The Mathematics of the Chinese Calendar

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ABSTRACT

The purpose of this paper is to present a clear description of the rules for the Chinese calendar. For most of the years, the Chinese calendar is quite simple, but in some exceptional cases, it can become very complicated. We will analyze one such case, namely the year 2033, in details. We have also used a computer to test the validity of some popular claims about the Chinese calendar.

INTRODUCTION

When people mention the date of the day, we usually refer to the Gregorian date. Hardly anybody knows the answer for the Chinese calendar. However, it is important that we do not lose the traditional Chinese culture and recognize the beauty of it.

Calendar computation was a crucial event in ancient China, because correct predictions could directly affect the relationship between the Heaven and the Emperor. Due to discovery of many astronomical theories and enhancement of skills to predict and observe astronomical events, rules for the Chinese calendars changed throughout history. From the very primitive ones inscribed on oracle bones, the calendar has developed to the form we use now, which is based on the actual positions of the sun and the moon. Sadly to say, sources are relatively scarce and incomplete, especially in the Western world.

As mentioned above, not many people, including the *Feng Shui* tellers in Singapore that we randomly interviewed, understand the rules of the Chinese calendar nor the basic computation of the important festivals such as Chinese New Year. We have therefore worked on and created some computer commands in Mathematica, based on the codes from the book of Dershowitz and Reingold [6]. We have put the program on the Internet for the appreciation of the public.

NOTATION

To avoid confusion and for convenience sake, the following notations are adopted:

Notation	**Definition**	**Notation**	**Definition**
Mn	‘n’th new moon	Mn+	New Moon after Mn
Mn++	New Moon after Mn+	Zn	‘n’th zhongqi

Table1: Notations used in the paper

BASIC KNOWLEDGE OF A CHINESE CALENDAR

There are many calendars used in the world, including the Gregorian, Jewish and Indian calendars. How do we classify these calendars? One of the methods is to divide the calendars into lunar, solar and lunisolar calendar. A lunar calendar follows the moon and the (mean or true) synodic month. A typical example is the Muslim calendar. A solar calendar, on the contrary, uses days to approximate the tropical year. The Gregorian calendar is solar. Lunar calendars cannot predict seasons, while solar ones cannot tell when the new moon is. This may be a reason why a more complicated kind of calendar, lunisolar calendar, was developed. The Chinese and Jewish calendars are lunisolar. They use months to approximate the tropical year. A leap month (also called intercalary month) is inserted every two or three years.

Another way to classify calendars is to separate them into astronomical and arithmetical calendars. The latter can be computed using arithmetical equations and easily converted into other calendars. An astronomical calendar is more complicated since it is determined by many

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variables that exist in the universe. The Gregorian and Jewish calendars are arithmetical, while the Chinese and Indian calendars belong to the astronomical type. Note that the Indians use some old methods to approximate true sun and moon, while the Chinese use the actual true motion of the planets for computations.

ASTRONOMY 1,2, 3

As most of us know, the earth revolves around the sun along its elliptical orbit in a clockwise motion and rotates around an axis, which is 23.5 degrees tilted with respect to the ecliptic. Every year, the Earth's axis lies perpendicularly to the radial line from the sun to the earth two times (called the equinoxes), and it points directly to the sun two times (called the solstices). These points on the ecliptic are called seasonal markers, namely, the vernal (spring) equinox, summer solstice, autumnal equinox and winter solstice. The seasonal markers do not fall at the vertices of the eclipse due to a phenomenon called precession, which states that the earth's axis is revolving in a circle slowly with a period of about 26,000 years.

As a result, we have two definitions of "year". One is the tropical year, which is the mean time from one vernal equinox to the next (approximately 365.2421896698 days). The actual time can vary from the mean by up to 10 minutes, but this length is decreasing by about one second every century. Tropical year is a basic unit of the Chinese calendar. Another form of year is the sidereal year, which is the time it takes for the earth to complete one revolution with respect to the stars. This takes about 365.25636 days, around 20 minutes longer than the tropical year.

Another fundamental concept for calendar theory is that the motion of the sun is not uniform along its ecliptic. This is explained by Kepler's Second Law, which states that a planet sweeps out equal areas in equal time. Since the sun is not situated at the centre of the eclipse, the earth is actually moving faster when it is nearer to the sun, and is moving slower when it is further from the sun. Together with the complex motion of the moon, it gives rise to variation of the period of one synodic moth, which is the time from one new moon (conjunction) to the next, ranging from 29d 6h 26m (29.27 days) to 29d 20h 6m (29.84 days) [4]. A lunar year is equal to 12 mean lunar months (354.36707 days), which is about 11 days shorter than a tropical year.

