

The Mathematics of Sundials

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Abstract

What is a Sundial?

A sundial is a device which uses the position of the Sun to indicate time.

You Need to Know

1. The *celestial sphere* is an imaginary sphere surrounding the Earth.
2. *Zenith* is the point on the celestial sphere that is directly over our heads at a given time.
3. The *meridian* is the imaginary circle passing through the north and south points on our horizon and through the zenith.
4. The *north celestial pole* and the *south celestial pole* are the imaginary points in the sky directly above the geographic north and south poles, respectively.
5. The *celestial equator* is the imaginary circle around the sky directly above the Earth's equator.
6. The *ecliptic* is the imaginary path that the Sun follows on the celestial sphere over the course of a year.
7. The *vernal (spring)* and *autumnal (fall) equinoxes* are the two points where the ecliptic and the celestial equator intersect.
8. The *summer* and *winter solstices* are the two points where the Sun's angular distance from the celestial equator is at a maximum magnitude of 23.5° .

Celestial Coordinate Systems

1. The Altitude-Azimuth System

The *altitude* of a star is how many degrees above the horizon it is (anywhere from 0° to 90°).

The *azimuth* of a star is how many degrees along the horizon it is.

2. The Longitude-Latitude System

The lines on a map of the Earth that run east-west parallel to the equator are lines of latitude and when projected onto the sky, they become *lines of declination*.

Parts of a Sundial

1. The *gnomon* is usually a rod or a triangular piece of metal or wood on a sundial.
2. The *style* is the sloping edge of the gnomon.
3. The *hour lines* are the numbered time lines that the shadow falls along.
4. The *nodus* is a "marker" along the gnomon to get an exact point on the shadow.
5. *Dial furniture* are the markers other than the hour lines on the dial plate. It is there to provide other information, such as the date and declination of the Sun.

Background knowledge

The gnomons of most sundials have to be placed parallel to the Earth's axis for easy calibration of the hour lines because when the gnomon is at an angle to the Earth's axis,

1. The shadow cast by the gnomon at a given hour points in different directions, depending on the seasons.
2. The angle covered by the shadow during a certain time interval depends on the seasons.

Different Classification of Hours

In the course of history, classification of time has changed several times.

1. Unequal or Temporary Hours

The time from sunrise until sunset, as well as the night arc, was divided into twelve equally long parts. The "hours" of any one day were equal, but the "hours" of the winter were short and the "hours" of summer long.

2. Equinoctial Hours

The time beginning with the passing of the low meridian, about midnight, until the passing of the next low meridian, was divided into 24 hours of equal length.

3. Italian Hours

Counting of the 24 equal hours began at *sunset*.

4. Babylonian Hours

Counting of the 24 equal hours began at *sunrise*.

Equation of Time – Analemma

The *equation of time* is the difference in time between what your watch reads and the position of the Sun.

The *analemma* is the path that the Sun takes if we mark the position of the Sun in the sky at the same time everyday, say sometime around noon.

There are two independent reasons why the Sun takes this strange path:

1. The Earth is tilted on its axis 23.5° in relation to the plane of its orbit around the sun.
2. The Earth does not orbit the Sun in a circle, but in an ellipse.

It is simply the sum of these two effects that causes the analemma.

Different Types of Sundials

There are two ways to classify sundials.

The first way is to group sundials into three main classes:

1. Sundials having gnomon parallel to Earth's axis
2. Sundials depending on altitude of the Sun
3. Sundials depending on azimuth of the Sun

The second way is to group sundials into two main classes:

1. Pole-style sundials
2. Nodal sundials

A *pole-style sundial* has a gnomon which casts a line-shaped shadow on a set of hour lines. *Equatorial, horizontal and vertical sundials* belong to this class.

For a *nodal sundial*, the time is read from the shadow of a fixed point or node.

Equatorial Sundials

The dial plate is in the plane of the celestial equator. They can be classified into two types:

1. Equatorial disk dials
2. Armillary dials

Equatorial Disk Dials

This is the only dial that has faces on both upper and lower sides.

Armillary Dials

The armillary dial is an extension of the equatorial dial, with two rings representing the celestial equator and the meridian. A third ring is usually added to represent the horizon.

Horizontal Sundials

It is one of the most commonly used sundials because it can be used to tell the time whenever the Sun is shining. Some other types of sundials, such as vertical sundials, can only be used during restricted hours.

Vertical Sundials

Vertical sundials can be classified into two types:

1. Vertical direct sundials
2. Vertical declining sundials

Vertical direct sundials are dials that faced directly toward one of the cardinal points of the compass. They are categorized into four types:

1. Vertical direct north dial
2. Vertical direct south dial
3. Vertical direct east dial
4. Vertical direct west dial

Vertical declining sundials are dials that are attached to vertical walls that do not directly face north, south, east and west. They can be categorized into four types:

1. Southwest decliners
2. Southeast decliners
3. Northwest decliners
4. Northeast decliners

The gnomon is twisted out of the vertical in order for it to be parallel to the Earth's axis.

Altitude Sundials

Sundials depending on the altitude of the Sun are called *altitude sundials*.

The best well-known altitude sundial is the *shepherd's dial*. It is one of the simplest and most widely used portable dials.

Azimuth Sundial

Sundials depending on the azimuth of the Sun are called *azimuth sundials*.

The analemmatic dial is the only representative of the azimuth sundial. It is not commonly used because its style requires daily setting.

Portable sundials

Prior to the early seventeenth century, pocket watches were uncommon, expensive and unreliable. The traveler who wished to keep track of time was forced to rely on a portable sundial. The following are some of the more commonly used portable sundials:

1. Shepherd's dial
2. Tablet dial
3. Cubic dial
4. Universal ring dial
5. Perforated ring dial

Dial Furniture

Dial furniture are additional markers on a sundial to provide information other than to tell time. These information can be:

1. Time of year (Sign of Zodiac)
2. Month of year
3. Time of sunrise
4. Length of daylight hours
5. Elevation of the Sun above the horizon
6. Compass bearing of the Sun
7. Temporary hours

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