring the solar year from a fixed point on the ecliptic: the *tropical (sayana)* system and the *sidereal (nirayana)* system. I will explain these terms later in the first and second section. Prior to that, I will introduce some basic astronomical concepts that are required to understand the fundamental units of time, namely the day, month and the year.

The third section introduces the two schools of *panchang* (calendar) makers who are responsible for all Indian calendric information pertaining to celebration of festivals, performance of rituals as well as on all astronomical matters. The 'Old' School base their calculations on an ancient astronomical treatise called the *Surya Siddhanta* while the 'Modern' School uses the modern *Astronomical Ephemeris* to obtain the true positions of the luminaries as observed in the sky. I will explain and highlight the underlying differences between the two schools in detail.

Finally, the last section comprises computer codes written to produce true longitude values of the Sun and the Moon, calculated based on modern methods. They are modified from the computer codes originally written by Nachum Dershowitz and Edward M. Reingold in Lisp, but converted to Mathematica by Robert C. McNally. Their calculations are based on old Siddhantic methods**.** I will discuss the calculations in detail.

1. **Basic facts about astronomy**

This section provides the necessary prerequisites for understanding the differences between the measure of the solar year under the tropical and sidereal system.

1.1 **The ecliptic, celestial equator and equinoxes**

The Earth *revolves* anti-clockwise around the Sun in an elliptical orbit, and the plane of orbit is called the plane of the *ecliptic*. The Earth also *rotates* anti-clockwise on its own axis. The revolution of the Earth around the Sun causes the succession of years while the rotation of the Earth causes the continuous cycles of night and day.

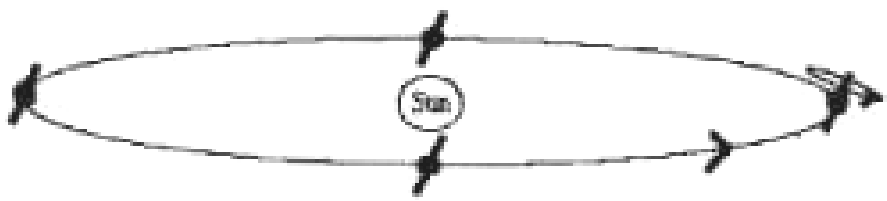

Figure 1: The plane of the ecliptic

The Earth's axis is inclined to the pole of the plane of the ecliptic at 23.5°, and this is reflected in the ecliptic being inclined to the celestial equator (the projection of the Earth's equator on the celestial sphere) at the same angle. As the Earth revolves around the Sun, the two positions where the projection of the Earth's axis onto the ecliptic plane is pointing directly towards the Sun are called the *June (Summer) and December (Winter) solstices.*

On the other hand, the two positions where the radial line from the Sun to the Earth is perpendicular to the Earth's axis are the *March (Vernal or Spring) and September (Autumn) equinoxes.* Equivalently, the March and September equinoxes are the points where the ecliptic intersect the celestial equator from south to north and north to south respectively. The March equinox occurs on or about 21 March while the September equinox occurs on about 23 September. The solstices and equinoxes are called the *seasonal markers.* Refer to Figure 2 below to see an illustration.

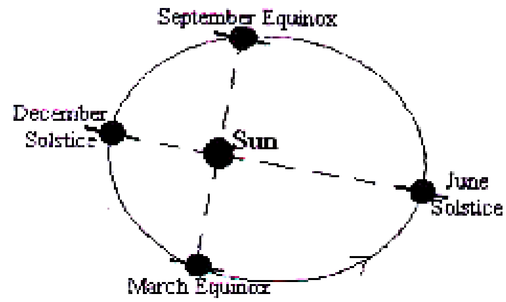

Figure 2: The seasonal markers

The motion of the Sun (or the Earth) along the ecliptic is not uniform. This is a consequence of *Kepler's Second Law*, which says that planets sweep out equal areas in equal time. This means that the Earth moves faster along the orbit near *perihelion*, the point on the orbit where the Earth is closest to the Sun, and slower when it around the *aphelion,* the point where the Earth is farthest away from the Sun.

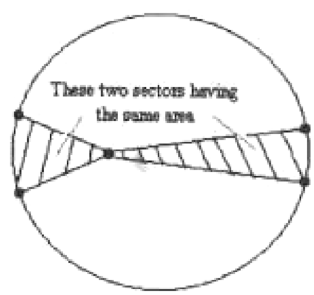

Figure 3: Kepler's first two laws

A common mistake is to think that the June solstice and aphelion (or December solstice and perihelion) coincide all the time. This is not true. This is due to a phenomenon called *precession of the equinoxes*.

1.2 Precession of the equinoxes

The Earth, apart from having motions of rotation and revolution, has another motion called precession. Under the gravitational attractions of the Sun, the Earth's axis makes a very slow conical clockwise motion, with a period of 25,800 years around the pole of the ecliptic and maintains the same inclination to the plane of the ecliptic. This causes the March equinox to slide westward along the ecliptic at a rate of approximately *50.2''* per year. Mathematically, 1° longitude corresponds to 60 elapsed *minutes*, denoted by *60'*. 1 minute corresponds to 60 elapsed *seconds*, denoted by *60''*. Hence, the March equinox recedes westwards by less than 1° per year. We call this *precession of the equinoxes*.

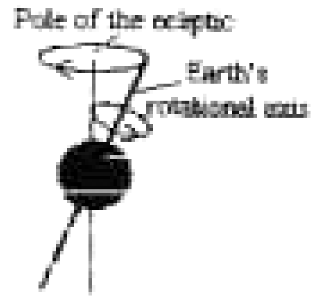

Figure 4: Precession of the equinoxes

The precessional motion of the equinox has a very important bearing on calendar making, and it is the cause of difference between the tropical and sidereal system of measuring a solar year.

1.3 Measuring a solar year

The measure of the solar year is the time period of the successive return of the Sun to the same reference point on the ecliptic. One such point is taken to be the March (Vernal) equinox point, for which the solar year measured under this system is the time interval between two successive March equinoxes. This is known as the *tropical* or *sayana* year. However, due to precession of the equinoxes, this point is not fixed. Recall that the March equinox point is consistently receding westwards at a rate of around 50.2'' per year and hence, the Earth makes a revolution of less than 360° around the Sun to return to the next March equinox. The tropical or sayana year measures 365.2422 days, slightly shorter than 365.25 days.

