

# The Chinese Calendar of The Later Han Period

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## ABSTRACT

The purpose of our project is to explain the Chinese calendar of the Later Han period, *Si Fen Li* ( ), and reformulate the formulas of the calendar to forms that are suitable for computer implementation. The *Hou Han Shu* ( ) does not give explanations for the formulas of the calendar. In our report, we discuss and explain some of the formulas for the calendar as described in [1]. In addition, we determine the pattern of big/small months and the distribution of leap years. To our knowledge this has never been discussed before.

## 1. INTRODUCTION

### 1.1 The Chinese Calendar

The Chinese calendar has been used for more than four millenniums. It is a combination of two calendars, a solar calendar and a lunisolar calendar; therefore, its calculations are based on calculations of the positions of the sun and moon. There are two definitions of year in Chinese calendar, the *sui* (the solstice year from one winter solstice to the next) and the *nian* (the Chinese year from one Chinese New Year to the next).

The modern Chinese calendar has the following rules:

- i) A Chinese year can contain 12 months or 13 months
- ii) Winter solstice falls in month 11.
- iii) The day on which a new moon occurs is the first day of the new month. Hence a month has 29 or 30 days.
- iv) Every two or three years has a leap month.
- v) A leap month is the first month in a leap *sui* that has no *zhong qi* ( ). It takes the same number as the previous month.

### 1.2 The Si Fen Li

The Later Han *Shi Fen Li* successfully replaced the *Tai Chu Li* in 85A.D. and it was used until 263A.D., for a total of 179 years.

The *Si Fen Li* has the following rules:

- i) Length of a year =  $365 \frac{1}{4}$  days<sup>2</sup>

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- ii) 7 leap months in 19 years (19 years with a total of 235 months).
- iii) This calendar used mean values for the motion of the sun and the moon (*ping qi* and *ping shuo*) to determine the lengths of *jie qi* and month.
- iv) The year name for the initial year of the calendar (*li yuan*) is *geng chen* ( ).
- v) The *ji nian* for 1A.D. is 9282.

Some important cycles of the *Si Fen Li*:

- i) One *yuan* is 4560 years.
- ii) One *ji* is 1520 years (one third of a *yuan*).
- iii) One *bu* is 76 years (one twentieth of a *ji*).
- iv) One *zhang* is 19 years (one fourth of one *bu*).

## 2. EXPLANATIONS OF SOME FORMULAS

The following formulas are some calculation methods of the calendar. We will give a brief explanation below for each formula. Q and R in those formulas are the quotient and remainder respectively. The computations are based on the astronomical year that starts with the new moon on or before winter solstice.

(I) Calculation of *ru bu* (method to determine the position of a given year)

Formula 1:  $Ji\ nian = 4560 \times Q_1 + R_1$

$Q_1 + 1$  is the number of *yuan* ( ) of the year since one *yuan* has 4560 years and we count *yuan* from 1 but  $Q_1$  starts at 0.

Formula 2:  $R_1 = 1520 \times Q_2 + R_2$

$Q_2 + 1$  is the number of *ji* ( ) of the year since one *ji* has 1520 years and we count *ji* from 1 but  $Q_2$  starts at 0.

Formula 3:  $R_2 = 76 \times Q_3 + R_3$

$Q_3 + 1$  is the number of *bu* ( ) of the year since one *bu* has 76 years and we count *bu* from 1 but  $Q_3$  starts at 0.  $R_3$  is the *ru bu* year of the current *bu*. The period for a *ru bu* year is from the winter solstice of the previous Chinese year (month 1 to month 12) to the winter solstice of the next Chinese year.

(II) Calculation of *tian zheng* (method to determine whether the *ru bu* year is leap year)

Formula 4:  $(R_3 - 1) \times 235 = 19 \times Q_4 + R_4$

235 is the number of months in a *zhang*,  $Q_4$  is the completed months from the beginning of the current *bu* until the winter solstice of the previous year (Chinese year, not *ru bu* year).  $R_4$  is the *run yu* of the year.

(III) Calculation of the position of leap month (approximate position)

Formula 5:  $(19 - R_4) \times 12 = 7 \times Q_5 + R_5$

If  $R_5 < 4$ , then the  $(Q_5 + 1)$ -th month of the *ru bu* year is a leap month. The leap month is the  $(Q_5 + 2)$ -th month of the *ru bu* year if  $R_5 \geq 4$ .

By a simple calculation, we can see that the interval between a *zhong qi* and the new moon after it will be  $\frac{3409}{3760}$  days longer than the next such interval. After a few months, a month will not contain *zhong qi* and it is called leap month (this only happens in a leap *ru bu* year). Its number follows the number if the month before.

#### (IV) Calculation of the length of a month

Formula 6:  $Q_4 \times 27759 = 940 \times Q_6 + R_6$

If  $R_6 \geq 441$ , then the month is a big month (30 days); otherwise, it is a small month (29 days).

We know that the average length of a month in *Si Fen Li* is  $29\frac{499}{940}$  days. The first month of a *bu* has length  $29\frac{499}{940}$  days, which is less than 1, so the month is a small month. The next month has length  $30\frac{58}{940}$  days, so it is a big month.  $R_6$  is actually the numerator of the month before the winter solstice of the previous year, if it is more than 440, then the month on which the winter solstice falls in will have length more than 30 and hence it's a big month.

