Undergraduate Research Opportunity Program in Science

STRINGS OF SHORT MONTHS AND LONG MONTHS IN THE CHINESE CALEDNAR

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Contents:

1. Preface

Today, people are paying less and less attention to calendars; even the most commonly used ones. For personal experience, friends of mine celebrated my 20th birthday on a wrong day. Since my birthday is on the 24th of the fifth month in the Chinese calendar, and is on July 6 in the Gregorian calendar, but the 24th of the fifth month in the Chinese calendar is not always July 6 in the Gregorian calendar (see Table 1). My 20th birthday was of the very case. Traditionally speaking, the Chinese were celebrating their festivals and birthdays referring to the Chinese calendar; but nowadays, the Gregorian calendar has begun to be commonly used in the Chinese society. In this way, some of Chinese keep on using the Chinese calendar, while others are using the Gregorian calendar. But only a few people can understand both of the two well. Due to this reason, misunderstanding frequently happens. In Singapore, the two most commonly used calendars are the Chinese calendar and the Gregorian calendar. To reduce misunderstandings to happen, help people familiarize with the two calendars and make them be able to efficiently manage the time with respect to different calendars, my project seems necessary and essential. It is mainly about the Chinese calendar, which is more interesting than the Gregorian calendar. At the last, here I should thank my supervisor Prof Helmer Aslaksen, who gave me great help and valuable guides. Also I should appreciate the support and help of my friends who contributed a lot to this project as well. Without them, I would not even be able to start.

	The equivalent date in the Gregorian calendar	
24th of the fifth month in the Chinese calendar	July 6	1980
	June25	1981
	July 14	1982
	July 4	1983
	:::::: ::::::	
	July 17	1998
	July 7	1999
	June 25	2000
	July 14	2001

Table 1. The time in the Gregorian calendar which is equivalent to the 24th of the fifth month in the Chinese calendar

2. Introduction --- A quick course in astronomy

To start the project, we first have a brief of some related astronomical and calendrical knowledge.

2.1 Some useful definitions:

The Earth revolves counterclockwise around the Sun (when viewed from the north celestial pole) in an elliptical orbit with the Sun as one of its foci. The following are the related definitions:

1. The *ecliptic plane* is the plane, in which the earth revolves around the Sun. (Figure 1)
2. The *March (or spring) and the September (or autumn) equinox* are the points on the Earth's orbit, at which the projection of the Earth's axis onto the plane of the ecliptic is perpendicular to the line joining the Sun and the Earth. (Figure 1)
3. The *June (or Summer) and December (or Winter) solstices* are the points on the Earth's orbit, at which the projection of the Earth's axis onto the plane of the ecliptic points directly towards the Sun. (Figure 1)
4. The point where the Earth is farthest from the Sun is called *aphelion.* (On the Earth's orbit)
5. The point where the Earth is closest from the Sun is called *perihelion*. (On the Earth's orbit)

Approximate dates	
Perihelion	January 4
Spring equinox	March 21
Aphelion	June 4
Summer solstice	June 22
Fall equinox	September 23
Winter solstice	December 22

Table 2. The approximate date in the Gregorian calendar

6. The time it takes for the Earth to complete one revolution with respect to the Stars is called *a sidereal year*.
7. In the modern definition, the time it takes for the Sun's mean longitude to increase by 360^o is called *a tropical year*.
8. The *sidereal month* is the time it takes for the Moon to move completely around the celestial sphere once as observed from the Earth.
9. The *synodic month* (or *lunation*) is the mean time from one new Moon (conjunction) to the next.
10. The point where the Moon is farthest from the Earth is called *apogee.* (On the Moon's orbit)
11. The point where the Moon is closest from the Earth is called *perigee.* (On the Moon's orbit)

12. The *anomalistic month* is the time between two consecutive perigees.
13. *A lunar year* is a year consisting of 12 mean lunar months.
14. The amount of "flattening" of the ellipse is measured the *eccentricity*. Thus, in Figure 2, the ellipses become more eccentric from left to right. A circle may be viewed as a special case of an ellipse with zero eccentricity, while as the ellipse becomes more flattened the eccentricity approaches one. Thus, all ellipses have eccentricities lying between zero and one.

