Calendars, Interpolation, Gnomons and Armillary Spheres in the Work of Guo Shoujing (1231-1314)

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Abstract

Introduction

The purpose of this project is to discuss the mathematical development on the method of interpolation done by the famous Chinese astronomer, Guo Shoujing (郭守敬), which was recorded in his Shou Shi calendar (授時歷) compiled in the Yuan Dynasty (元朝, 1260-1368 A.D). This project also goes further to discuss the astronomical instruments: the gnomon and the armillary sphere, that were used by Guo Shoujing for astronomical observations, and the innovations he made to these instruments to improve on their accuracy. This project also touches on the developments of the Chinese calendar throughout its history, and clarifies some of the actual occurances of important developments on the Chinese calendar.

The Developments of the Chinese Calendar Throughout its History

Many developments had been done on the Chinese calendar throughout its history, and they were all largely due to the new discoveries on the various phenomenon that contradicted to the formal beliefs of Chinese astronomers. These discoveries, and the developments done on the Chinese calendar, can be summarised as follows:
1) Inconsistent motion of the moon: first discovered by Jia Kui (賈逵, ?-92 A.D) in the Eastern Han period (西漢朝, 25-200 A.D); first discussed in the Sifen calendar (后漢四分歷, 85 A.D) compiled in the Eastern Han Dynasty (25-200 A.D)
2) Inconsistent motion of the sun: first discovered by Zhang Zixin (張子信, 6th century A.D) during the North and South Dynastry (南北朝, 386-589 A.D); first mentioned in the Huang Ji calendar (皇极歷, 600 A.D) of Sui Dynasty (隋朝, 589-618 A.D); commonly accepted after the Da Yan calendar (大衍歷, 729

A.D) of the Tang Dynasty (唐朝, 618-907 A.D)
3) Ping shuo (平朔): introduced in the first Chinese calendars during the Shang Dynasty (商朝, 1523-1027 B.C)
4) Ding shuo (定朔): first proposed in the Yuan Jia calendar (元嘉歷, 445 A.D) of the North and South Dynasty (386-589 A.D); first used in the Wu Yin calendar (戊寅歷, 619 A.D) of the Tang Dynasty (618-907 A.D); commonly accepted after the Lin De calendar (麟德歷, 665 A.D) of the Tang Dynasty (618-907 A.D)
5) Ping qi (平气): first introduced in the Zhuan Xu calendar (顓頊歷, 174 B.C) of the Warring States (春秋戰國時代, 770-221 B.C)
6) Ding qi (定气): first introduced in the Huang Ji calendar (600 A.D) of Sui Dynasty (589-618 A.D); commonly accepted in calendars compiled in the Qing Dynasty (清朝, 1645 A.D onwards)
7) Precession (sui cha, 歲差): first discovered by Zu Chongzhi (祖衝之, 429-500 A.D) during the North and South Dynasty (386-589 A.D); first mentioned in the Da Ming calendar (大明歷, 510 A.D) of the North and South Dynasty (386-589 A.D)

The Method of Interpolation

The method of interpolation was discovered by the Chinese as early as the Sui Dynasty, and was mainly used to approximate the complex motion of the sun and the moon after their motions were found to be inconsistent. The Chinese astronomer, Liu Zhuo (劉焯, 544-610 A.D), came up with the *equal interval second difference method of interpolation*, recorded in his Huangji calendar as follows:
Suppose that the time interval between observations is l and the results of the observations are $f(l), f(2l), f(3l), \ldots, f(nl), \ldots$, then the predicted observational result at a certain time $nl+s$, where $0 < s < l$, is

$$f(nl+s) = f(nl) + \frac{s}{2l}(\triangle_1 + \triangle_2) + \frac{s}{l}(\triangle_1 - \triangle_2) - \frac{s^2}{2l^2}(\triangle_1 - \triangle_2),$$

