

Calendars in Singapore

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ACKNOWLEDGMENTS

Taking up honours was never part of my options throughout my undergraduate years.

I was probably the only eager beaver who wanted to end my academic life and step into the working world as soon as I can. This project, among a few other things, caused me to change my mind. I would like to thank my supervisor, Assistant Professor Helmer Aslaksen, for proposing such an interesting project, thereby giving me the opportunity to work under him. I appreciate his continued patience in helping me in the completion of this thesis.

I am grateful to my fellow classmates for their supportive sharing of ideas and feedback. Special thanks to Shuqing, Tianzhen, Shirlyn, Bingquan and Jho, all of whom, despite their own heavy commitments, were always there to encourage me in times when I felt like giving up. Their friendship has made honours year a memorable one for me.

Finally, but most importantly, I would like to thank my parents for being so understanding of my hectic school life and forgiving me for spending so little time with the family during the past few months. And to my sister, for offering her help in many ways.

SUMMARY

The calendar is an important tool used in time keeping. Without calendars, we would not be able to plan into the future. Without calendars, I probably would not know when this thesis has to be handed in. Since ancient times, different cultures or races devised their own rules and developed their own types of calendars. The calendars that we use today are the result of numerous reformations of those rules. However, few people would actually make the effort to delve into those rules. For those interested, to start reading on the wide selection of books, articles or journals etc can be quite intimidating for the beginners. This thesis hopes to serve as a guide for them. Chapter 1, as the title suggests (no offence intended), is an introductory course for anyone who is interested in calendars but have little or no knowledge whatsoever about the topic. As such, I have tried to keep the content of this chapter simple and not too technical. The chapter starts off by introducing some basic concepts of astronomy and calendrical theory. We briefly discuss the motions of the sun, moon and earth as well as give the definitions of the day, month and year. Next, we take a look into how the different calendars are classified into the solar, lunar or lunisolar calendars. Finally, we describe the rules and touch on the holidays of the four main calendars used in Singapore: the Gregorian, the Muslim, the Chinese and the Indian calendars. Chapter 2 takes the reader into the more technical aspects of the topics covered in Chapter 1. In particular, the Gregorian and the Chinese calendars will be covered in greater detail here. We also present a number of functions for calculations for the Chinese calendar.

STATEMENT OF AUTHOR'S
CONTRIBUTIONS

As much as I hate to mention it, it is in my opinion that I have not made any substantially original contribution in this thesis. However, I have, in my best effort, tried to present the content in such a way that my peers would find it easy to understand. The other thing that might be worth mentioning is that I was able to learn and do some simple programming using Mathematica. I wrote the functions included in the appendix, particularly the ones mentioned below, with the help of some brief outline given by my supervisor:

lengthOfChineseYear

lengthOfMonthsBetween

bigSmallConsecutiveBetween

stringsOfMonthsBetween

dayOfChineseFullMoonBetween

doubleSpringDoubleRain

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CHAPTER 1

CALENDARS FOR DUMMIES

1.1 Very Simple Astronomy

Calendars are based on the units of the day, month and year. These units arise due to the rotation of the earth on its axis, the revolution of the moon around the earth and the revolution of the earth around the sun respectively.

The earth & the sun

The earth performs two motions simultaneously. It rotates counterclockwise about an axis that is tilted 23.5° . To an observer on earth, the sun is seen as rising from the east and setting in the west. The earth also revolves counterclockwise around the sun in an elliptical orbit. The plane of this orbit is called the plane of the ecliptic. This is shown in the diagram below:

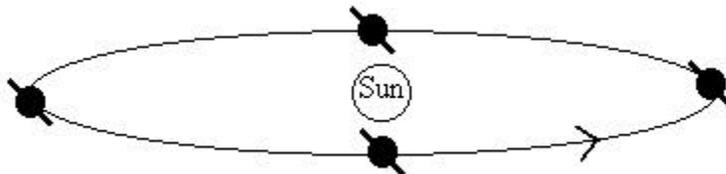


DIAGRAM 1: THE ECLIPTIC PLANE

The two positions where the projection of the earth's axis points directly towards the sun are the June and December solstices. On the other hand, the two positions where the radial line from the sun to the earth is perpendicular to the axis are the March and

September equinoxes. The point where the earth is closest to the sun is called perihelion. This takes place in early January. The point where the earth is farthest from the sun is called aphelion and this happens in early July.

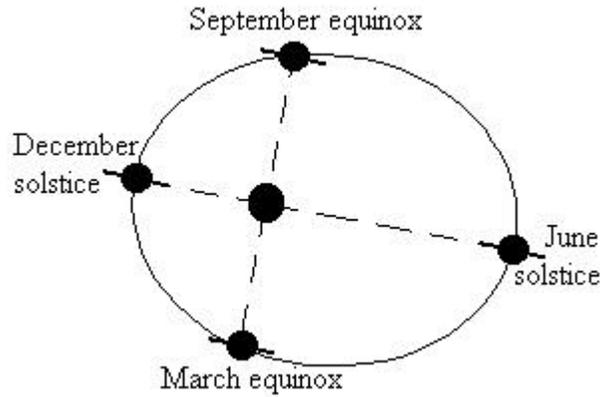


DIAGRAM 2: SOLSTICES AND EQUINOXES

The moon

The moon, our nearest neighbour, shines by reflected sunlight. There are two motions of the moon of interest to us. Because of the earth's rotation, the moon rises from the east and sets in the west like the sun. At the same time, the moon also revolves counterclockwise around the earth. This revolution causes us to see different phases of the moon. When the moon is directly between the sun and the earth (conjunction), we call this the new moon. At that time, the moon rises and sets approximately at the same time as the sun. During the few days after conjunction, the waxing crescent appears but it cannot be seen during the day due to the brightness of the sun. As the moon moves counterclockwise in its orbit around the earth, it begins to rise (and set) after the sun does and we gradually see more of the moon. Full moon occurs when the moon is aligned with the sun and the earth but on the opposite side of the earth (opposition). At that time, the moon rises at sunset and sets at sunrise.

The second motion of the moon is its revolution around the sun. The moon does so together with the earth as a system.

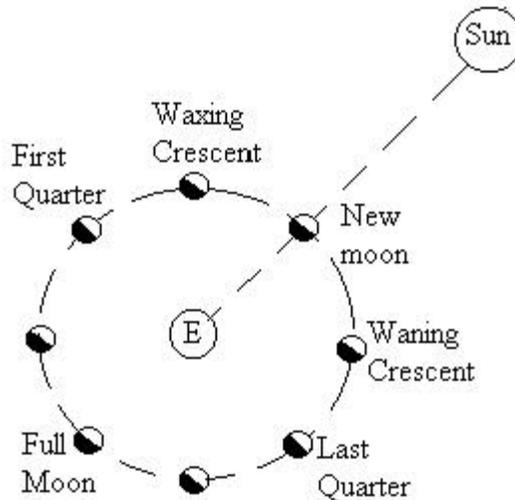


DIAGRAM 3: PHASES OF THE MOON

The day, month and year

The earth's rotation causes the sun, moon and the stars to move across the sky from east to west in the course of a day. The mean *solar* day is the time period from one noon transit of the sun to the next. It is of length 24 hours.