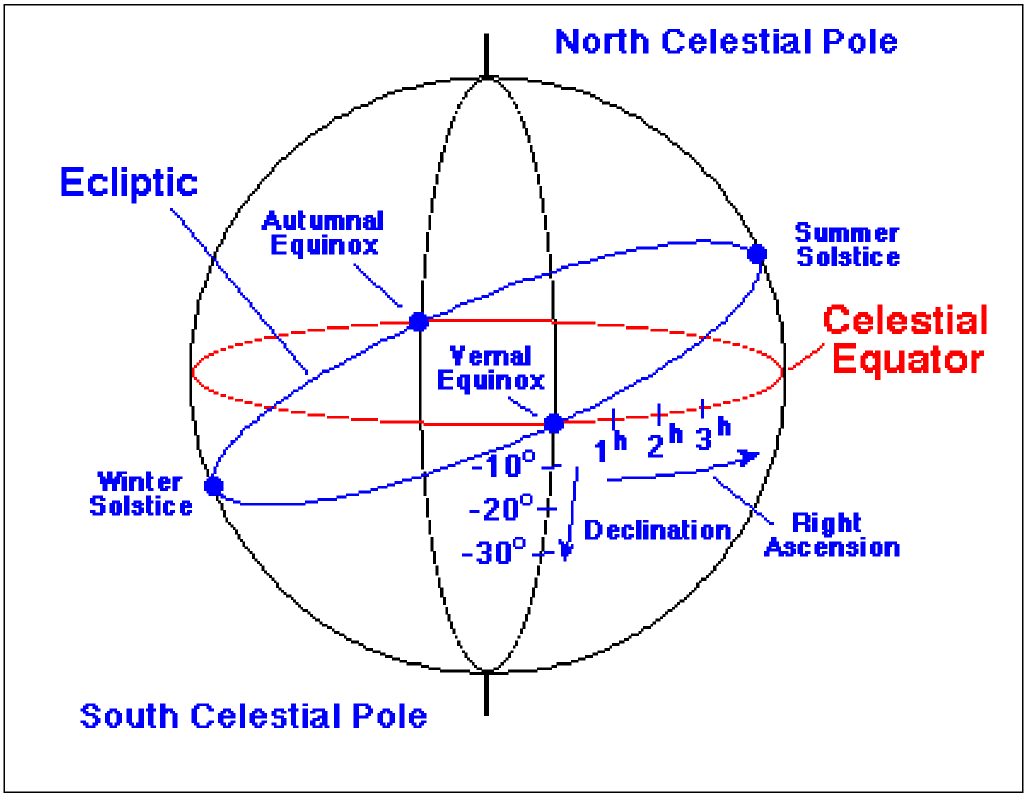

Figure 1: The celestial sphere

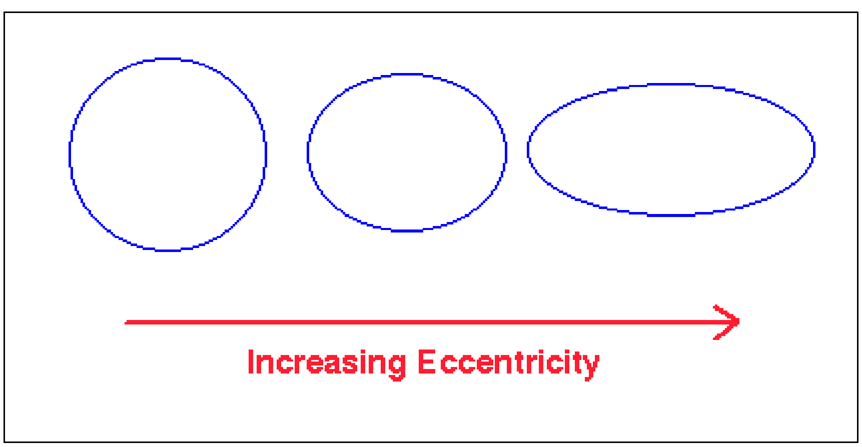

Figure 2. Means of eccentricity

2.2 Some useful data:

1 sidereal year	approximately equals	365.25636 days.
1 tropical year	approximately equals	365.24219 days.
1 sidereal month	approximately equals	29.32 days.
1 synodic month	approximately equals	29.53059days.
1 anomalistic month	approximately equals	29.55455 days.
1 lunar year (12 months)	approximately equals	354.36707 days.
1 Metonic cycle	consists of	19 tropical years (6939.6018 days)
	or	235 mean lunar months(6939.6884 days).