THE METONIC CYCLE

235 mean lunar months (6939.6885 days) are roughly equal to nineteen tropical years (6939.6018 days), differing by around 2 hours. An error of one day will only occur after 220 years. From the mathematical point of view, 7 leap (intercalary) months have to be added within these 19 years, since 235=19(12)+7. To verify the validity of this cycle, you may like to know that for most people, they are actually celebrating their 19th, 38th, 57th …Gregorian and Chinese birthdays on the same day. Check it out.

THE TWENTY-FOUR JIE QI (節气)

春雨惊春清谷天
夏滿芒夏暑相連
秋处露秋寒霜降
冬雪雪冬小大寒

This is a poem that the Chinese made to help them memorize the sequence of the 24 solar terms (jie qi) in the Chinese calendar [7]. Solar terms are determined by the tropical year. In Chinese, they are called jie qi (節气). The equinoxes and solstices form a subset of the jieqi. According to the Chinese calendar, “Chun fen” (Z2) marks the 0^{o} of the ecliptic. From that point onwards, every 15^{o} on the ecliptic indicates another jieqi. The even ones are the major solar terms, called the zhongqi (中气). The odd ones are minor solar terms, called jieqi (節气).

Although the Chinese astronomer, Zhang Sui (张遂) [2] had a concept similar to Kepler's Second Law, the TaiYanLi (太衍历) he made during the Tang Dynasty still follows the traditional use of the mean sun, (平气). Under this system, there are 30.44 days between two zhongqi, which is longer than the length between two new moons. Therefore, it was not surprising to have no zhongqi in one month.

However, since the last reform of the Chinese calendar in 1645, Chinese astronomers started to use the true sun, (定气) [3]. The length between two zhongqi now ranges from 29.44 days to 31.44 days. Exceptional cases arise when there are two zhongqi in one month, in addition to months without any zhongqi.

Jie Qi No	Hanyupingying Name	Chinese character	Meaning	Gregorian date (rough)
J1	Li Chun	立 春	Beginning of spring	February 4
Z1	Yu Shui	雨 水	Rain water	February 19
J2	Jing Zhe	惊 蛰	Waking of insects	March 6
Z2	Chun Fen	春 分	Spring equinox	March 21
J3	Qing Ming	清 明	Pure brightness	April 5
Z3	Gu Yu	谷 雨	Grain rain	April 20
J4	Li Xia	立 夏	Beginning of summer	May 6
Z4	Xiao Man	小 满	Grain full	May 21
J5	Mang Zhong	芒 种	Grain in ear	June 6
Z5	Xia Shi	夏 至	Summer solstice	June 22
J6	Xiao Shu	小 暑	Slight heat	July 7
Z6	Da Shu	大 暑	Great heat	July 23
J7	Li Qiu	立 秋	Beginning of autumn	August 8
Z7	Chu Shu	处 暑	Limit of heat	August 23
J8	Bai Lu	白 露	White dew	September 8
Z8	Qiu Fen	秋 分	Autumnal equinox	September 23
J9	Han Lu	寒 露	Cold dew	October 8
Z9	Shuang Jiang	霜 降	Descent of Frost	October 24
J10	Li Dong	立 冬	Beginning of winter	November 8
Z10	Xiao Xue	小 雪	Slight snow	November 22
J11	Da Xue	大 雪	Great snow	December 7
Z11	Dong Zhi	冬 至	Winter solstice	December 22
J12	Xiao Han	小 寒	Slight cold	January 6
Z12	Da Han	大 寒	Great cold	January 20

Table 2: The 24 Jie qi

<u>THE RULES OF THE CHINESE CALENDAR</u>

Rule 1 *Calculations are based on the 120^o meridian East*

Before 1929, calculations were based on the location of Beijing. However, from 1928, China adopted the standard time zone, based on 120^o meridian East, which is approximately the position of Nanjing, where the Purple Mountain Observatory (the main centre for calendrical calculations in China) is located.

Rule 2 *The day on which a new moon occurs is the first day of a new month*

Zhongqi and the new moon denote just an instant. According to the rule, if n-zhongqi happens to occur before the new moon time on the same day, the new month would have n-zhongqi. Length of a month (29/30 days) is determined astronomically.

New Moon	**Next New Moon**	**Length**
Jun 1 1300	Jul 1 0100	30 days
Jun 1 0100	Jun 30 1300	29 days

Table 3: Length of a month (assuming the month has 29.5 days)

In the modern Chinese calendar, there can be up to four big months (大月) or three small months (小月) in a row. A recent example is the four big months in late 1990 to early 1991.