Formula 7:  $R_6 + 499 = 940 \times Q_7 + R_7$

If  $R_7 \geq 441$ , then the next month is a big month

The explanation for this formula is similar to the previous formula.

#### (V) Calculation of the 24 *jie qi* of the year (method to find the day name of each *jie qi*)

Formula 8:  $(R_3 - 1) \times 168 = 32 \times Q_8 + R_8$

The total number of days before the winter solstice of the previous year is  $(365 \frac{1}{4}) \times (R_3 - 1)$ . Because of the sixty cycle, we use  $(5 \frac{1}{4}) \times (R_3 - 1)$  ( $360 = 6 \times 60$ ) to determine the day name for the winter solstice.  $Q_8$  is the integral part of  $(5 \frac{1}{4}) \times (R_3 - 1)$ .

Formula 9:  $Q_8 = 60 \times Q_9 + R_9$

$R_9$  is the remainder of 60 divides  $(365 \frac{1}{4}) \times (R_3 - 1)$ , so the number for day name of the winter solstice is  $R_9$  + the number for the first-day name of the current *bu*.

Formula 10:  $R_8 + 7 = 32 \times Q_{10} + R_{10}$

The interval between two adjacent *jie qi* is  $15 \frac{7}{32}$ , therefore its fraction is  $7/32$ . The  $R_8$  in formula 8 is the numerator of the fraction for the winter solstice and the  $R_{10}$  is the numerator of the fraction for the next *jie qi*.

Formula 11:  $R_9 + 15 + Q_{10} = 60 \times Q_{11} + R_{11}$

The day-name number of the next *jie qi* is the quotient of 60 divides  $15 + Q_{10}$  + the number for day name of the winter solstice.

### **3. REFORMULATED METHODS AND ANALYSIS**

The formulas of the *Si Fen Li* stated in [1] are quite troublesome. By ignoring the 60-daays cycle of the Chinese calendar, we have reformulated some methods to determine date and time for full moon, length of a month and *jie qi*.

We use term “Accumulated Days” to help our calculation. It is the total number of days since the beginning of the current *bu* cycle. It is a fractional number.

Let  $AcD$  denote by “Accumulated Days” and  $\lfloor n \rfloor$  denote by greatest integral number smaller than  $n$ .

**(I) Calculation of date & time of full moon and length of a month**

$AcD$  for the full moon of  $Y$  month in  $X$  *ru bu* year:

$$AcD_1 = (\lfloor (X-1) \times 12 \frac{7}{19} \rfloor + (Y-1) + \frac{1}{2}) \times 29 \frac{499}{940}$$

$AcD$  for the new moon of  $Y$  month in  $X$  *ru bu* year:

$$AcD_2 = (\lfloor (X-1) \times 12 \frac{7}{19} \rfloor + (Y-1)) \times 29 \frac{499}{940}$$

**Date of the full moon of  $Y$  month in  $X$  *ru bu* year =  $AcD_1 - AcD_2 + 1$**

**Time of the full moon of  $Y$  month in  $X$  *ru bu* year =  $(AcD_1 - \bar{e} AcD_1 \hat{u}) \cdot 24$  hours**

$AcD$  for the new moon of  $(Y+1)$  month in  $X$  *ru bu* year:

$$AcD_3 = (\lfloor (X-1) \times 12 \frac{7}{19} \rfloor + (Y)) \times 29 \frac{499}{940}$$

**Length of  $Y$  month in  $X$  *ru bu* year =  $\bar{e} AcD_3 \hat{u} - \bar{e} AcD_2 \hat{u}$**

**(II) Calculation of date & time of *Jie qi***

$AcD$  for the  $Y$  *zhong qi* in  $X$  *ru bu* year:

$$AcD_4 = ((X-1) \times 12 + (Y-1)) \times 30 \frac{7}{16}$$

$AcD$  for the new moon just before  $Y$  *zhong qi* in  $X$  *ru bu* year:

$$AcD_5 = \lfloor AcD_4 \div 29 \frac{499}{940} \rfloor \times 29 \frac{499}{940}$$

**Date of the  $Y$  *zhong qi* in  $X$  *ru bu* year =  $\bar{e} AcD_4 \hat{u} - \bar{e} AcD_5 \hat{u} + 1$**

**Time of the  $Y$  *zhong qi* in  $X$  *ru bu* year =  $(AcD_4 - \bar{e} AcD_4 \hat{u}) \cdot 24$  hours**

The occurrence of the odd *jie qi* is in between two adjacent *zhong qi*.

**(III) Determination of leap month**

$AcD$  for the new moon of  $Y$  month in  $X$  *ru bu* year:  $AcD_6 = AcD_1$

$AcD$  for the new moon of  $(Y+1)$  month in  $X$  *ru bu* year:  $AcD_7 = AcD_2$

If  $\lfloor AcD_6 \rfloor = \lfloor AcD_7 \rfloor$  and  $AcD_6 \neq \lfloor AcD_6 \rfloor$ ,  $Y$  month in  $X$  year is a leap month.

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