where $\triangle_1 = f(nl+l) - f(nl)$, $\triangle_2 = f(nl+2l) - f(nl+l)$. Later during the Tang Dynasty, the Chinese astronomer, Yi Xing (一行, 683-727 A.D) independently came up with the *unequal interval difference interpolation* formula, given as follows:
Assume that L_1, L_2 are two unequal time intervals and at times $w, w+L_1, w+L_1+L_2$, the results of observations are $g(w), g(w+L_1), g(w+L_1+L_2)$, then the predicted result at time $w+s$ is

$$g(w+s) = g(w) + s\left(\frac{\triangle_1 + \triangle_2}{L_1 + L_2}\right) + s\left(\frac{\triangle_1}{L_1} - \frac{\triangle_2}{L_2}\right) - \frac{s^2}{L_1 + L_2}\left(\frac{\triangle_1}{L_1} - \frac{\triangle_2}{L_2}\right),$$

where $\triangle_1 = g(w + L_1) - g(w)$, $\triangle_2 = g(w + L_1 + L_2) - g(w + L_1)$. Both formulas are found to be equivalent to the Newton's divided difference method of interpolation, degree 2. During the Yuan Dynasty, Guo Shoujing was able to improve on both of the methods, by using the method of tabulation. Guo Shoujing used the so-called accumulated difference of the degrees moved by the sun with respect to the earth in a day, defined as the difference in the actual degree moved by the sun in a day and the degree moved by the sun in a day with its motion taken to be constant, and used the method of tabulation to come up with the following table recorded in his Shou Shi calendar:

No of days after the winter solstice	Accumulated Difference	First Difference	Second Difference	Third Difference
0	0			
		510.8569		
1	510.8569		-4.9386	
		505.9183		-0.0186
2	1016.7752		-4.9572	
		500.9611		-0.0186
3	1517.7363		-4.9758	
		495.9853		-0.0186
4	2013.7216		-4.9944	
		490.9909		
5	2504.7125			

Using the above table, Guo Shoujing was able to obtain the result of the accumulated difference n days after the winter solstice, and clearly, he had done this tabulations on the basis of a *third* degree formula of interpolation. The table contained the well-known recursive nature of the forward difference used in Newton's forward difference method of interpolation, which indicated that Guo Shoujing had grasped the concept of the method of interpolation.

Astronomical Tools Used by Guo Shoujing

Gnomon was an astronomical tool simple in its construction: it is a rod of a certain height placed vertically on flat ground. Chinese astronomers had made use of the length of its noon shadow to determine the time of the solstices. However, they faced the problems of the insignificant changes in the length of its noon shadows around the time of the solstices, and the indistinct shadow produced by a long gnomon, which brought about difficulties in the determination of the actual length of the gnomon's noon shadow. Guo

Shoujing was able to solve both of the problems by making use of a crossbar fixed at the top of his 40 feet gnomon, and a device called a *shadow definer* (影符) invented by him. The shadow definer made use of the principle of the pin-hole camera, which was able to produce a distinct shadow of the crossbar within a bright spot of light. This distinct shadow was then able to point out the actual reading for the length of the 40 feet gnomon's noon shadow on a measuring scale.

Armillary spheres are astronomical instruments that ancient Chinese astronomers used to determine the positions of stars with respect to certain celestrial coordinates. There are basically two types of armillary spheres: the equatorial armillary sphere and the ecliptic armillary sphere. Chinese astronomers had always been using the equatorial armillary sphere due to their loyality to the use of the equatorial coordinates. However, the sphere can be troublesome to use at times, so Guo Shoujing simplified it into a new observational tool called the *equatorial torquetum*. The equatorial armillary sphere has the various rings corresponding to the various great circles of the celestrial sphere arranged into a spherical structure, but the equatorial torquetum has the various rings rearranged into different positions for easy usage. Also, the equatorial torquetum has some new features like the pole determining circle to determine the moment of culmination of the pole star.

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