New Moon		**Length of the month**
Date	Time	
18 Oct 1990	2336	29day 17hour 29min
17 Nov 1990	1705	29day 19hour 17min
17 Dec 1990	1222	29day 19hour 28min
16 Jan 1990	0750	29day 17hour 42min
15 Feb 1990	1532	

Table 4: Four big months in a row

If the new moon occurs at a time very close to midnight, the determination of the length of a month can be very tricky. A wrong calculation can cause much confusion for the exact date of some Chinese festivals. For example, in 1978, calendars in Hong Kong and Taiwan had the seventh month as a short month, whereas the Revised Version used in China put it as a long month. Thus, people celebrated the Mid-Autumn festival, which is supposed to be on the fifteenth of the 8th month each year in the Chinese calendar, on different days [5].

It is a common wrong concept that the full moon always falls on the 15th of the month of a lunar month. In fact, it can occur on the 14th, 16th or even 17th of the month.

Rule 3 *A leap sui is a sui that contains 13 months*

We have two definitions of a year in the Chinese calendar, a sui (岁） and a nian（年). A sui is the solstice year from a winter solstice to the next, equivalent to a tropical year. A nian refers to the time period from one Chinese New Year to the other. The length of a nian varies from 353 to 355 days in a normal year, and 383 to 385 days in a leap nian.

Taking the length of a sui as 365 days, we can divide it into 12 whole months and 11 days, or 11 whole months and 40 days. If there is no new moon on the same day as the winter solstice, and the first new moon is more than 12 days after the winter solstice, we will have a normal year. Otherwise, due to the pigeonhole principle, we will get at least one month without a zhongqi, which means that we would have a leap sui.

Rule 4 *In a leap sui, the first month that does not contain a zhongqi is the leap month.*

Rule 5 *Winter solstice always occurs in month 11. It determines the numbering of the months in a normal sui. In a leap sui, the leap month duplicates the number of the previous month.*

The number of a month is usually the same as that of the zhongqi it contains, provided that it is not a leap month or a month in a year, which has month(s) with no or two zhongqi.

Other than the above mentioned rules, there are a few claims in the Chinese calendar

Claim 1 *It is rare to have leap 11th month, and there is no leap 12th or 1st month.*

In fact, any month can be a leap month. The claim holds in the 21st century for most of the time. It will not necessarily be true many years (in fact, around 10,500 years) later, due to the effect of precession, when the Earth travels fastest in summer and relatively slower in winter.

Z11	Z12	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Z8	Z9	Z10	Z11
29.44	29.59	29.97	30.47	30.97	31.34	31.44	31.29	30.89	30.37	29.89	29.55	

Table 5: Length between zhongqi

However, it is still possible to have some exceptions to the claim, based on reasonable calculations. Assuming that one month consists of 29.53 days, and then we take the start of a new sui to be 0. From that point onwards, we begin to count days.

Zhongqi/Month	Z11	M12	Z12	M12+	Z1	M12++
Day no.(for leap 11)	-0.01	0.01	29.43	29.54		
Day no.(for leap 12)	-0.01	0.48	29.43	30.01	59.02	59.54

Table 6: Leap month after 11th month, 12th month

Notice that Z11 and M12 occur on different days and Z12 on the same day as M12. By rule2, Z12 is taken by M12+, and so M12 is actually leap 11th month. Examine the second row of the table. M12 occur a bit later this time than the previous case. M12+ and Z12 fall on different days. M12 is a normal 12th month in that nian. Z1 and M12++ fall on the same day. By rule 2 again, M12++ grabs Z12, leaving M12+ without a zhongqi. In this case, M12+ becomes leap 12th month [1]. Using our computations in Mathematica, we predict that year 3358 is a year that has leap 12th month. However, since accurate astronomical predictions cannot be very reliable more than 100 years ahead, this is only a theoretical result based on today's information.

Zhongqi/Month	Z11	M12	Z12	M1	Z1	M1+	Z2	M1++
Day no.	-0.04	0.48	29.40	30.01	58.99	59.54	88.96	89.07

Table7: Leap month after 1st month

By similar reasoning as above, since M1++ takes Z2, M1+ does not have a zhongqi. Therefore, it becomes a leap 1st month. We believe that there is a wrong computation for the calendar of 1651. 1651 should have a leap 1st month, instead of a leap 2nd month as recorded. Another theoretical prediction is that a leap 1st month will occur in 2262.

Claim 2 *Chinese New Year is the second new moon after winter solstice.*

This claim only holds if Claim 1 holds. Exceptions are explained above.