The *sidereal* month is the time it takes for the moon to complete one revolution around the earth. Its length is about 27d7h43m11.5s (27.3217 days). However, during this time, the earth-moon system has also revolved about 27° around the sun. Hence, with respect to the sun, the moon has not completed one revolution. The time interval between two successive new moons is the *synodic* month, of length 29d12h44m2.8s (29.5306 days). This is a mean value. The actual length of successive new moons can vary by up to 7 hours because of complicated interactions between the earth, the moon and the sun.

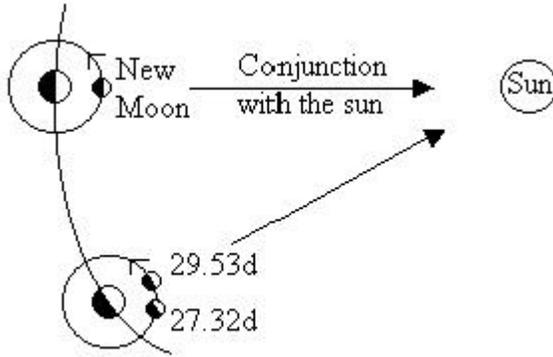


DIAGRAM 4: SIDEREAL AND SYNODIC PERIODS OF THE MOON

There are two ways to measure the year. The *tropical* year is the time interval between two successive March equinoxes. Its length is about 365d5h48m46s (365.2422 days). However, the position of the equinoxes and solstices are not fixed in the orbit. The sun and the moon's pull on the earth causes the earth's rotational axis to rotate clockwise in a slow circle with a period of about 25 800 years. This phenomenon, known as precession of the equinoxes, causes the position of the March equinox to move backwards along the ecliptic. As such, the tropical year is about 20 minutes shorter than the *sidereal* year, which is the actual time it takes for the earth to revolve once around the sun relative to the stars. The sidereal year is of length about 365d6h9m10s (365.2564days).



Diagram 5: Precession

The celestial sphere

The model introduced at the beginning of the chapter is known as the heliocentric model since it has the sun as the center. Ancient civilizations, however, used the geocentric model. This model views the earth as being in the middle of a large sphere called the celestial sphere as shown below:

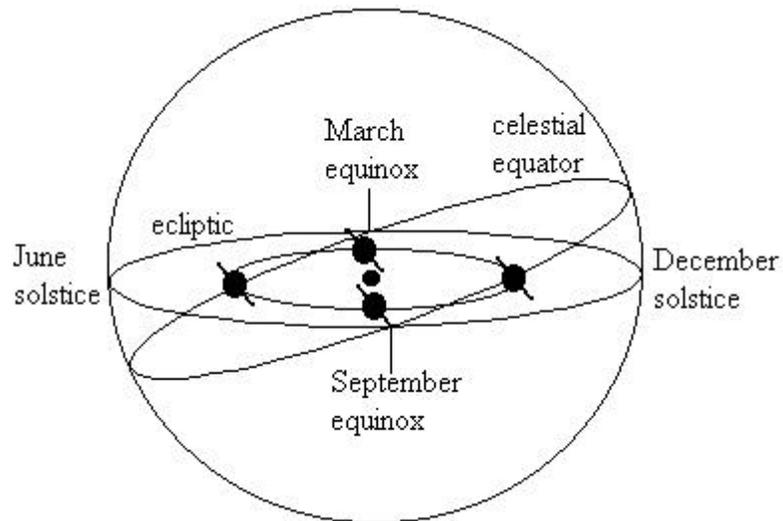


DIAGRAM 6: THE CELESTIAL SPHERE

The celestial equator is an extension of the earth's equator. The sun's path across the celestial sphere is called the ecliptic. The plane of this ecliptic makes an angle of 23.5° with the celestial equator. The points where the ecliptic intersects the celestial equator are the equinoxes while the points where the ecliptic and the celestial equator are farthest apart are the solstices. We can adopt either model for our calendar discussion.

1.2 Basic Calendrical Theory

The natural units of the calendar are the day, month and year. These three periods are incommensurable, meaning none of them is an integral multiple of any of the others.

Ancient astronomers tried to find such relations, and came up with different ways to structure days into larger units of weeks, months, years and cycle of years, resulting in different types of calendars. These calendars approximate the tropical year, the lunar month or both.

Classification of calendars

There are three ways to classify calendars.

1. Solar Calendar

The basic unit of solar calendars is the day. Solar calendars approximate the tropical year using days. Most solar calendars are divided into months but these months ignore the lunar events. The Gregorian calendar is a solar calendar. A common Gregorian year consists of 365 days. Every fourth year is a leap year consisting of 366 days unless it is a century year that is not divisible by 400.

2. Lunar Calendar

The basic unit of lunar calendars is the lunar month. Lunar calendars ignore the sun and the tropical year and hence do not keep in line with the seasons. The Muslim calendar is a lunar calendar. Each year consists of 12 lunar months. Each month has an average length of about 29.5 days. This amounts to about $12 \times 29.5 = 354$ days a year, about 11 days short of the tropical year.

3. Lunisolar Calendar

Like the lunar calendar, the basic unit of lunisolar calendars is the lunar month. Lunisolar calendars approximate the tropical year with years consisting of a whole number of months. The Chinese calendar is a lunisolar calendar. It is based on the moon and consists of 12 months that begin at the new moon. A 13th month is

occasionally inserted by fixed rules so that the calendar stays in line with the seasons.

There is an alternative classification of calendars. *Arithmetical* calendars are based on arithmetical rules and formulae. An example is the Gregorian calendar. On the other hand, calendars that are based on astronomical calculations are known as *astronomical* calendars. Examples are the Chinese, Indian and Islamic calendars.

We will study the four main calendars used in Singapore: the Gregorian, the Chinese, the Islamic and the Indian calendars. We will also touch on the public holidays in Singapore. Secular holidays are holidays not related to any religion or traditional culture. In Singapore, the secular public holidays are New Year's Day on January 1st, Labour Day on May 1st and National Day on August 9^h. In addition, there are eight religious or traditional public holidays; two Christian, two Muslim, two Chinese and two Indian. We will start off our discussion with the calendar we are most familiar with, the Gregorian calendar.

1.3 Basics of the Gregorian calendar

The Gregorian calendar is used in most countries today. The day begins at midnight and there are 12 months in each year. The names and length of each month of the year are as follows:

(1) January	31 days	(7) July	31 days
(2) February	28{29} days	(8) August	31 days
(3) March	31 days	(9) September	30 days
(4) April	30 days	(10) October	31 days
(5) May	31 days	(11) November	30 days
(6) June	30 days	(12) December	31 days

A common year consists of 365 days while a leap year consists of 366 days, with the extra day being added to February. The length of a Gregorian year is determined by the following rule:

A year is a leap year if it is divisible by 4 and is not a century year; or
if it is a divisible by 400.