The other such point is taken to be a fixed point on the ecliptic with reference to a fixed background star. The solar year measured is the actual time taken for the Earth to revolve once around the Sun with respect to this fixed star. This is known as the *sidereal* or *nirayana* year. The mean length of the sidereal year is about 365.2564 days. This is about 20 minutes longer than the length of the tropical year.

A calendar is a system of organizing units of time such as days, months and years for the purpose of time-keeping over a period of time. Different calendars are designed to approximate these different units of time. In general, there are three such distinct types of calendars:

The *solar* calendars: A solar calendar uses days to approximate the tropical year by keeping it synchronised with the seasons. It follows the motion of the Sun and ignores the Moon. One example is the Gregorian calendar, where a common solar year will consist of 365 days and a leap year, 366 days. However, this is different for the Indian solar calendars, which uses days to approximate the sidereal year instead.

The *lunar* calendars: A lunar calendar uses lunar months to approximate the lunar year. A lunar month is the time interval between two successive *new moons* (or *full moons*, for the Indian lunisolar calendars) and each month has an average length of 29.5 days. This amounts to about 12 x 29.5 = 354 days a year, which is shorter than the tropical year by about 11 days. A lunar calendar follows the Moon and ignores the Sun. The Islamic calendar is an example of a lunar calendar.

The *lunisolar* calendars: A lunisolar calendar uses lunar months to approximate the tropical *or* the sidereal year. Since 12 lunar months are about 11 days shorter than the tropical year, a leap month (or *intercalary*) month is inserted about every third year to keep the calendar in tune with the seasons. The Indian lunisolar calendars, for example, are made to approximate the sidereal year, and not the tropical year.

2. The workings of the Indian solar and lunisolar calendars

This section gives an overview of the workings of the Indian solar and lunisolar calendars. A detailed description of the calendars can be pursued in the book by Chatterjee [1].

2.1 The Indian solar calendars

The regional Indian solar calendars are generally grouped under four schools, known as the *Bengal*, *Orissa*, *Tamil* and *Malayali* School. I will only touch on the rules of these schools briefly in Section 2.1.2 and not go into full details. Full details can be pursued in reference book by Chatterjee [1]. Recall that the Indian solar calendar is made to approximate the sidereal year. With this in mind, I will explain the basic structure of the Indian solar calendar.

2.1.1 Measuring a solar year

The Indian solar calendar is made to approximate the sidereal or nirayana year. The nirayana year is the time taken for the Sun to return to the same fixed point on the ecliptic which is directly opposite to a bright star called *Chitra*. The longitude of Chitra from this point is 180°. In order to assign a firm position to this initial point for astronomical purposes, this fixed initial point is taken to be the March equinox point of 285 A.D.. In other words, the starting point of the nirayana year coincided with the March equinox in the year 285 A.D.. This occurred on March 20, 285 A.D. at around 22 53 hrs, I.S.T.[1]. The celestial longitude of Chitra from the March equinox then was around 179°59'52'', which for all calendrical calculations is taken to be 180°.

However, due to precession of the equinox, the March equinox recedes westwards along the ecliptic each year and by January 1 2001, it has shifted by nearly 23°51'26'' from the initial point. But since the nirayana year approximates the sidereal year, precession of the March equinox does not affect the length of the Indian solar year.

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[1] Indian Standard Time. Ahead of Universal Time (U.T.) , or Greenwich mean time, by 5 h 30 min.

2.1.2 Measuring a solar month

The nirayana year comprises 12 solar months and they are directly linked with the 12 *rasi* divisions. A rasi is defined to be a division that covers 30° of arc on the ecliptic. The first rasi (*Mesha* rasi) starts from the same point that starts the nirayana year. The entrance of the Sun into the rasis is known as *samkranti*. A solar month is defined to be the time interval between two successive samkrantis.

The diagram below shows the name of the rasi (in black) with the name of the corresponding solar month (inner circle). Most Indian solar calendars start with Mesha rasi and end with Mina rasi.