Claim 3 *Chinese New Year is the new moon closest to li chun, the beginning of spring.*

Usually this rule is correct, since lichun always falls around Feb 4, and Chinese New Year falls within Jan 21 and Feb 21. However, in year 1985, li chun is actually closer to the last month of the previous nian. Similar occasion will happen again in 2148.

SUI 2033, CHINESE Y2K?

Month	Date	Time	Zhongqi	Date	Time
M7	26-7-2033	1611	Z7	23-8-2033	0300
M8	25-8-2033	0538	Z8	23-9-2033	0050
M9	23-9-2033	2138	Z9	23-10-2033	1026
M10	23-10-2033	1527	Z10	22-11-2033	0814
M11	22-11-2033	0938	Z11	21-12-2033	2144
M11+	22-12-2033	0245	Z12	20-1-2034	0825
M12	21-1-2033	1800			

Table 8: Times for zhongqi and new moon

Year Month	2033 M7	2033 M8	2033 M9	2033 M10	2033 M11	2033 leapM11	2033 M12	2034 M1	2034 M2
Zhongqi no	1	0	1	1	2	0	2	0	1

Table 9: Distribution of zhongqi in 2033/34

Year 2033 is a very special year in the Chinese calendar. Extreme cases of the distribution of zhongqi are illustrated in this year, making computations very complex. It is not until early 1990s that Chinese astronomers realize that they have placed the leap month wrongly for 2033. What makes year 2033 exceptional is that there is a month with two zhongqi, and there are 3 times when the zhongqi and the new moon fall on the same day. From Table 8, it can be noticed that M8 takes no zhongqi, M9 takes Z8, M10 takes Z9, M11 holds both Z10 and Z11 at the same time. This is the root of all the troubles. Last time, astronomers took the month after the 7th month in 2033 to be leap 7th month, which makes 2033 a leap sui and 2034 a normal sui.

According to rule 5, the old way to place the leap month is incorrect. Since M11 takes Z11, the winter solstice, M11+ would become leap 11th month, as it does not contain a zhongqi. In other words, 2034 is a leap sui, rather than 2033. The “loss” of a zhongqi in M8 is compensated by the fact that M11 in 2033 has 2 zhongqi. This makes M8 in 2033 a “fake” leap month.

DISCUSSION/CONCLUSION

We have attempted to check the frequency of the occurrence of Chinese New Year on different dates, having a hypothesis that this Chinese festival would have a higher tendency on some dates. However, after collecting data from an appreciable period of time, we realize that there is no long-time significance in this hypothesis. Besides, as mentioned in the text, due to the ever-changing astronomical events and many minor variables, computations for the data 100 years later may not be very accurate.

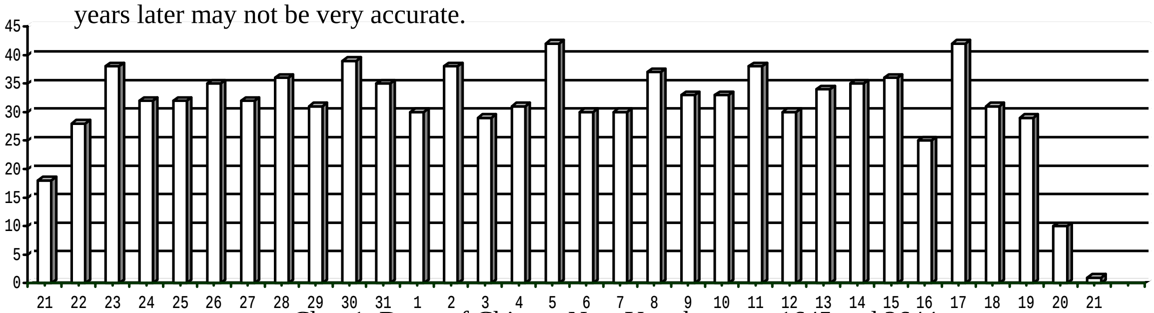

Chart1: Dates of Chinese New Year between 1645 and 2644

We have successfully pointed out some exceptions to the claims in the Chinese calendar. Therefore, our approach reminds us of the good old attitude of all scientists: be doubtful, make hypothesis, check so as to make new discoveries. Rules for the Chinese calendar can be near perfect However, we may notice some exceptions if we are thoughtful and careful enough.

Future investigations in this area may include topics like checking the validity of the Metonic Cycle, finding the maximum number of long/small months in a row. These are not done due to time constraints. Ideally, a user-friendly computer programme for computing the Chinese New Years and other important dates can be introduced for the public usage.

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