Hence, 1700, 1800 and 1900 for example are common years while 2000 is a leap year. This is the difference between the Gregorian calendar and its predecessor, the Julian calendar, in which all century years were leap years. The Julian calendar has a cycle of 4 years consisting of $4 \times 365 + 1 = 1461$ days, giving an average year of length 365.25 days. In comparison, the Gregorian calendar has a cycle of 400 years consisting of $400 \times 365 + 97 = 146\,097$ days. The average length of a year is hence 365.2425 days. This is a closer approximation of the tropical year, of length about 365.2422 days. Here, we are comparing with the modern definition of the tropical year, something that will be mentioned in Chapter 2.

Since the Gregorian calendar is an approximation to the tropical year, the solstices and the equinoxes will stay almost constant. Each common year is a bit shorter than the tropical year, so the March equinox, for example, will move forward a quarter day in the calendar for three years in a row. The leap year will then even it out, moving the equinox back by one day in the calendar. The equinox thus performs a “four step dance”: three small steps forward and one long step back. The old Julian calendar kept the rhythm but the Gregorian calendar will “miss a beat” three times every 400 years.

In Singapore, the two religious holidays based on the Gregorian calendar are Good Friday and Christmas Day. Christmas is fixed on December 25th. For Good Friday, we need to first determine Easter Sunday. Good Friday is then the Friday before Easter Sunday. Easter Sunday is the first Sunday after the first full moon occurring on or after the day of the March equinox. The Sunday *after* the full moon was specified intentionally to decrease the possibility of celebrating Easter on the same day as the Jewish Passover. It is difficult to compute the actual date of the March equinox and the time of the full moon. Instead, an approximate date is used. We will explain the details of computing Easter in Chapter 2.

1.4 Basics of the Muslim calendar

The Islamic calendar is a strictly lunar calendar. The day begins at sunset and each year consists of 12 lunar months with no leap months. The names of the 12 Islamic months are:

- | | |
|---------------------|------------------|
| (1) Muharram | (7) Rajab |
| (2) Safar | (8) Sha baan |
| (3) Rabi-ul-Awal | (9) Ramadhan |
| (4) Rabi Athani | (10) Shawwal |
| (5) Jamada Alula | (11) Thul Qi'dah |
| (6) Jamada Athaniah | (12) Thul Hijjah |

Since the Islamic calendar is not related to the tropical year, Islamic dates move about 11 days backward each year. Hence, Hari Raya Puasa falls about 11 days earlier each year. In addition, the Islamic year, being shorter, can be completely contained in the Gregorian year. This means that there can be two Hari Raya Pugas

in a Gregorian year. This happens in year 2000 with one Hari Raya Puasa occurring in early January and the next in late December.

Western sources often mention an arithmetical Islamic calendar in which each year consists of 12 months and the number of days in each month alternate between 29 and 30. An extra day is intercalated in the last month according to a fixed system. However, this calendar is not currently used in any Muslim community. Instead, an astronomical calendar based on lunar visibility is used. In the Islamic calendar, the first sighting of the new moon marks the beginning of the lunar month. Now, a new moon is normally not visible until it is more than 24 hours old. Hence, Islamic months usually start one or two days after the Chinese months. In addition, the instant when a new moon can first be observed depends on the latitude and the longitude of the observer. Bad weather conditions also contribute to uncertainty in the sightings. As such, it is not uncommon for festive celebrations to be held on different days by communities living in different areas of a country.

The two Islamic public holidays in Singapore are Hari Raya Puasa and Hari Raya Haji. Hari Raya Puasa falls on the first day of the 10th Islamic month while Hari Raya Haji falls on the 10th day of the 12th Islamic month.

1.5 Basics of the Chinese calendar

Introduction

The Chinese calendar is a lunisolar calendar with the day beginning at midnight. Lunar months form the basis of this calendar. There are 12 months in a common year. A 13th month is intercalated from time to time to keep in the calendar in line with the seasons.

The Chinese divide the ecliptic into 24 equal regions of 15° each. These regions form the 24 solar terms or *jié qì* 's (), an important principle required to explain some of the rules of the Chinese calendar. The even terms are called major solar terms or *zhōng qì* 's (), while the odd ones are called minor solar terms or *jié qì* 's (). Here, we see that the word *jié qì* is used either as a generic term for all 24 solar terms or to refer specifically to the odd ones. The names of the *jié qì* 's are given in Table 1. It is important to note that the given dates are only approximate as the Gregorian calendar is an approximation of the tropical year. The insertion of leap days and the fact that the Gregorian year is a little longer than the tropical year causes the dates to shift. The rules for the Chinese calendar have changed many times. In this section, we will highlight the rules for the current Chinese calendar that has been used since the last calendar reform in 1645.

Rule 1 - Calculations are based on the meridian 120° east.

Before 1929, the computations were based on the meridian in Beijing ($116^{\circ}25'$). But in 1928, China adopted the standard time zone based on 120° east.

The Chinese Month

Rule 2 - The day on which a new moon occurs is the first day of the new month.

According to this rule, Chinese months begin on the day of the new moon and end on the day before the next new moon. The length of each month is either 29 (short or “small” month,) or 30 days (long or “big” month,). There can be up to four long months or three short months in a row. In addition, if a *zhōng qì* and a new moon fall on the same day, the *zhōng qì* is considered as falling in the new month, even though it may have occurred earlier in the day than the new moon.

Index	Chinese name	English name	Solar Longitude	Approximate Gregorian date
J1	Lì chūn	Beginning of spring	315 ⁰	February 4
Z1	Yǔ shu	Rain water	330 ⁰	February 19
J2	Jǐng zh	Waking of insects	345 ⁰	March 6
Z2	Chūn fēn	March equinox	0 ⁰	March 21
J3	Qīng míng	Pure brightness	15 ⁰	April 5
Z3	Gǔ y	Grain rain	30 ⁰	April 20
J4	Lì xià	Beginning of summer	45 ⁰	May 6
Z4	Xiǎo mǎn	Grain full	60 ⁰	May 21
J5	Máng zhòng	Grain in ear	75 ⁰	June 6
Z5	Xià zhì	June solstice	90 ⁰	June 22
J6	Xiǎo sh	Slight heat	105 ⁰	July 7
Z6	Dà sh	Great heat	120 ⁰	July 23
J7	Lì qiū	Beginning of autumn	135 ⁰	August 8
Z7	Chè sh	Limit of heat	150 ⁰	August 23
J8	Bái lù	White dew	165 ⁰	September 8
Z8	Qiū fēn	September equinox	180 ⁰	September 23
J9	Hán lù	Cold dew	195 ⁰	October 8
Z9	Shuāng jiāng	Descent of frost	210 ⁰	October 24
J10	Lì dōng	Beginning of winter	225 ⁰	November 8
Z10	Xiǎo xu	Slight snow	240 ⁰	November 22
J11	Dà xu	Great snow	255 ⁰	December 7
Z11	Dōng zhì	December solstice	270 ⁰	December 22
J12	Xiǎo hán	Slight cold	285 ⁰	January 6
Z12	Dà hán	Great cold	300 ⁰	January 20