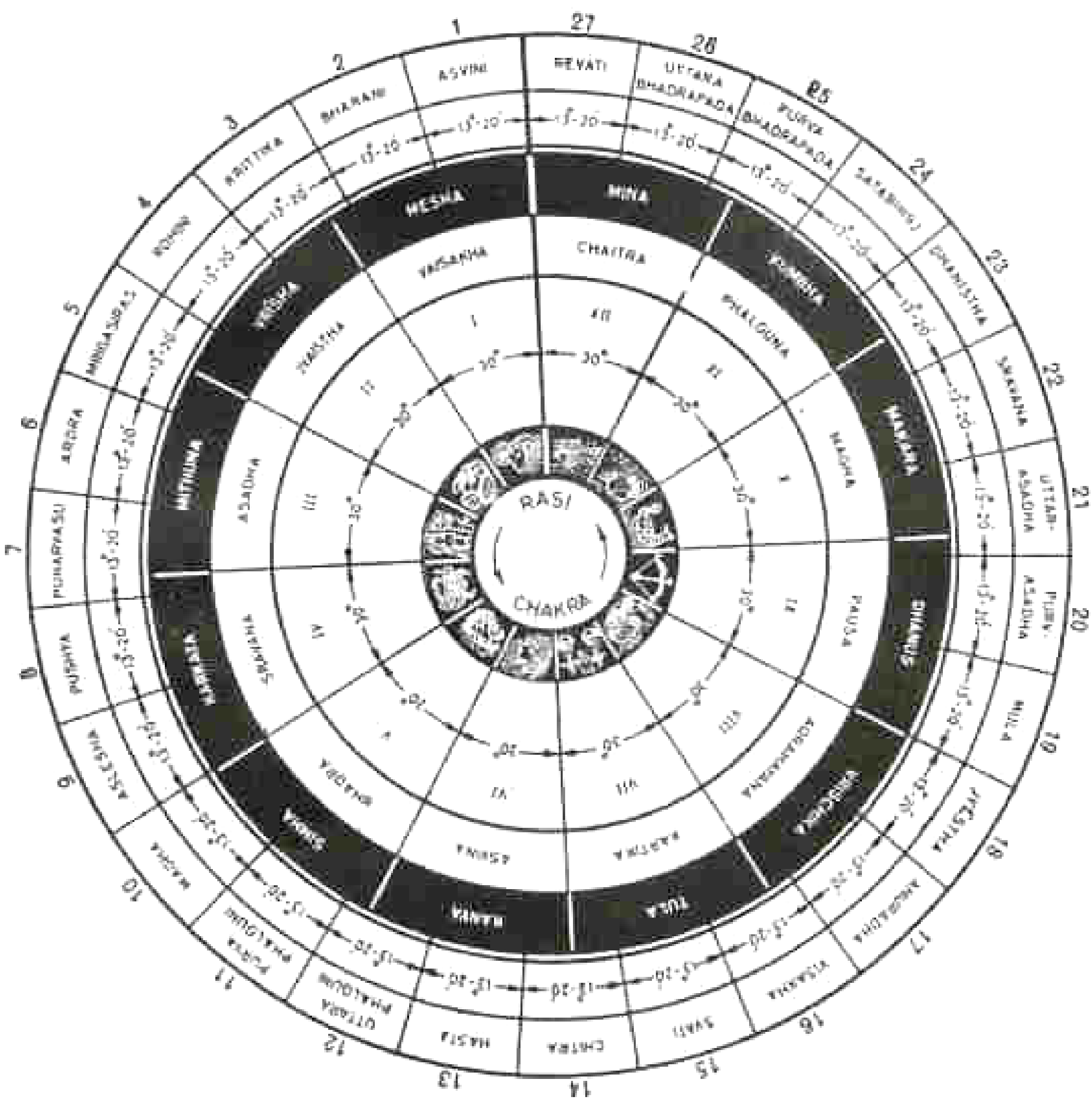

Figure 5: The Indian calendric system:

Relative dispositions of solar months and rasi divisions

However, as mentioned earlier, the solar calendar is grouped into 4 different schools. Hence, the start of the nirayana year may differ according to the rules of differing schools. For example, the Malayali calendar starts the calendar year not at the solar month corresponding to *Mesha* rasi, but corresponding to the *Simha* rasi.

Recall that the solar month is the time interval between two successive samkrantis. However, samkranti can occur at any time of the day and hence it is not advisable to start a solar month at the concerned samkranti. Instead, the beginning of a solar month is chosen to be from a *sunrise*[2] that is *close to* its concerned samkranti. There are four different conventions for the four different schools with respect to the rules of samkranti:

The *Orissa* rule: The solar month begins on the same day as samkranti.

The *Tamil* rule: The solar month begins on the same day as samkranti if samkranti falls before the time of sunset on that day. Otherwise, the month begins on the following day.

The *Bengal* rule: The solar month begins on the following day if samkranti takes place between the time of sunrise and midnight on that day. If samkranti occurs after midnight, the month begins on the *day after the next*.

The *Malayali* rule: The solar month begins on the same day as samkranti if samkranti occurs before the time of *aparahna* on that day. Aparahna is the point at 3/5th duration of the period from sunrise to sunset. Otherwise the month starts on the following day.

The mean length of a solar month is about 30.4369 days, but the actual time taken by the Sun to traverse the rasis can vary from 29.45 days to 31.45 days. However, whichever rules of samkranti the calendar follows, the length of solar months will always fall between 29 to 32 days. Solar months with their corresponding rasis near the aphelion will most probably have 32 days while solar months linked to rasis near the perihelion will probably have 29 days.

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[2] The Hindu solar day starts with sunrise.

I have briefly described the workings of the Indian solar calendar by explaining the ways how the solar year and solar month is measured. Next, I will touch on the Indian lunisolar calendars. Again, I will just give an overview and not go into details.

2.2 The Indian lunisolar calendars

The basic unit of a lunisolar calendar is the lunar month, which is the time interval either from one *new moon* to the next or one *full moon* to the next. The lunisolar calendar based two successive new moons is called the *amanta* calendar. The lunisolar calendar based on two successive full moons is called the *purnimanta* lunisolar calendar. I will only touch on the amanta calendar in my report.

2.2.1 The Amanta month

The amanta month is a lunar month that takes the time interval from one new moon to the next. Each amanta month (and hence the lunar year) is expressed in an integral number of days. In general, the months of the amanta lunar calendar are named after the solar month in which the moment of its defining initial new Moon falls. The solar month is taken to start from the exact time of the concerned samkranti to the next samkranti. Hence, an amanta month can start from any day of the concerned solar month.

The amanta lunisolar calendar generally starts with the solar month Chaitra and ends with Phalguna. This is unlike the Indian solar calendar which generally starts from the solar month Vaisakha and ends with Chaitra. Hence, the 12 months of the lunar year are as follows:

1.Chaitra	2.Vaisakha	3.Jyaishtha	4.Ashadha	5.Sravana	6.Bhadra
7.Asvina	8.Kartika	9.Agrahayana or Margasirsha			10.Pausha
11.Magha	12.Phalguna				

As mentioned earlier, since the lunar year consisting of 12 amanta months is shorter than the solar nirayana year, leap months have to be added occasionally so that the calendar approximates the nirayana year. Such leap or intercalary months are called *adhika* months, and the occurrence of adhika months cannot be determined by arithmetical rules as in the Gregorian calendar. The rules that govern the insertion of adhika months will not be elaborated on in detail.

Some months in the amanta lunisolar calendar can have *skipped* days or *repeated* days, which makes the numbering of days in the month slightly more complicated than usual. This is due to an item in the Indian lunisolar calendar called *tithi*.

2.2.2 Tithi

A *tithi* is defined to be the time required for the longitude of the moon to increase by 12° over the longitude of the Sun. We call it the *lunar day* sometimes. A lunar month can be divided into 30 tithis, of which 15 are *sukla paksha* (bright half) and 15 are *krishna paksha* (dark half). Sukla paksha is the time period from new moon to the next full moon, while Krishna paksha is the time period from full moon to the next new moon. Tithis of each paksha are serially numbered 1 to 15, designated by the prefix 'S' for sukla paksha and 'K' for krishna paksha. The name of the 15 tithis along with their serial number is as follows:

(S)	(K)	1.	Pratipada	(S)	(K)	8.	Ashtami
(S)	(K)	2.	Dvitiya	(S)	(K)	9.	Navami
(S)	(K)	3.	Tritiya	(S)	(K)	10.	Dasami
(S)	(K)	4.	Chaturthi	(S)	(K)	11.	Ekadasi
(S)	(K)	5.	Panchami	(S)	(K)	12.	Dvadasi
(S)	(K)	6.	Sashthi	(S)	(K)	13.	Trayodasi
(S)	(K)	7.	Saptami	(S)	(K)	14.	Chaturdasi
					(S)	15	Purnima
					(K)	30.	Amavasya

The days of the month of the lunar calendar are numbered in accordance with the serial number of the tithi at sunrise. When it is said that the day is *Asvina sukla dvitiya*, it is meant to be the second tithi of sukla paksha of the lunar month of Asvina. The duration of a tithi may vary from 26.78 hours to 19.98 hours and this can sometimes result in a tithi period covering two successive sunrises or no sunrise at all. In the former case, the serial number of the days will be repeated (since it covers two consecutive sunrises) and in the latter case, the serial number of the days will be omitted. For example, if tritiya tithi covers two successive sunrises, then the days of the lunar month will be numbered as 1,2,3,3,4 etc. When the same tithi falls between two sunrise (or it does not cover any sunrise) then days of the month will be numbered 1,2,4,5 etc.