2.3 Useful phases and facts:

1). The phases of the Moon:

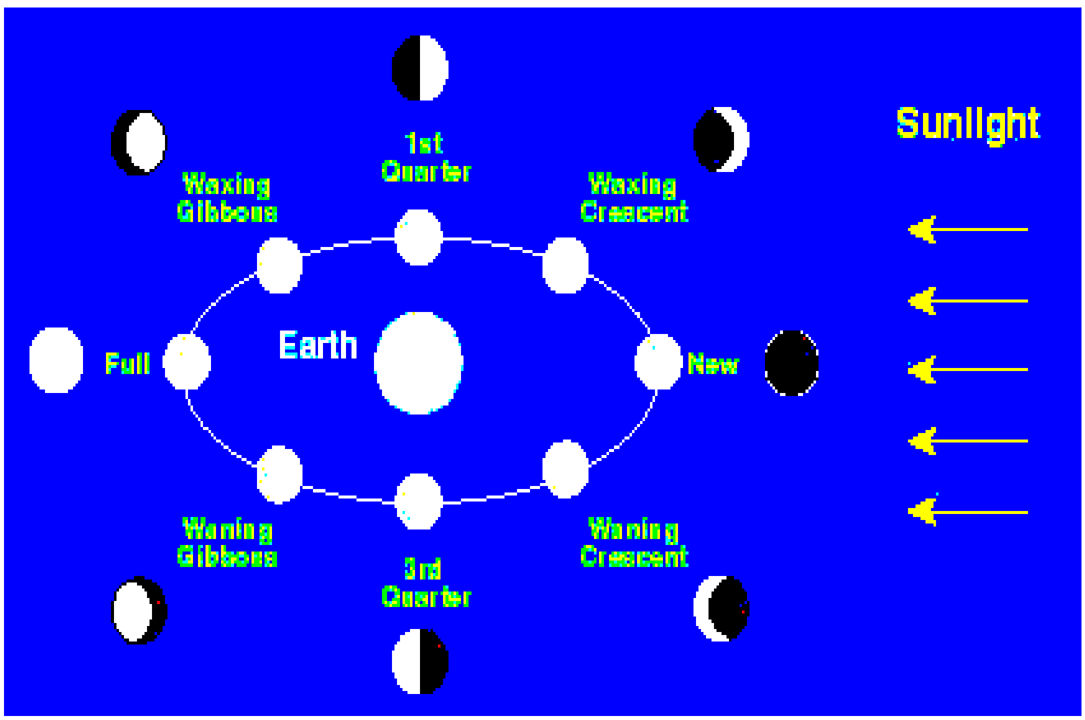

Figure 3. Phases of the Moon

2). Precession of the Earth's Rotation axis:

The Earth's rotation axis is not fixed in space. Like a rotating toy top, the direction of the rotation axis executes a slow precession with a period of 26,000 years (see following figure).

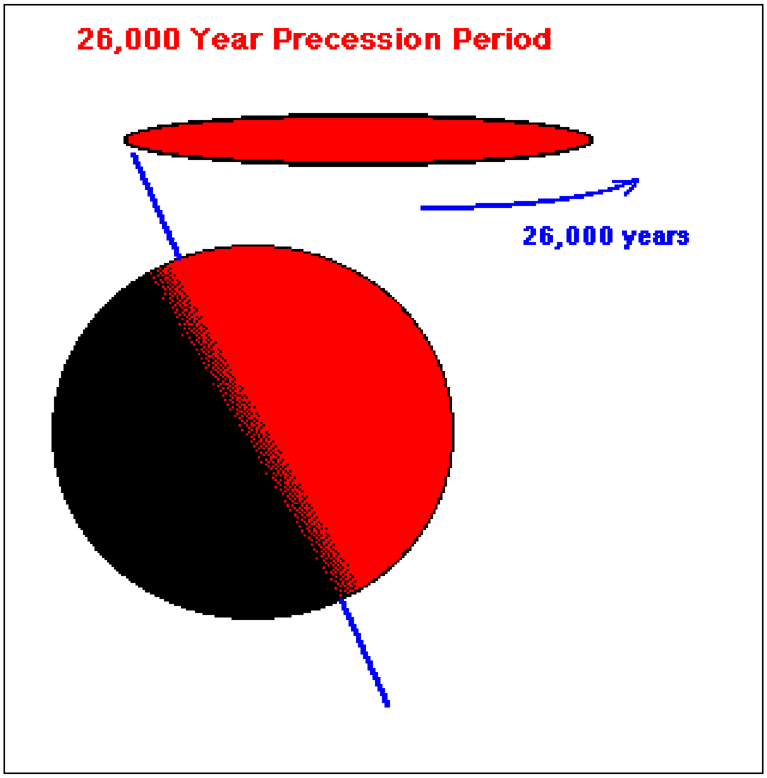

Figure 4. Precession of the Earth’s Rotation axis

Pole Stars are Transient

Thus, Polaris will not always be the Pole Star or North Star. The Earth's rotation axis happens to be pointing almost exactly at Polaris now, but in 13,000 years the precession of the rotation axis will mean that the bright star Vega in the constellation Lyra will be approximately at the North Celestial Pole, while in 26,000 more years Polaris will once again be the Pole Star.