Table 1: The 24 *jié qì* 's

The Chinese Year

The Chinese define two types of years. The first, analogous to the definition of the tropical year, is the *sui* (). A *sui* is defined to be the solstice year from one December solstice to the next. On the other hand, a *nián* () is the Chinese year from one Chinese New Year to the next. Since a Chinese year can contain 12 or 13 months, each of 29 or 30 days, the length of a *nián* can be 353, 354, 355 days or 383, 384, 385 days in the case of leap years. It is important at this point to clarify some of the terminology that will surface in later sections. The Chinese year 2033 means the *nián* from Chinese New Year 2033 to Chinese New Year 2034, while the *sui* 2033 is the *sui* from December solstice 2032 to December solstice 2033.

A Chinese year is a leap year if it contains 13 months. For example, 2033 is a leap year since the *nián* 2033 contains 13 months. On the other hand, we have the following rule for the *sui*:

Rule 3 – A sui is a leap sui if it contains 12 complete months.

Naming the Months

Chinese months are numbered from one to twelve.

Rule 4 – The December solstice (dōng zhì) falls in the eleventh month.

The date of the December solstice can be computed and the month containing this day is the 11th month. In a normal *sui*, provided there are no months with two *zhōng qì*'s, there is exactly one *zhōng qì* in each month. In this case, the number of the month follows from the number of the *zhōng qì*. In the event of a leap *sui*, which has more months than a normal *sui*, there will be at least one month without any *zhōng qì*. In rare cases, there can even be one or more months with two *zhōng qì*'s and

hence at least two months without any *zhōng qì*. In this case, the following rule applies:

Rule 5 – In a leap suì, the first month that does not contain a zhōng qì is a leap month, rùn yuè (闰月)

A leap month duplicates the number of the preceding month. For example, if the leap month is after the fifth month, it is called the leap fifth month (闰五).

Chinese New Year

If there are no leap months after months 11 or 12, then Chinese New Year is the second new moon after the December solstice. Otherwise, Chinese New Year will fall on the third new moon after the December solstice. The possible dates of Chinese New Year are between January 21st and February 21st. As shown in Table 2 below, dates between January 22nd and February 19th are common, January 21st and February 20th are rare and February 21st is extremely rare. In Singapore, Chinese New Year is celebrated on the first two days of the first Chinese month.

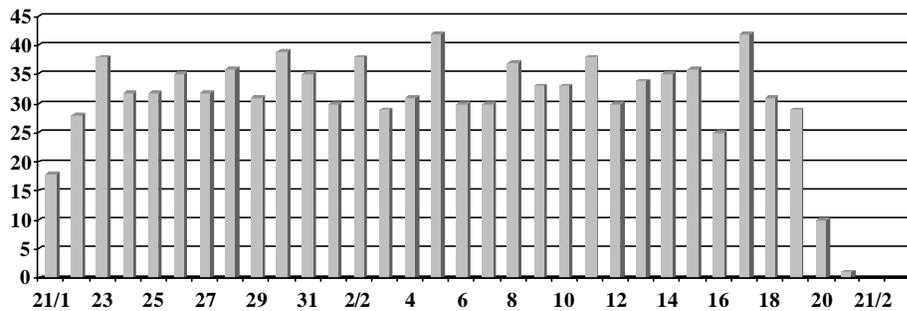


Table 2: Distribution of the dates of Chinese New Year between 1645 and 2644

1.6 Basics of the Indian calendar

Introduction

The Indian calendar is probably the most complicated calendar in common use in the world today. There are over 30 different calendars in use in India, both solar and lunisolar calendars. For civil purposes, some states in India use a solar calendar while some states use a lunisolar calendar. But all states use a lunisolar calendar for religious purposes. There are many regional variations, but we will follow the Tamil conventions. This means that we will not discuss the full-moon to full-moon calendars that are common in northern India.

The solar calendar

The solar calendar is based on the sidereal year. The ecliptic is divided into 12 equal regions of 30° each, which are known as *rasis*. This is similar to the *zhōng qì* 's used in the Chinese calendar. The names of the 12 *rasis* are:

- | | |
|---------------|---------------|
| (1) Mesha | (7) Tula |
| (2) Vrishabha | (8) Vrischika |
| (3) Mithuna | (9) Dhanus |
| (4) Karkata | (10) Makara |
| (5) Simha | (11) Kumbha |
| (6) Kanya | (12) Mina |

The entry of the sun into each *rasi* marks the start of a new month and the time of entry into a *rasi* is called the *samkranti*. The time it takes for the sun to move through one *rasi* is called the sidereal solar month. The number of days in each solar

month varies from 29 to 32. This is because the sun does not move at a constant speed along the ecliptic.

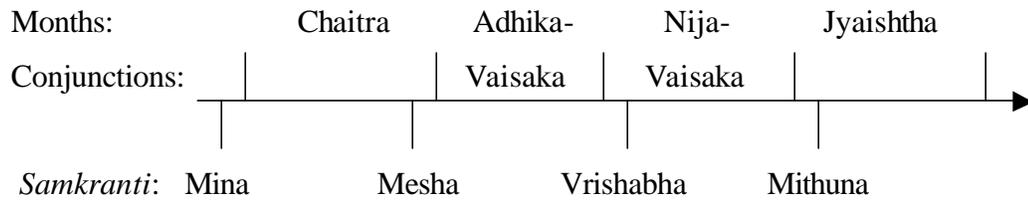
Each day begins at sunrise. According to Tamil conventions, the new month begins on the same day if the *samkranti* takes place before sunset. Otherwise, the month begins on the following day. The sidereal year begins on the first day after the *mesha-samkranti*.