I have briefly explained the workings of the Indian lunisolar calendar, highlighting some of the more important phenomena and terminology that would enable easy understanding of the material in later sections. I have omitted some items in the lunisolar calendar in my report, such as the elaboration of the *adhika* month, *kshaya* month and also the *purnimanta* lunisolar calendar.

3. The Surya Siddhanta and the Astronomical Ephemeris

The Indian solar and lunisolar calendars are widely used, with their local variations, in different parts of India. They are important in predicting the dates for the celebration of various festivals, performance of various rites as well as on all astronomical matters. The modern Indian solar and lunisolar calendars are based on close approximations to the *true* times of the Sun's entrance into the various rasis. This is unlike the old Indian calendars used before 1100 A.D. where *mean* times are used. I will not talk about the old Indian calendars in this report.

3.1 The Surya Siddhanta

Though used with local variations in different parts of India, the Indian solar and lunisolar calendars are still based on common calendrical principles found mainly in an ancient and well-known astronomical treatise called the *Surya Siddhanta*. The Surya Siddhanta is the first Indian astronomical treatise where rules were laid down to determine the *true* motions of the luminaries, which conforms to their actual positions in the sky. It was not known who composed the Surya Siddhanta or when it was first compiled, but it was believed to have come into use as early as 400 A.D.. However, the astronomical values and true positions obtained by the *siddhantic* methods are not very accurate. This is primarily due to the fact that when the original Surya Siddhanta was written around 400 A.D., the knowledge of positional astronomy was not so advanced as now, where modern, sophisticated and scientific methods are adopted to record and calculate all astronomical events and positions. The inaccuracy of the Surya Siddhanta can be seen in the measurement of the sidereal year. The modern mean length of the sidereal year as mentioned in Section 2 is around 365.2564 days, but the siddhantic length of the sidereal year is 365.258756 days, which is longer than the correct mean length by about 3 minutes 27 seconds.

Although the astronomical values given in the Surya Siddhanta are inaccurate, conservative *panchang* (calendar) makers still use the formulae and equations found in the Surya Siddhanta to compile and compute their panchangs. The panchang is an annual publication published in all regions and languages in India containing all calendrical information on religious, cultural and astronomical events. It exerts great influence on the religious and social life of the people in India and is found in almost every household. Followers and users of the Surya Siddhanta make up the ‘Old School’, which stands in opposition to the ‘Modern School’.

3.2 The Astronomical Ephemeris

The ‘Modern School’ comprises followers of the *Astronomical Ephemeris*, from which they obtain true positions of the Sun and other luminaries in the sky. Unlike the followers of the Surya Siddhanta, they use modern, advanced methods to determine astronomical events and data needed for making the Indian calendars and hence their modern panchangs. For example, they use the correct mean length of the sidereal year (365.2564 days) as the basis for all longitudinal calculations, which can be very crucial in predicting the positions of the Sun and moon.

With the differences of the Surya Siddhanta and the Astronomical Ephemeris in mind, I will now briefly introduce computer codes from the Lisp package Calendrica.m written by Nachum Dershowitz and Edward M. Reingold, converted into Mathematica by Robert C. McNally. Their functions are based on the Surya Siddhantic rules. I will only focus on the solar and lunar longitudes. As explained later in the next section, I will implement algorithms modified from existing algorithms by Dershowitz and Reingold, but now use *ephemeric* (modern) methods as the basis of my calculations.

4. Computer codes: Algorithms based on ephemeric rules

This section comprises computer codes that produce longitude values, which are modified from the computer codes written by Dershowitz and Reingold. As mentioned earlier, they wrote their algorithms based on old siddhantic methods, and hence it has caused their outputs to be different from those obtained if modern ephemeric methods were to be used. I will modify their computer codes using ephemeric rules to obtain outputs that will more accurately determine present-day longitude values and other important astronomical information.

Before I go into the actual computer codes, I will explain certain important terminology that will be essential for thorough understanding of the codes.

4.1 Fixed day numbers

Over the centuries, humans have attempted to specify dates through an enormous variety of methods such that they can find the most convenient and simplest way to reckon time. However, the huge amount of time that has elapsed since human civilization and the diversity in cultural practices in different parts of the world has led to different dates of references from which dates are to be specified.

4.1.1 The fixed calendar: R.D. dates

For a computer implementation, the easiest way to reckon time is simply to count the number of days that has elapsed since day one. We first fix an arbitrary starting point as day one and specify a date by giving a day number relative to that starting point. Dershowitz and Reingold chose the *midnight* of January 1, 1 of the Gregorian calendar as the 'day one', abbreviated as R.D[3]. 1. Hence, Monday November 12, 1945 on the Gregorian calendar corresponds to an integral value of R.D. 710,347. This same R.D. value also corresponds to October 30, 1945 A.D. in the Julian calendar.

Our objective is to make sure that we can convert any date of any calendar into a *fixed* R.D. date so that there is only one common way of counting days. All that is required for calendrical conversion is to be able to convert each calendar to and from this fixed calendar. Since some calendars begin their day at midnight and others at

[3] *Rata die*, or fixed date.

sunrise or sunset, we fix the time of day at which conversions are performed to be at *noon*.