Precession of the Equinoxes

Since the rotation axis is precessing in space, the orientation of the Celestial Equator also precesses with the same period. This means that the position of the equinoxes is changing slowly with respect to the background stars. This *precession of the equinoxes* means that the right ascension and declination of objects changes very slowly over a 26,000-year period. This effect is negligibly small for casual observing, but is an important correction for precise observations.

After we get some familiar with the related astronomy, next we will have a look at different kinds of calendars:

2.4 Basic calendrical concepts:

Generally speaking, there are two ways to classify calendars. For convenience we will illustrate the classification in Table 3.

A \ B	1.Arithmetical	2.Astronomical
1.Solar	Gregorian	French Revolutionary
2.Lunisolar	Jewish	Chinese
3.Lunar	Civil Muslim	Religious Muslim

Table 3: Classification of calendars

A1. Solar calendars

A solar calendar uses days to approximate the tropical year. Being specific, we take the Gregorian calendar as an example.

The Gregorian calendar:

One year = 365 days (for non-leap years)
366 days (for leap years)

Another feature for the Gregorian calendar:

The lengths of every month, except February, are fixed. They are either 30 or 31 days. The length of February for non-leap years, is 28 days, while for leap years is 29 days.

To define leap years in the Gregorian calendar:

Year n is a leap year if n is divisible by 4, but not by 100;
or n is divisible by 400.

Reasons to define the leap years in this way:

One tropical year approximately equals 365.2419 days,
very close to 365.25 days.

But in normal non-leap years, there are only 365 days; in this way, every four years, there will be about 1 day shorter than the actual tropical years. To cover this shortage making the calendar in tune with the tropical years, we should add one extra day to February about every four years. This added extra day is called the leap day, and the year with the added day is called the leap year. For simplicity, we define year n as a leap year if n is divisible by 4. However, this definition will arise some problem; since every 400 years, we can only define 97 leap years. This is because the value of the tropical year is a bit less than 365.25 days. In this way three years, which are divisible by 4 in every 400 years will be fake leap years. For simplicity, we define the leap years in the way explained above.

What is more, the Gregorian calendar is performing a “four-step dance”. To clearly understand this phenomenon, we should have a look at the equinoxes or the solstices --- “three small steps forward” (since non-leap years are shorter than the tropical years) and “one big step backward” (since the leap years are a bit longer). In this way, we keep the Gregorian calendar in tune with the seasons.

Errors in the Gregorian calendar:

As we know, the Gregorian calendar is an approximation to the tropical years. From the above, we know the calendar has a cycle of 400 years, with years of average length about 365.2425 days. This is about 27 seconds longer than the current value of the tropical year (365.2419 days). And this 27-sencond error has accumulated. So every 3200 years, the Gregorian calendar will be in error by one day with respect to the seasons. Due to this reason, a further modification has been suggested: years evenly divisible by 4000 are not leap years. This would reduce the error to 1 day in 20,000 years. However, this last proposed change has not been officially adopted; there is still plenty of time to consider it, since the error would not have an effect until the year 4000.

A2. Lunisolar Calendars

A lunisolar calendar uses months to approximate the tropical year. Notice that the lunisolar calendar is not simply a lunar calendar! The traditional Chinese name for the lunisolar calendar is in fact yin yang li (阴阳历). In other words we can say that lunisolar calendars are solar calendars that just happen to use the lunar months as the basic units rather than the solar days. Actually the calendar is a complex combination and modification of the lunar and solar calendars. Examples are the Jewish and Chinese calendars. In the calendars, a lunar year consisting of 12 lunar months is about 11 days shorter than the tropical year, thus a leap month will be inserted about every three years in order to keep the calendar in tune with the seasons. The big question is how to do this. Two ways are recommended:

a). Traditional method used in ancient time

In ancient Israel, the religious leaders would determine the date for Passover each spring by seeing if the roads were dry enough for the pilgrims and if the lambs were ready for slaughter. If not, one extra month would be added, and the process would be tried next month. As same as the Israel, an aboriginal tribe in Taiwan would go out to sea with lanterns near the new moon at the beginning of spring. If the migrating flying fish appeared, there would be fish for New Year's dinner; otherwise they would try it next month, which means one month was added. Both of these methods are referring to the nature by simple observations and determinations, and the added extra month was defines as the leap month. Obviously this method is based on no theoretical support. So we will come to the other more theoretical method.

b). The way of using the Metonic cycle

By comparing 19 tropical years and 235 synodic months, we find their lengths are almost equal (mentioned in the introduction part). And 235 synodic months = 19 x 12 ordinary months + 7 added months. It follows that we get a fairly good lunisolar calendar if we insert 7 leap months in each 19-year period. But the exact rules for this intercalation can be quite tricky. This method of Metonic cycle is used in the Jewish calendar, and was also used in the Chinese calendar before 104 BCE.