The lunisolar calendar

Like the Chinese calendar, the Hindu lunisolar calendar contains 12 lunar months in a common year and a 13th month is sometimes intercalated to keep the calendar in line with the seasons. Like the solar calendar, the day begins with sunrise. The year starts with the beginning of the month *Chaitra*. Each month begins on the day of a new moon. This system is called the *amanta* system. The alternative is to use the *purnimanta* system in which the month is from one full moon to the next. The names of the 12 lunar months are:

- | | |
|-----------------|---------------|
| (1) Chaitra | (10) Pausha |
| (2) Vaisaka | (11) Magha |
| (3) Jyaishtha | (12) Phalguna |
| (4) Ashadha | |
| (5) Sravana | |
| (6) Bhadra | |
| (7) Asvina | |
| (8) Kartika | |
| (9) Margasirsha | |

The rule of intercalation of months in the Hindu calendar is similar to the Chinese system. In addition, months can be extracalated (skipped) in the Hindu calendar. The length of a solar month varies from about 29.318 to 31.644 days while the length of a lunar month varies from about 29.305 to 29.812 days. The variation can lead to two possibilities. A lunar month can be completely contained in a solar month. This means there is no *samkranti* in the lunar month. When this happens, this lunar month is a leap month and the following month is given the same name as the leap month. The prefix *adhika*, meaning “added” and *nija*, meaning “normal” distinguish the two months. For example, we can have:

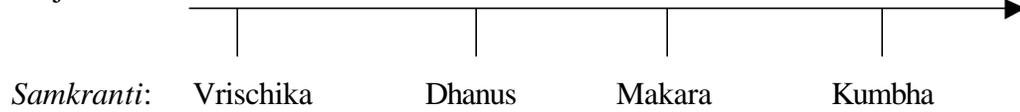


The Indian leap-month system is different from the Chinese system in the following way. In both systems, the month without any *samkranti* or *zhōng qì* is a leap month. However, in the Hindu calendar, the month after the leap month is the “normal” month and it shares the same name as the leap month. For the Chinese calendar, the leap month takes the same number as the month preceding it.

In the rare event, a solar month can be completely contained in a lunar month. This means there is no new moon in the solar month. This normally occurs near perihelion, when the sun moves faster round the ecliptic. When this happens, the next lunar month is extracalated or skipped (called *kshaya*). For example, the following can happen:



Conjunctions:



Recall that in the Chinese calendar, the leap month is the first month (of a leap *sui*) without a *zhong qi*. We will see in Chapter 2 that the other months of a leap *sui* without any *zhong qi* are called “fake” leap months and these are treated as normal months in the naming of the Chinese months. This means it is possible to intercalate only one month in the Chinese system. In the Indian calendar, any month without a *samkranti* is a leap month. Hence it is possible to intercalate more than one month which may result in “too many months” in a year. The Hindu system takes care of this by having skipped months to compensate for the leap months.

In the Hindu calendar, the days in a lunar month are also named. The *tithi* or the lunar day is the time taken for the angular separation between the sun and the moon to change by 12° . There is a total of 30 *tithis*: from *S1* to *S15* and from *K1* to *K14* and the last being *K30*. Each day of a lunar month is assigned the ordinal number of the *tithi* at the time of sunrise. As the true motions of the sun and the moon are not uniform, both the *tithi* and the solar day vary in length. Therefore, as in the case of intercalation and extracalation of months, a *tithi* number may be repeated or skipped. It is sometimes possible for a *tithi* to begin and end within two consecutive sunrises. This means there is no sunrise in the *tithi*. Such a *tithi* is known as a *ksaya tithi* and the ordinal number of this *tithi* is omitted. On the other hand, it is also possible to have two sunrises during a *tithi*. Such a *tithi* is known as an *adhika tithi* and the

ordinal number of this *tithi* is repeated. For example, we can have the sequence of days of the month to be: 1, 2, 4, 5, etc or 1, 2, 3, 4, 4(*adhika*), 5, etc.

A lunar month is of an average length of about 29.53 days. This is usually shorter than a solar month. Hence, it is quite common to have leap months. However, since the synodic period is subject to large variations, it is also possible to have skipped months but this, in comparison, is a rare event. For the case of days, since the average length of a *tithi* is about 23.62 hours, which is shorter than a solar day, it is quite common to have skipped days. However, the actual length of a *tithi* varies from about 19.983 to 26.783 hours. Hence, in the rare event when the *tithi* is longer than the solar day, there will be leap days.

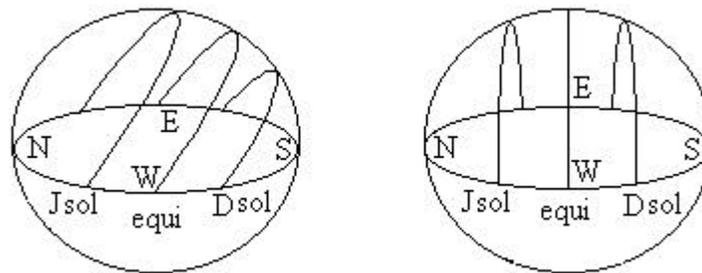
The *tithi* plays an important part in the determination of Hindu religious festivals. It is one of the five main items listed in the *pancanga*, the Hindu almanac. Deepavali, the “Festival of Lights” falls on the last day of the lunar month *Asvina*. Vesak Day, one of the major Buddhist holidays, is celebrated by many Hindus and Chinese in Singapore. Traditionally, it was celebrated on the 8th or 15th day of the fourth month. However, since the 1950’s, the Singapore Buddhist Federation has followed the guidelines of the World Buddhist Fellowship Council and celebrates it on the first full moon in May.

CHAPTER 2

CALENDARS FOR THE INQUISITIVE MIND

2.1 Some Technical Aspects of Astronomy *The Seasons*

As mentioned in Chapter 1, the earth's axis is tilted at an angle of 23.5° . The tilting of the axis and the movement of the earth around the sun (or equivalently, the sun moving along the ecliptic) give rise to the seasons. The equinoxes and the solstices are called the seasonal markers. At the equinoxes, the sun rises due east and sets due west, and the day and night are almost equal in length. After the March equinox, the sun moves north until the June solstice. The days lengthen and the sun rises further and further north until it "stands still" (from which the word solstice has been derived). In the northern hemisphere at the June solstice, the sun rises at its most northerly position and the day is the longest. The sun then turns southward. The days shorten and the sun rises further and further south.



Beijing

Singapore

Diagram 7: The sun's path across the sky

Kepler's Laws

Kepler's first two laws of planetary motion are:

- 1. The orbit of the planet about the sun is an ellipse with the sun at a focus.*
- 2. The radius vector (the line that joins a planet to the Sun) sweeps out equal areas in equal intervals of time.*

The first law accounts for the variation in the distance of the earth from the sun. The second accounts for the variation in the earth's velocity along its orbit. It follows from Kepler's Second Law that the earth moves faster along the orbit near perihelion.

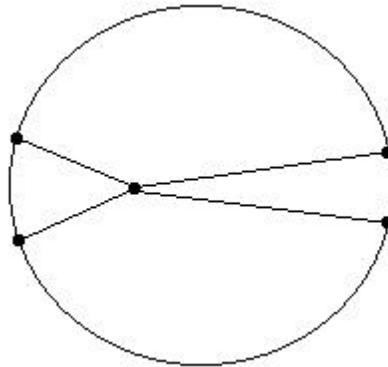


Diagram 8: Kepler's second law

The Metonic cycle

As mentioned before, a lunar month has a mean length of 29.53 days. 235 lunar months equals about $235 \times 29.53 = 6939.6884$ days while 19 tropical years are about $19 \times 365.2422 = 6939.6018$ days. The difference is about two hours. This is called the Metonic cycle after the Greek astronomer Meton who discovered it in 432 B.C.E. Since $235 = (12 \times 12) + (7 \times 13)$ months, we can use the Metonic cycle to obtain a calendar with 12 years of 12 months and seven leap years of 13 months. The

Metonic cycle was known in China by at least 500 B.C.E. and was called the zhāng () cycle. The Metonic cycle is used in the ecclesiastical calculation of Easter.