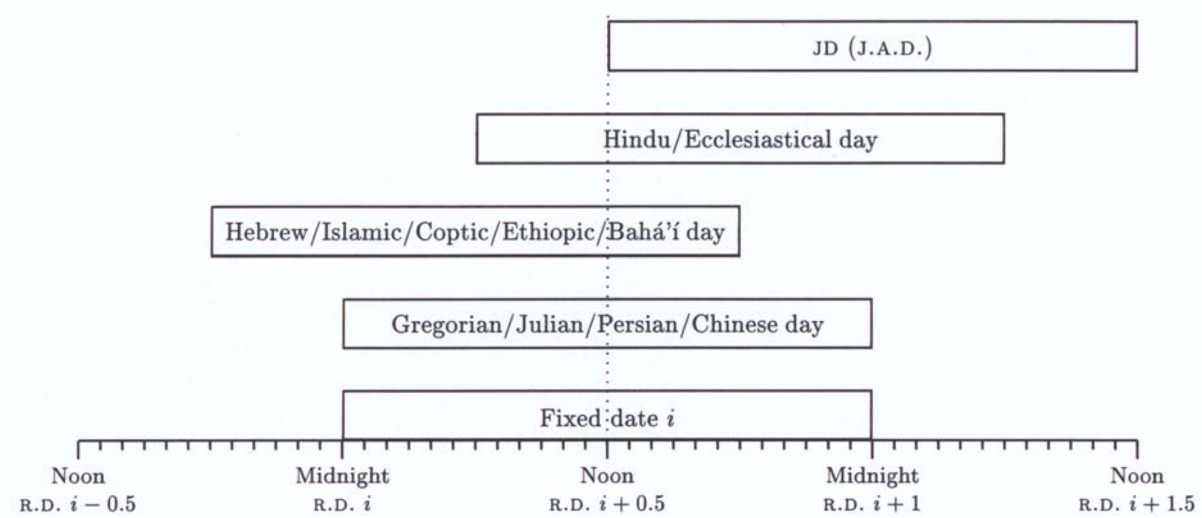

Figure 6: Meaning of 'day' in various calendars

Conversion from a date on a calendar to an R.D. date is done at noon

Hindu date corresponding to fixed date *i* is the Hindu day that begins at sunrise in the morning of fixed *i* and ends at sunrise of the morning of fixed date *i* +1

For each calendar, we can write a function to convert a given date **[date]** on that calendar to the corresponding fixed R.D. date as follows:

ToFixed[date]

where the input **[date]** can be dates in the form **Gregorian[3, 20, 2001**] or **Julian[3, 21, 285]**. The former means March 20, 2001 of the Gregorian calendar and the latter means March 21, 285 A.D. in the Julian calendar. Putting all together, we have

ToFixed [Gregorian[3, 20, 2001]
ToFixed [Julian[3, 21, 285]

where the first function returns an integral R.D. value reflecting the number of days that has elapsed since **Gregorian[1, 1, 1]**. The second function returns another integral R.D. value reflecting the number of days since the start of the Julian calendar, which is January 1, 1 A.D.

Some astronomers however, do not use **Gregorian[1, 1, 1]** as the first day which they count their days. Instead, they specify dates based on *julian day numbers* as explained below.

4.1.2 Julian day number

Some astronomers attempt to reckon time by specifying moments in time using julian days or *jd*. We use the abbreviation jd instead of 'julian' to distinguish clearly between the jd numbers in our functions and the dates in the Julian calendar. Jd numbers can be expressed as a rational number, where the fractional part gives the fraction of a julian day *beyond noon*. However, we do not use julian days directly, because we want our civil days to begin at civil midnight. Hence, we use fractional days in our calculations, but bearing in mind that our day starts from midnight.

As mentioned earlier, the midnight of January 1, 1 of the Gregorian calendar was the 'day one' from which most astronomers count their days, abbreviated R.D.1. On the noon of R.D. 0, the julian day number was jd 1,721,425. Hence, counting backwards the number of days, jd 1 corresponds to the noon of R.D. –1,721,424. But remember that days are measured from the midnight of R.D. dates. Thus jd 1 corresponds to the midnight of R.D. –1,721,424.5. In mathematical notation, we have

JDStart = -1,721,424.5

JDFromMoment[ToFixed [date]] = [ToFixed[date]] - JDStart

where **JDStart** is basically the 'day one' of the *julian day number*. This should not be confused with the Julian calendar. **JDFromMoment** takes an R.D. date at any moment in time and gives an output measuring the number of R.D. days it has elapsed since **JDStart**.

4.2 Epochs

Every calendar has an *epoch* or simply the starting date. This date is *not* the date in which the calendar was adopted, but it is rather a hypothetical starting point for the first day of the first year of the calendar. As an illustration, we look at the Gregorian calendar. The Gregorian calendar was devised and adopted in the 16^{th} century, but its epoch is January 1, 1, as mentioned earlier. The epoch of the jd number is -1,721,424.5. We take a brief look at the epoch for the Hindu calendar.

4.2.1 Hindu epoch

There are various epochs that are, or have been used as starting points for reckoning time and counting days in India. We will only focus on two of the more widely-used Hindu epochs. The first is the *Kali Yuga* ('Iron Age') epoch, abbreviated K.Y. The first day of year K.Y. 0 corresponds to January 23, -3101 of the Gregorian calendar, or February 18, 3102 B.C. of the Julian calendar. The R.D. date is R.D. –1,132,959.
In mathematical notation, we have

KYHinduEpoch = Gregorian[1, 23, -3101]

Chatterjee, the author of reference [1], did not use the K.Y. era for establishing the Hindu epoch. Instead, he took March 20, 285 A.D. of the Julian calendar, the day when the March equinox point coincided with the fixed initial point on the ecliptic opposite the bright star Chitra that starts the nirayana solar year. Hence we define his Hindu epoch in the following way:

HinduEpoch = Julian[3, 20, 285]

To count the number of days that has elapsed since the Hindu K.Y. epoch, we simply perform the subtraction of the R.D. date of the K.Y. Hindu epoch from the R.D. date at any given moment in time.

HinduDayCount[ToFixed[date]] = [ToFixed[date]] – [ToFixed[KYHinduEpoch]]

For Chatterjee's epoch, we define the number of days elapsed since then as

HinduDayCount[ToFixed[date]] = [ToFixed[date]] – [ToFixed[HinduEpoch]]

I have explained the basic terminology and concepts that forms the foundation for understanding the next section. All the concepts discussed in Sections 4.1 and 4.2 will be used extensively in the computation of longitude values later.

