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Symmetry Groups in Arts and Architecture

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ABSTRACT

The use of geometric principles of symmetry for the description and understanding of decorated forms represented the union of two normally separate disciplines-mathematics and designs. The only limitation to the types of designs that can be described by these principles is that they must consist of regularly repeated patterns. In this paper, we will see how to use the geometric principles of crystallography to develop a descriptive classification of patterned design. This description of designs by their geometric symmetries makes it possible to study their functions systematically and meaning within cultural contexts. We are interested in how Mathematics can play a part in arts and architecture and even our daily life. We will be using Ming (1368-1644) blue and white porcelains as the study subject of frieze patterns. We will classify the designs by their symmetries and hope to gain insights from these designs in their cultural context. To add a local flavour to the paper, we also take a look at the Peranakan porcelains.

FOUR ISOMETRIES

It is necessary to know the underlying structure of aesthetically pleasing plane figures. By a plane figure, we mean any subset of the plane i.e. we will assume that the figures we are going to study can be flattened out into a plane. To understand the underlying structure, we need to examine the symmetries of the plane. Symmetry of a figure is an isometry that maps the figures back onto it. Isometries are transformations that do not distort the shapes of objects in the process of moving them. By this, we mean that it is a *distance-preserving transformation* of the plane onto itself. They are four isometries: translation, reflection, rotations and glide refection.

Translation

A *translation* of a plane figure means to move it without rotating or reflecting it in a given direction. There are no fixed points and every point moves by exactly the same distance, *d* in a translation

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Reflection

A *reflection* is a mapping of all points of the original figure onto the other side of a "mirror that the distance between the image (the figure that is mapped from the original one) and the mirror line is the same as that between the original figure and the mirror line. The mirror line is known as the *axis of reflection*. *Reflections* are isometries that have *infinitely many* fixed points

Rotation

We completely specify a *rotation* when we know its *centre* and *angle of rotation*. A figure with an angle of rotation è is said to have an *order of rotation n* if $n = 360^{o}/_{è}$ and *n* is a natural number. We define a *symmetry region* as a subset of the figure that generates the whole figure by rotations.

Glide Reflection

A *glide reflection* is a combination of two transformations: a reflection and a translation. Whether the original figure is reflected or translated first does not affect the final result; the final figure generated will be the same.

SYMMETRY GROUPS

In our context, we will call figures with at least one (non-trivial[3]) symmetry *designs*. We will call designs that have a translation symmetry *patterns*. Each pattern has a *basic unit, which* is a smallest region of the plane such that the set of its images under translations of the pattern generates the whole pattern. We call designs that do not admit translation symmetry *finite designs*. They are two other types of patterns, namely *frieze patterns* that admit translation in only one direction and *wallpaper patterns*, which admit translations in two or more directions frieze patterns and wallpaper. We are concerned with frieze patterns in this paper only.

Frieze Patterns

The symmetry groups of frieze patterns are named in the form of a four-symbol notation *pxyz*. They are seven classes for frieze patterns. Each name begins with the letter p. The following shows how to derive the rest of the four-symbol notation for each symmetry group for frieze patterns.

x = { m if there is vertical reflection
{ 1 otherwise
y = { m if there is horizontal reflection
{ a if there is a glide reflection but no horizontal reflection
{ 1 otherwise
z = { 2 if there is a half turn
{ 1 otherwise

[3] The trivial symmetry is the identity transformation, which maps the figure back to itself.

MING BLUE AND WHITE PORCELAINS

In the long history of world ceramics, there has been no single ware more appreciated and imitated than Chinese blue and white porcelain. As a ceramic tradition, it has the longest continuous development in ceramic history and has acquired a worldwide reputation unmatched by any other Chinese art form. It is perhaps the best-known category of all decorative arts. This is the reason why we are looking at Ming porcelains.

Key Findings

We are able to find all seven types of frieze patterns on the porcelains from the Ming Dynasty using the classification of the symmetries. We find that frieze pattern type pm11 (vertical refection only) appears most frequently on the porcelains and that p1m1 (horizontal reflection only) is a rare pattern that has only appeared once. The frequency of the rest of the patterns seems to be quite even showing that the decorators of the porcelains frequently use these frieze patterns without obvious preference.

We are able to see that the frieze patterns are mostly distributed at the top rim (33%), followed by body (24%) and base (19%). Both the top and the foot ring share the same percentage (12%).

We also find that all seven types of frieze patterns are present on the body of the porcelains with pm11 type been frequently used. Beside the body of porcelains, the top rim has six out seven types of the frieze pattern and the different types of frieze seem to be more evenly spread out unlike on the body where pm11 is the distinct type. Unlike the top rim, we find that the decorators only use two types of frieze patterns, p111 (only translation) and pm11 at the base. pm11 is the distinct pattern type for the top, body and base portion of the porcelains. One interesting finding is that we do not find p112 (half turns only) and p1a1 (glide reflection only) frieze pattern at the top and the base of the porcelain at all but we can find pm11 in all five parts of the porcelains.

We also find that, except for the period of Tianqi and Chongzhen, do not have frieze patterns at the base and that foot ring, all the other periods have frieze on the all parts of the porcelain. We also see a gradual decline in the frieze pattern being used on the body of the porcelains towards the end of Ming dynasty. This may be a result in the increase use of decorating using narrative themes on the porcelains. We also see an increase in the frieze patterns found at the foot rings from Yuan to Xuande period but this trend dies off at the end of the Ming dynasty. This may be due to change of the design of the porcelains.

Peranakan Porcelains

We also went to the Asian Civilisation Museum to have a look at the Peranakan porcelains. Peranakan is a Malay word that simply means, “ born locally”. From about six hundred years ago, Chinese traders came from China and settled in what we now known as Malaysia, Indonesia and Singapore. They brought with them their traditions from China but slowly developed a different way of life in their new country. Their offspring became known as the Paranakan Chinese.

Key Findings

We are able to find only six out of the seven classes of frieze patterns. We find that pm11 is the most preferred frieze pattern on the Peranakan porcelains. We are not able to find any p1m1 frieze pattern in this case. We find that half of the frieze patterns are distributed at the base of the porcelains and only 3% of the frieze patterns can be found at the foot ring. We find that pm11 dominates four of the five parts of the porcelains.

We can only find one frieze pattern that is p111 at foot ring. Unlike the Ming porcelains, where we can only find two frieze types at the base area, we can find four types at the base of Peranakan porcelains. Also in the Ming period, we are not able to see any p112 frieze type on the top that we can find in the Peranakan porcelains.

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