2.2 More About the Gregorian calendar

The tropical year

The Gregorian calendar is used to approximate the tropical year. However, there is some confusion over the definition of the tropical year. In the past, the tropical year was defined as the time interval between two successive March equinoxes. For simplicity, this is the definition given in Chapter 1. Table 3 below shows that the time interval between two successive seasonal markers is dependent on the choice of the seasonal marker. Hence, if the year is defined as the time interval between two successive seasonal markers, then different starting points adopted for the definition of the year will give different lengths of the year.

	Mean time interval (days)	
	YEAR 0	YEAR 2000
Between two March equinoxes	365.242137	365.242374
Between two June solstices	365.241726	365.241626
Between two September equinoxes	365.242496	365.242018
Between two December solstices	365.242883	365.242740

Table 3: Mean time interval between solstices and equinoxes

The variation in Table 3 is caused by precession and Kepler's Second Law. Recall that the earth's axis rotates clockwise in a circle by a small amount every year. If we think in terms of the December solstice, this rotation causes the axis to point directly towards the sun a bit earlier than a year ago. This means the December solstice will

happen a bit earlier than a year ago (regression along the ecliptic). The same argument holds for the March equinox except that now, the axis is perpendicular to the radial line. This means the earth covers one orbit minus a small piece. The time it takes to cover the extra piece depends on where in the orbit this small piece is. According to Kepler's Second Law, the earth moves faster near perihelion. Presently, perihelion is in early January, near the December solstice. It follows that the December solstice regresses slower than the March equinox. This means that DS1 (December solstice) is closer to DS2 (the next December solstice) than ME1 (March equinox) is to ME2 (the next March equinox). The time interval between two successive December solstices is hence longer than the time interval between two successive March equinoxes.

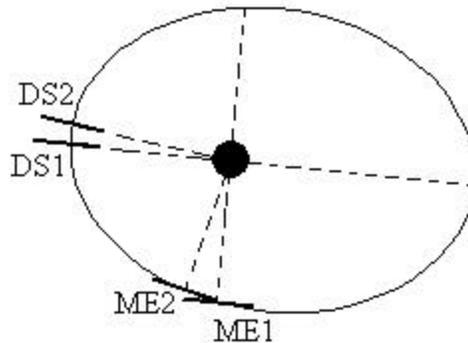


Diagram 9: Regression of the December solstice and the March equinox

The more accurate definition adopted nowadays for the tropical year is the time interval needed for the sun's mean longitude to increase by 360° . It is currently about 365.24219 days. This is almost equal to the average of the current mean time intervals given in Table 3: $(365.242374 + 365.241626 + 365.242018 + 365.242740) / 4 = 365.24219$ days.

Easter

Easter Sunday is the first Sunday after the first full moon occurring on or after the day of the March equinox. As mentioned before, it is difficult to compute the actual date of the March equinox and the time of the full moon. Instead, the ecclesiastical moon based on the Metonic cycle is used and the March equinox is assumed to fall on March 21st.

Easter Sunday in year y falls on day d in month m where d and m are computed as follows (all remainders from division are dropped).

$$c = y / 100$$

$$n = y - 19 \times (y / 19)$$

$$k = (c - 17) / 25$$

$$i_1 = c - c / 4 - (c - k) / 3 + 19 \times n + 15$$

$$i_2 = i_1 - 30 \times (i_1 / 30)$$

$$i_3 = i_2 - (i_2 / 28) \times \{1 - [i_2 / 28] \times [29 / (i_2 + 1)] \times [(21 - n) / 11]\}$$

$$j_1 = y + y / 4 + i_3 + 2 - c + c / 4$$

$$j_2 = j_1 - 7 \times (j_1 / 7)$$

$$l = i_3 - j_2$$

$$m = 3 + (l + 40) / 44$$

$$d = l + 28 - 31 \times (m / 4)$$

2.3 Delving into the Chinese calendar

The fundamental rules of the Chinese calendar have been introduced in Chapter 1.

We now further our discussion with more detailed explanations of some of the rules.

We also provide some computational functions related to the Chinese calendar.

About 10 years ago, it was discovered that there was an error in the Chinese calendar for 2033. The traditional calendar claimed that the leap month should follow the seventh month. We shall see that, in actual fact, it follows the 11th month.

Length of the Chinese year

Using Mathematica, we have written a function that computes the length of a Chinese year starting in a given Gregorian year. An extension of this is a function that computes the length of the Chinese years between two Gregorian years. Table 4 shows the distribution of the length of the Chinese years between 1911 and 2110:

353 days	354 days	355 days	383 days	384 days	385 days
1	84	41	5	66	3

Table 4: Distribution of the length of Chinese years between 1911 and 2110

Length of the Chinese month

The length of the Chinese month is determined astronomically. There is a function to compute the moment of the first new moon on or after a given date. Recall that the length of a lunar month can vary between about 29.25 and 29.75 days with a mean of 29.53 days. Suppose the synodic period is 29.5 days. If a new moon occurs at 1pm on May 1st, the next new moon will be at 1am on May 31st. This means the month has 30 days (long or “big” month,). On the other hand, if a new moon occurs at 1am on May 1st, the next new moon will be at 1pm on May 30th. In this case, the month has 29 days (short or “small” months,).

New moon	Next new moon	Length
May 1 st 1pm	May 31 st 1am	30 days
May 1 st 1am	May 30 th 1pm	29 days

Table 5: Length of months

As in the case of years, we have written a function to compute the length of the Chinese months between two dates. In addition, we have written a function to test for

the alternation between the long and short months. For example, between 1900 and 2100, the ratio of the number of non-alternating months to the number of alternating months is about 0.3203. This suggests that the length of the months usually alternates.

Because the motion of the moon is highly irregular, it is possible to have up to four big months or three small months in a row. We can search for these strings of months with one of the functions written. The following is an example of four big months in a row:

New moon	Length
1990 Oct 18 23h 36m	29d 17h 20m
1990 Nov 17 17h 5m	29d 19h 17m
1990 Dec 17 12h 22m	29d 19h 28m
1991 Jan 16 7h 50m	29 d 19h 28m
1991 Feb 15 1h 32m	

Table 6: String of four big months

The full moon

Many of us have the notion that every full moon falls on the 15th day of the Chinese month. This is normally true but due to the complexity of the motion of the moon, full moon can fall on the 14th, 16th or 17th as well. We have written a function to compute the actual day of occurrence of the full moon.