4.3 The siddhantic and ephemeric codes: A comparison

In this section, I will compare the codes written by Dershowitz and Reingold that use siddhantic methods with those written using modern methods. However, astronomical data given in the old Surya Siddhanta are inaccurate and complicated. Thus, I will apply modifications to these codes using modern, ephemeric rules. To distinguish between their codes and mine, I will place an extra 'e', abbreviated for '*ephemeric*', in front of all codes written by them. For example, the codes

HinduSiderealYear **HinduSolarLongitude** **HinduLunarLongitude**

will become

***e*HinduSiderealYear** ***e*HinduSolarLongitude** ***e*HinduLunarLongitude**

and similarly this will apply to all other codes not mentioned here.

4.3.1 Hindu sidereal year and the Hindu epoch

The length of the sidereal year given by Dershowitz and Reingold (D.R. Rule) is based on the Surya Siddhanta and it is longer than the correct mean length by about 3 minutes 27 seconds. They define the length of the sidereal year and the Hindu epoch (K.Y.) in the following notation:

HinduSiderealYear = 365 + 279457/1080000

= 365.258756

HinduEpoch = FixedFromJulian[3102, 2, 18]

However, we use the modern, correct of the sidereal year in our calculations. In addition, we use the *March equinox point* of March 20, 285 A.D. as our Hindu epoch.

eHinduSiderealYear = 365.2564

eHinduEpoch = Julian[3, 20, 285]

However, recall from Section 2.1.1 that the March equinox of March 20, 285 A.D. occurred at 22 53 hrs I.S.T., ahead of the Universal Time (U.T.) by 5 h 30 min. Hence, since we use U.T. in our calculations, we have to compensate a value defined by

$$\textbf{diff} = \frac{5.5}{24}$$

so that there will be a consistent time frame for all our calculations. This value is important in our calculations later.

4.3.2 Solar longitude and Hindu solar longitude

The solar longitude measures the position of the Sun with respect to a reference point on the ecliptic from which all measurements are to be taken. For calendars following the tropical system, this reference point will be the March equinox point, and bearing in mind this point will recede westwards at a rate of 50.2’’ per year due to precession of the equinoxes. The Indian calendar approximates the sidereal year. Hence, they have a different point of reference from which *Hindu solar longitude* is to be measured. This point is the point on the ecliptic when the March equinox occurred on March 20, 285 A.D., which is also the Hindu epoch defined by Chatterjee.

The solar longitude at any given moment in time is the position of the Sun relative to the March equinox point. This means that the solar longitude at the March equinox is always taken to be 0°. We input the moment in time as an R.D. date in mathematical notation as follows:

SolarLongitude[JDFromMoment[ToFixed[date]]]

This gives the solar longitude on a specified date. According to Dershowitz and Reingold, they define solar longitude as follows, which we will not go into details:

SolarLongitude[JDFromMoment[ToFixed[date]]]
= [Longitude + Aberration[JDFromMoment[ToFixed[date]]] +
Nutation[JDFromMoment[ToFixed[date]]]] mod 360

The *Hindu solar longitude,* as mentioned earlier, measures the position of the Sun with respect to a fixed initial point. Dershowitz and Reingold used the K.Y. epoch as this starting point. This means that the Hindu solar longitude will be 0° at this point (and not at the March equinox). They defined the function for finding the Hindu solar longitude based on the Surya Siddhanta as follows:

HinduSolarLongitude[t]

$$= \textbf{TruePosition}[t, \textbf{HinduSiderealYear}, \frac{14}{360}, \textbf{HinduAnomalisticYear}, \frac{1}{42}]$$

$$= \textbf{MeanPosition}[t, \textbf{HinduSiderealYear}] -$$

$$\textbf{HinduArcsin}[\textbf{HinduSine}[\textbf{MeanPosition}[t, \textbf{HinduAnomalisticYear}]] \times [\frac{14}{360} -$$

$$|\textbf{HinduSine}[\textbf{MeanPosition}[t, \textbf{HinduAnomalisticYear}]]| \times \frac{14}{360} - \frac{14}{360}]] \bmod 360$$

where the input is any moment t in time in the form of R.D. date. The function given by them makes use of items in the Surya Siddhanta such as the Hindu sine values, the

Hindu anomalistic year, as well as the siddhantic length of the Hindu sidereal year. It also involves the size and rate of change of the *epicycle*, which I will not go into further details again. However, the Hindu solar longitude obtained by the D.R. Rule may be slightly inaccurate because *true* longitude values are not used.

We shall attempt to modify the complicated functions given above by Dershowitz and Reingold. I have defined the **eHinduSiderealYear** and **eHinduEpoch** previously, and these will be used in the modified functions. Our objective will be to make use of *true* solar longitude values to calculate Hindu solar longitudes. Similarly, we will calculate the Hindu lunar longitude by using *true* lunar longitude values.

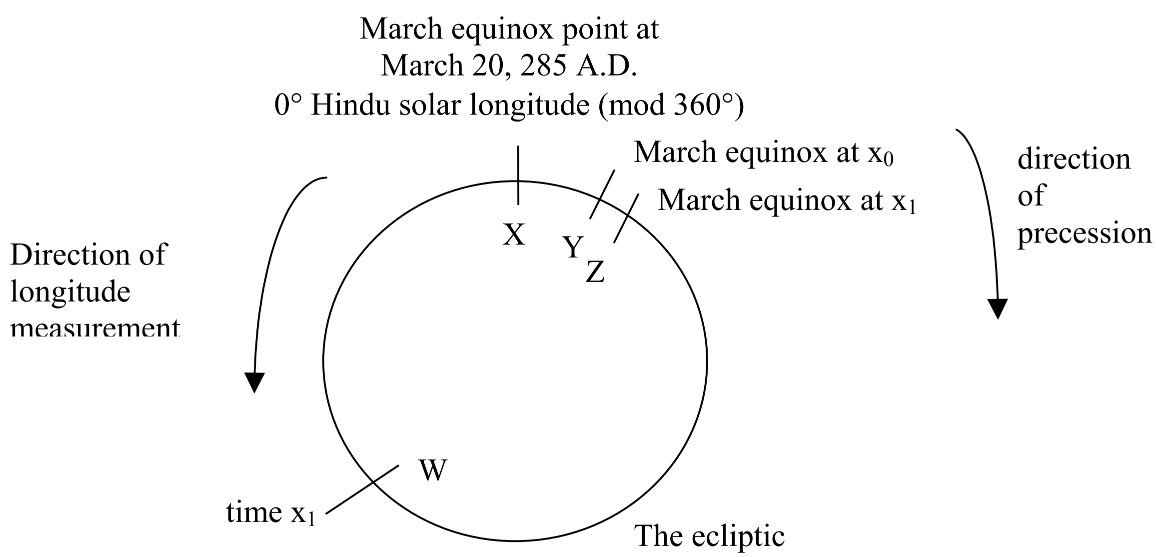

Figure 7 : Calculation of Hindu solar longitude using modern solar longitude

We define point X to be the *fixed* initial point from which all Hindu solar longitude is to be measured. This is the March equinox point on the ecliptic which occurred on March 20, 285 A.D. The Hindu solar longitude will be *0°* (mod 360°) whenever the Sun completes *x revolutions* along the ecliptic and returns to this fixed point X after *x* elapsed sidereal years.

Suppose we define x_0 to be the moment in *time* when the Sun returns to point X after s_o elapsed sidereal years, the expected output for the Hindu solar longitude at time x_0 should be 0° as shown:

eHinduSolarLongitude[x_0] = 0

The March equinox occurred on point X at March 20, 285 A.D.. However, recall the precessional motion of the equinoxes which occur at a rate of 50.2'' per year. The March equinox recedes clockwise and will no longer occur at point X. For example, when the Sun returns to point X after s_0 elapsed sidereal years, the March equinox may have receded to point Y by that time.

Recall that Hindu solar longitudes are measured from point X, and solar longitudes are measured from the March equinox at that time. If the Sun is at point X at time x_0 after s_0 *sidereal* years, we will get different outputs as shown:

eHinduSolarLongitude[x_0] = 0
SolarLongitude[x_0] = YX

Suppose now the Sun has moved from point X to point W. Similarly, the March equinox will have receded a small value from point Y to point Z. We want to measure the Hindu solar longitude XW at *time* x_1. We make use of the solar longitudes ZW and YX as follows:

SolarLongitude[x_0] = YX
SolarLongitude[x_1] = ZW
eHinduSolarLongitude[x_1] = ZW – YX

However, the Hindu solar longitude XW obtained here does not take into account the distance ZY attributed to the precessional motion of the equinox. The distance ZY is the distance between the March equinox at time x_0 and the March equinox at time x_1. We know that precession of the equinoxes occur at a rate of about 50.2'' in a *tropical* year (365.2422 days). Hence, from time x_0 to time x_1, the equinox will have moved a distance of

$$\mathbf{ZY} = \frac{x_1 - x_0}{365.2422}\,\mathbf{(50.2''/3600'')}$$

and hence we can measure the Hindu solar longitude XW in full as follows:

eHinduSolarLongitude[x_1] = ZW – YX – ZY

In formal mathematical notation, we define the times $\mathbf{x_1}$, $\mathbf{x_0}$ and **diff** as follows:

$$\textbf{diff} = \frac{5.5}{24}$$

x_1 = JDFromMoment[ToFixed[date]]

x_0 = Floor[(JDFromMoment[ToFixed[date]]-
JDFromMoment[ToFixed[eHinduEpoch[]] – diff]) / eHinduSiderealYear[]] x
eHinduSiderealYear[] + JDFromMoment[ToFixed[eHinduEpoch[]] – diff]

PrecessionDistance = ZY

Hence, we define the Hindu solar longitude at any date, in full notation as shown:

eHinduSolarLongitude[date_]
:= SolarLongitude[x_1] - SolarLongitude[x_0] – PrecessionDistance

Putting the date as March 20, 285 A.D. (Julian), we get an output of 0.223921° as shown

eHinduSolarLongitude[Julian[3, 21, 285]] = 0.223921

which is close to the expected output of 0°.

However, when we input the same date using Dershowitz and Reingold's function from Calendrica.m, we get

Calendrica`Private`HinduSolarLongitude[HinduDayCount[ToFixed[Julian[3, 20, 285]]]] / 60[4]
= 2.95308

We get a slight discrepancy of approximately 2.7°. This may be due to the formula we have devised for finding out **eHinduSolarLongitude**, which did not take into account other subtle celestial motions. However, our approximation is close enough. We now try an input of March 20, 2001 (Gregorian). We get an output 336.549°, just about 1° away from that obtained by Dershowitz and Reingold

eHinduSolarLongitude[Gregorian[3, 20, 2001]] = 336.549

Calendrica`Private`HinduSolarLongitude[HinduDayCount[ToFixed[Gregorian, 20, 2001]]]] / 60 = 335.326

This result is very accurate as the March equinox at 2001 will have receded about $\left(\frac{2001-285}{25{,}800}\right)$ x 360° = 24°, resulting in an expected output of about 336°. It is observed that our output corresponds to the expected output.

To see a similar pattern, we now define the Hindu epoch to be the *start of the K.Y. era*, which is January 23, -3101 of the Gregorian calendar. In addition, we also define the length of the Hindu sidereal year as the *incorrect* length of 365.258756 days. The reason why we chose this two values, is because these two values are used correspondingly in the Surya Siddhanta, as well as by Dershowitz and Reingold. Mathematically, we define:

eKYHinduEpoch[] = Gregorian[1, 23, -3101]
eKYHinduSiderealYear = 365.258756

---

4 Dershowitz and Reingold calculate HinduSolarLongitude in arcminutes. Recall 1° longitude corresponds to 60'. Our output is in degrees.

We define in a similar fashion the Hindu solar longitude from this epoch, however, changing every term **eHinduEpoch** to **eKYHinduEpoch** and **eHinduSiderealYear** to **eKYHinduSiderealYear**. The *expected* output is 0° at this epoch. However our output gives 0.223466°, not far from 0°.

eKYHinduSolarLongitude[Gregorian[1, 23, -3101]] = 0.223466

Comparing it with the value obtained by Deshowitz and Reingold,

Calendrica`Private`HinduSolarLongitude[HinduDayCount[ToFixed[Gregorian [1, 23, -3101]]]] / 60
= 2.1209

our value gives a discrepancy of about 2°.

We now try an input of March 20, 2001 (Gregorian). We get an output of 333.426°, about 2° away from the value obtained by Dershowitz and Reingold, but still a close approximation:

eKYHinduSolarLongitude[Gregorian[3, 20, 2001]] = 333.426

Calendrica`Private`HinduSolarLongitude[HinduDayCount[ToFixed[Gregorian [3, 20, 2001]]]] / 60 = 335.326

I have tried to implement algorithms in Mathematica to approximate as accurately as possible the Hindu solar longitude by using *true* solar longitude values. A copy of it can be found at the appendix so that it can serve as a template for future usage.