14 th day	6
15 th day	306
16 th day	380
17 th day	124

Table 7: Day of full moon between 1984 and 2049

Table 7 shows the distribution of the day of the full moon between 1984 and 2049. We see that the most common day is the 16th day. The Mid-Autumn Festival is celebrated on the 15th day of the 8th month.

Fake leap months

According to rule 5, the first month of a leap *sui* that does not contain a *zhong qi* is a leap month. Now, recall from Chapter 1 that in the Indian calendar, a solar month can sometimes fall completely within a lunar month. In the Chinese calendar, this means that there can be two *zhong qi*'s in a lunar month. When this happens, a non-leap *sui* can have a month without a *zhong qi*. This month is called a “fake” leap month because it occurs in a non-leap *sui*. Fake leap months also occur when a leap *sui* has more than one lunar month without any *zhong qi*. The additional months without any *zhong qi* are the fake leap months. Fake leap months can be determined by one of the written functions. Table 8 shows all the fake months between 1800 and 2100. In 1832, 1851, 1870 and 1984, a month with two *zhong qi*'s caused a month to have no *zhong qi* in the next *sui*.

	Leap year	Leap <i>sui</i>	Leap month	Month with 2 <i>zhong qi</i> 's	Fake leap month
1832	Yes	Yes	9-leap	11	
1833	No	No			1
1851	Yes	Yes	8-leap	12	
1852	No	No			2
1870	Yes	Yes	10-leap	11	12
1984	Yes	Yes	10-leap	11	
1985	No	No			1
2033	Yes	No	11-leap	11, 12	8
2034	No	Yes			1

Table 8: Fake leap months

Double spring, double rain

A Chinese year is said to have “double spring”, *shuāng chūn* (), if it contains two *lì chūn*'s, one in the beginning and one at the end of the year. It is an easy exercise to see that this is equivalent to a leap year. Similarly, a year is said to have “double spring, double rain”, *shuāng chūn shuāng yǔ* () if it contains two *lì chūn*'s and two *yǔ shū* 's. We can use one of the written functions to determine which years have double spring, double rain. Between 1645 and 2100, it happens only 15 times: in 1699, 1832, 1851, 1984, 2033 and 2052.

The year 2033

The year 2033 is an exceptional year because both fake leap month and double spring, double rain will occur in 2033. Table 9 gives the times for *zhōng qì* 's and new moons in 2033. From Table 9, the distribution of the *zhōng qì* 's in 2033 is obtained in Table 10. Here, M6 represents the sixth month while Z6 represents the sixth *zhōng qì*. M11+ represents the leap month after the 11th month.

New moon	<i>Zhōng qì</i>
M6: Jun 27 th 2033, 5h5m	Z6: Jul 22 nd 2033, 19h51m
M7: Jul 26 th 2033, 16h11m	Z7: Aug 23 rd 2033, 3h0m
M8: Aug 25 th 2033, 5h38m	Z8: Sep 23 rd 2033, 0h50m
M9: Sep 23 rd 2033, 21h38m	Z9: Oct 23 rd 2033, 10h26m
M10: Oct 23 rd 2033, 15h27m	Z10: Nov 22 nd 2033, 8h14m
M11: Nov 22 nd 2033, 9h38m	Z11: Dec 21 st 2033, 21h44m
M11+: Dec 22 nd 2033, 2h45m	Z12: Jan 20 th 2034, 8h25m
M12: Jan 20 th 2034, 18h0m	

Table 9: Times for *zhōng qì* 's and new moons in 2033/34

Month	Number of <i>zhông qì</i> 's
2033 M7	1
2033 M8	0
2033 M9	1
2033 M10	1
2033 M11	2
2033 M11+	0
2033 M12	2
2034 M1	0
2034 M2	1

Table 10: Distribution of *zhông qì* 's in 2033/34

Here, M9 takes Z8, M10 takes Z9 and M11 takes Z10 and Z11 (winter solstice or *dông zhì*). The fact that M8 has no *zhông qì* is compensated for by the fact that M11 has two. Hence, the *sui* 2033, which begins at winter solstice in 2032, has only 11 complete months and is hence not a leap *sui*. But the *sui* 2034, which begins at Z11, has 12 complete months and hence is a leap *sui*. As such, M8 is not a leap month while M11+ is a leap month. Because of this, Chinese New Year will be the third new moon after Z11.

APPENDIX

```
lengthOfChineseYear[gYear_] :=
```

```
  Calendrica`Private`ChineseNewYear[gYear+1]-Calendrica`Private`ChineseNewYear[gYear]
```

```
lengthOfChineseYear[2000]
```

```
354
```

```
lengthOfChineseYearsBetween[gYear1_, gYear2_] :=
```

```
  Module[{L=Table[0, {6}], M=Table[0, {6}]},
```

```
    For[i=gYear1, i<= gYear2, i++,
```

```
      (Which[lengthOfChineseYear[i]==353,L[[1]]++,
```

```
        lengthOfChineseYear[i]==354,L[[2]]++,
```

```
        lengthOfChineseYear[i]==355,L[[3]]++,
```

```
        lengthOfChineseYear[i]==383,L[[4]]++,
```

```
        lengthOfChineseYear[i]==384,L[[5]]++,
```

```
        lengthOfChineseYear[i]==385,L[[6]]++);
```

```
      j=lengthOfChineseYear[i]; Print[i, ":", j] )];
```

```
  For[k=1,k<=3,k++,M[[k]]=ToString[352+k]<>":"<>ToString[L[[k]]];
```

```
  For[k=4,k<=6,k++,M[[k]]=ToString[379+k]<>":"<>ToString[L[[k]]];
```

```
  M
```

```
]
```

```
lengthOfChineseYearsBetween[2000,2005]
```

```
2000:354
```

```
2001:384
```

```
2002:354
```

```
2003:355
```

2004:384

2005:354

{353:0,354:3,355:1,383:0,384:2,385:0}

lengthOfMonthsBetween[fDate1_,fDate2_] :=

Module[{s=0,t=fDate1},

While[(t=Calendrica`Private`ChineseNewMoonOnOrAfter[t+1])<fDate2,

If[s==3,Print[t," : 3 short"],

If[(Calendrica`Private`ChineseNewMoonOnOrAfter[t+1]-t)==29,

(s++;Print["The month starting on ",Gregorian[t], " is short"]),

(s=0; Print["The month starting on ",Gregorian[t] ," is long"])]

]

]

]

lengthOfMonthsBetween[ToFixed[Gregorian[1,1,2000]],ToFixed[Gregorian[3,1,2000]]]