4.3.3 Lunar longitude and Hindu lunar longitude

Lunar longitude, by definition, is the position of the moon along its path of motion. Lunar longitudes are more difficult to compute than solar longitudes, because it is non-negligibly affected by the pull of the Sun, Venus, and Jupiter. Hindu lunar longitudes give the position of the moon based on Indian calendric rules. Without going in further detail, I will state the function given by Dershowitz and Reingold based on the Surya Siddhanta, for lunar longitudes and Hindu lunar longitudes at any moment t in time:

LunarLongitude[t] = [MeanMoon + Correction + Venus + Jupiter + FlatEarth + Nutation [t]] mod 360

$$\textbf{HinduLunarLongitude[t] = TruePosition[t, HinduSiderealMonth, } \frac{32}{360}\textbf{, HinduAnomalisticMonth, } \frac{1}{42}\textbf{]}$$

However, just as we have an ephemeric-based function for calculating Hindu solar longitudes, I have also attempted to devise an ephemeric-based function for calculating Hindu lunar longitudes. This function is almost similar to that of **eHinduSolarLongitude**.

We define

eHinduLunarLongitude
= LunarLongitude[JDFromMoment[ToFixed[date]]] –
SolarLongitude[Floor[(JDFromMoment[ToFixed[date]]-
JDFromMoment[ToFixed[eHinduEpoch[]] – diff]) / eHinduSiderealYear[]]
eHinduSiderealYear[] + JDFromMoment[ToFixed[eHinduEpoch[]] - diff]

Putting in the date March 20, 2001 in the Gregorian calendar, we have

eHinduLunarLongitude[Gregorian[3, 20, 2001]] = 279.614

and comparing with the private function by Dershowitz and Reingold, we have

Calendrica`Private`HinduLunarLongitude[HinduDayCount[ToFixed[Gregorian [3, 20, 2001]]]]/60
= 276.812

Again, we get a rather good approximation to the lunar longitudes using ephemeric values. However, due to time constraint, I have not come up with functions to compensate for distance covered due to precession. I have just used the un-corrected version to calculate the lunar longitudes as seen above.

I conclude my report with the end of this section. I hope that this report has given readers a clear idea of the workings of the Indian solar and lunisolar calendars, as well as give an introduction on how programming in Mathematica can help determine true positions of luminaries in the sky.

4.4. Appendix

```
<< AddOns`Calendrica`
Remove::"rmnsm": "There are no symbols matching \"\!\(\"Calendrica`*\"\)\"."
Remove::"rmnsm": "There are no symbols matching \\"\!\(\"Calendrica`Private`*\"\)\"."

<< AddOns`IndianCalendar`

SolarLongitude[JDFromMoment[ToFixed[Gregorian[3, 21, 2001]]]]
0.434329

SolarLongitude[JDFromMoment[ToFixed[Gregorian[3, 20, 2001]]]]
359.441

eHinduSiderealYear[] = 365.2564
365.256

eHinduTropicalYear[] = 365.2422
365.242

diff = N[5.5/24]
0.229167

eHinduEpoch[] = Julian[3, 20, 285]
Julian[3, 20, 285]

eHinduSolarLongitude[date_] :=
 SolarLongitude[JDFromMoment[ToFixed[date]]] –
SolarLongitude[Floor[(JDFromMoment[ToFixed[date]] -JDFromMoment[ToFixed[eHinduEpoch[] - diff]) /
 eHinduSiderealYear[]] * eHinduSiderealYear[] + JDFromMoment[ToFixed[eHinduEpoch[]] - diff]] -
   ((JDFromMoment[ToFixed[date]] –
(Floor[(JDFromMoment[ToFixed[date]] - JDFromMoment[ToFixed[eHinduEpoch[]] - diff]) / eHinduSiderealYear[]] *
eHinduSiderealYear[] + JDFromMoment[ToFixed[eHinduEpoch[]] - diff])) /
      eHinduTropicalYear[]) * 50.2/3600

eHinduSolarLongitude[Julian[3, 20, 285]]
0.223921

Calendrica`Private`HinduSolarLongitude[HinduDayCount[ToFixed[Julian[3, 20, 285]]]] / 60
2.95308
```

```
eHinduSolarLongitude[Gregorian[3, 20, 2001]]
336.549

Calendrica`Private`HinduSolarLongitude[HinduDayCount[ToFixed[Gregorian[3, 20, 2001]]]] / 60
335.326

eKYHinduEpoch[] = Gregorian[1, 23, -3101]
Gregorian[1, 23, -3101]

eKYHinduSiderealYear = 365.258756
365.259

eKYHinduSolarLongitude[date_] :=
 SolarLongitude[JDFromMoment[ToFixed[date]]] -
  SolarLongitude[Floor[(JDFromMoment[ToFixed[date]] - JDFromMoment[ToFixed[eKYHinduEpoch[]] - diff]) /
       eKYHinduSiderealYear]* eKYHinduSiderealYear
    + JDFromMoment[ToFixed[eKYHinduEpoch[]] - diff]] -
  ((JDFromMoment[ToFixed[date]] - (Floor[(JDFromMoment[ToFixed[date]] -
              JDFromMoment[ToFixed[eKYHinduEpoch[]] - diff])/
         eKYHinduSiderealYear] * eKYHinduSiderealYear + JDFromMoment[ToFixed[eKYHinduEpoch[]] diff])) /
     eHinduTropicalYear[]) * 50.2/3600

eKYHinduSolarLongitude[Gregorian[1, 23, -3101]]
0.223466

Calendrica`Private`HinduSolarLongitude[HinduDayCount[ToFixed[Gregorian[1, 23, -3101]]]] / 60
2.1209

eKYHinduSolarLongitude[Gregorian[3, 20, 2001]]
333.426

Calendrica`Private`HinduSolarLongitude[HinduDayCount[ToFixed[Gregorian[3, 20, 2001]]]] / 60
335.326
```

5. References:

1. S.K. Chatterjee, *Indian Calendric System*, Publications Division, Ministry of Information and Broadcasting, Government of India, 1998
2. Nachum Dershowitz and Edward M. Reingold, *Calendrical Calculations*, Cambridge University Press, 1997
3. Leow Choon Lian, *The Indian Calendar*, Honours Year Project 2000-2001, Department of Mathematics, National University of Singapore.
4. Helmer Aslaksen, *The Mathematics of the Chinese Calendar*, available at http://www.math.nus.edu.sg/aslaksen

6. Acknowledgements:

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