The month starting on Gregorian[1,7,2000] is short

The month starting on Gregorian[2,5,2000] is long

bigSmallConsecutiveBetween[gYear1_, gYear2_] :=

Module[{i1, i2, i3, j, A=0, S=0},

i1=Calendrica`Private`ChineseNewYear[gYear1];

i2=Calendrica`Private`ChineseNewMoonOnOrAfter[i1+1];

i3=Calendrica`Private`ChineseNewMoonOnOrAfter[i2+1];

j=Calendrica`Private`ChineseNewYear[gYear2];

```

While[i3<=j,
  If[i2-i1==i3-i2, S++, A++];i1=i2; i2=i3;
  i3=Calendrica`Private`ChineseNewMoonOnOrAfter[i3+1]];
Print["S=", S]; Print["A=",A]; Print["S/A = ",S/A]
]

bigSmallConsecutiveBetween[1900,2100]

S=600
A=1873
S/A=600/1873

stringsOfMonthsBetween[fDate1_,fDate2_]:=
Module[{s=0,l=0,t=fDate1},
  While[(t=Calendrica`Private`ChineseNewMoonOnOrAfter[t+1])<fDate2,
    Which[s==3,Print["THREE SHORT months in a row ending on ",
Gregorian[t]],
      l==4,Print["FOUR LONG months in a row ending on ",
Gregorian[t]],
      l==5,Print["FIVE LONG months in a row ending on ", Gregorian[t]]
    ];
  If[(Calendrica`Private`ChineseNewMoonOnOrAfter[t+1]-t)==29,
    (l=0;s++), (s=0;l++)
  ]
]
]
]

```

*stringsOfMonthsBetween[ToFixed[Gregorian[1,1,1990]],
ToFixed[Gregorian[1,1,2050]]]*

FOUR LONG months in a row ending on Gregorian[2,15,1991]

FOUR LONG months in a row ending on Gregorian[4,16,2037]

FOUR LONG months in a row ending on Gregorian[3,19,2045]

fullMoonAfter[jd_]:=FullMoonAtOrBefore[FullMoonAtOrBefore[jd]+45]

chineseFullMoonAfter[fDate_]:=

FixedFromJD[

LocalFromUniversal[

fullMoonAfter[

UniversalFromLocal[

JDFromMoment[fDate],

Calendrica`Private`ChineseTimeZone[fDate]]],

Calendrica`Private`ChineseTimeZone[fDate]]]

dayOfChineseFullMoonBetween[gYear1_, gYear2_]:=

Module[{d,t=Calendrica`Private`ChineseNewYear[gYear1],

t1=Calendrica`Private`ChineseNewYear[gYear2],L=Table[0,

{4}],M=Table[0, {4}]},

```

While[t< t1, d=chineseFullMoonAfter[t]-t+1;
    Print["The full moon falls on the ",d,"th day of the month starting
",Gregorian[t]];
    t=Calendrica`Private`ChineseNewMoonOnOrAfter[t+1];L[[d-13]]++
];
For[k=1,k<=4,k++,M[[k]]=Tostring[k+13]<>":"<>Tostring[L[[k]]]
];
Print[M];N[Sum[L[[i]] (i+13),{i,1,4}]/Sum[L[[i]] ,{i,1,4}]]
]

```

dayOfChineseFullMoonBetween[1999,2000]

The full moon falls on the 15th day of the month starting **Gregorian**[2,16,1999]

The full moon falls on the 15th day of the month starting **Gregorian**[3,18,1999]

The full moon falls on the 15th day of the month starting **Gregorian**[4,16,1999]

The full moon falls on the 16th day of the month starting **Gregorian**[5,15,1999]

The full moon falls on the 16th day of the month starting **Gregorian**[6,14,1999]

The full moon falls on the 16th day of the month starting **Gregorian**[7,13,1999]

The full moon falls on the 17th day of the month starting **Gregorian**[8,11,1999]

The full moon falls on the 16th day of the month starting **Gregorian**[9,10,1999]

The full moon falls on the 17th day of the month starting **Gregorian**[10,9,1999]

The full moon falls on the 16th day of the month starting **Gregorian**[11,8,1999]

The full moon falls on the 16th day of the month starting **Gregorian**[12,8,1999]

The full moon falls on the 15th day of the month starting **Gregorian**[1,7,2000]

{14:0,15:4,16:6,17:2}

15.8333

sui[fDate_] :=

Module[{c=CYear[Gregorian[fDate]],

s=Calendrica`Private`MajorSolarTermOnOrAfter[

ToFixed[Gregorian[December[],1,CYear[Gregorian[fDate]]]]],

If[fDate< Floor[s], c,c+1]

]

winterSolsticeOnOrBefore[fDate_] :=

Module[{s0=Calendrica`Private`MajorSolarTermOnOrAfter[

ToFixed[Gregorian[December[],1,CYear[Gregorian[fDate]]-1]],

s1=Calendrica`Private`MajorSolarTermOnOrAfter[

ToFixed[Gregorian[December[],1,CYear[Gregorian[fDate]]]]],

If[fDate< Floor[s1], s0,s1]

]

fakeLeapMonths[gYear1_, gYear2_] :=

Module[{c,t=Calendrica`Private`ChineseNewYear[gYear1],

t1=Calendrica`Private`ChineseNewYear[gYear2]],

While[t<t1, c=sui[t];

If[Calendrica`Private`NoMajorSolarTermQ[t],

If[chineseLeapSolsticeYearQ[c],

If[Calendrica`Private`PriorLeapMonthQ[winterSolsticeOnOrBefore[t],t-

1],

```
Print["The month starting ", Gregorian[t], " is a fake leap month in a
leap sui"]
```

```
],Print["The month starting ", Gregorian[t], " is a fake leap month in a
non-leap sui"]
```

```
],
```

```
];t=Calendrica`Private`ChineseNewMoonOnOrAfter[t+1]
```

```
]
```

```
]
```

```
fakeLeapMonths[2033,2035]
```

```
The month starting Gregorian[8,25,2033] is a fake leap month in a non-leap sui
```

```
The month starting Gregorian[2,19,2034] is a fake leap month in a leap sui
```

```
yuShui[gYear_]:=
```

```
Gregorian[Calendrica`Private`MajorSolarTermOnOrAfter[ToFixed[liChun[g
Year]]]]
```

```
doubleSpringDoubleRain[gYear1_,gYear2_]:=
```

```
Module[{i=gYear1},
```

```
While[i<=gYear2,
```

```
If[chineseLeapYearQ[i],
```

```
If[(Calendrica`Private`ChineseNewYear[i]<=ToFixed[liChun[i]] &&
```

```
Calendrica`Private`ChineseNewYear[i+1]>ToFixed[yuShui[i+1]]),
```

```
Print["The Chinese year starting in Gregorian year ", i, " has double
spring
```

double rain'']

]

];i++

]

]

doubleSpringDoubleRain[1645,2060]

The Chinese year starting in Gregorian 1699 has double spring double rain

The Chinese year starting in Gregorian 1832 has double spring double rain

The Chinese year starting in Gregorian 1851 has double spring double rain

The Chinese year starting in Gregorian 1984 has double spring double rain

The Chinese year starting in Gregorian 2033 has double spring double rain

The Chinese year starting in Gregorian 2052 has double spring double rain

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