Since the year in a lunisolar calendar is an approximation to the tropical year, the solstices and equinoxes stay relatively constant. The main movement is caused by the insertion of leap months. Each 12-month year is about 11 days shorter, so the solstices and equinoxes move forward 11 (or 10 or 12) days. But each 13-month leap year is about 19 days longer, so the solstices and equinoxes will jump back 19 (or 18 or 20) days. In this way, the solstices and equinoxes are performing a "3-step dance" in the calendar: "two small steps forward" and "one big step back". However, this dance is a bit offbeat. Two of the seven leap years in each 19-year cycle come after just one normal year (since 19 = 3x5 +2x2), so in that case the solstices and equinoxes change into a 2-step rhythm.

A3. Lunar Calendars

A lunar calendar is a calendar that ignores the Sun and the tropical year (and hence the seasons) but tries to follow the Moon and the synodic month. For illustration, we take the Muslim calendar as an example.

Muslim Calendar:

The Muslim calendar is based on the first visibility of the crescent Moon. One year in Muslim calendar is consisting of 12 lunar months. However, since 12 lunar months is about 11 days shorter than the tropical year, the Muslim years are moving forwards with the pace of 11 days per year with respect to the seasons, i.e. the Islamic holidays regress throughout the tropical year with a cycle of 32 or 33 years. The calendar is described with alternating days of 29 and 30 and a system of leap years (in which one extra day will be added in the last month as the leap years) by some sources.

B1. Arithmetical calendars

Arithmetical calendars are calendars based on the mathematical calculations to suit the tropical years. Examples are the Gregorian calendar and Jewish calendar. Predictions and conversions between different arithmetical calendars are in principle simple.

B2. Astronomical calendars

An astronomical calendar is instead a calendar that is defined directly in terms of astronomical events. Examples are the Islamic and Chinese calendars. Strictly speaking, there are two kinds of astronomical calendars. One is referring to the observations of the Sun and the other is referring to the motion of the Moon. However, many Indian calendars are semi-astronomical calendars using traditional formulas for approximating the true motions. In these calendars, errors are likely to develop, because the traditional methods for computations are not accurate enough.

3. The Chinese calendar

Today, the Chinese calendar is commonly used together with the Gregorian calendar in the Chinese society. It is a calendar based on astronomical observations of both the longitude of the sun and the phases of the moon. This means that principles of modern science have had an impact on the Chinese calendar. The Chinese calendar is a lunisolar calendar combining the solar/lunar calendar in which it strives to have its years coincide with the tropical years and its months coincide with the synodic months:

1). An ordinary year has 12 months; a leap year has 13 months (a leap month is added).

2). An ordinary year has 353, 354, or 355 days; a leap year has 383, 384, or 385 days.

In order to understand the Chinese calendar better, we will first have to know some terminology --- shuo (朔) and qi (气) ---used by the Chinese astronomers in the Chinese calendars. Firstly, we have the concepts of **"ping shuo" (平朔)** and **"ding shuo" (定朔)**. Ping shuo means that the motion of the Moon was taken to be constant when the astronomers were calculating the number of days in a month; and in this way we have the term called "mean Moon" (considering the Moon moving with the same speed). Ding shuo, on the other hand, means that the number of days in a month is computed based on the actual movement of the Moon (being inconsistent); and the first day of a month must be the day, on which the first visibility of a new moon occurred. Secondly, we have the concept of **24 jie qi (节气)** (Table 4) (where the even ones are zhong qi (中气), and the odd ones are called jie qi (节气)). It is a terminology used in the calendar to track the seasonal changes throughout a year. With the 24 jie qi, the farmers were able to know the time for planting the seeds of their crops, collecting their grown cops, and starting preparing themselves for winter, etc. The Chinese astronomers had defined the 24 jie qi in two different ways. Initially, they thought that the motion of the Sun with respect to the Earth was constant, and it moved in an average speed and completed one revolution in a year. Therefore they divided a year by 24, and the average value would be the number of days between the successive jie qi. Let us name this number "N" (N is approximately 15). The years will start from the winter solstice, and after every N days, it would change to the next jie qi. This method evenly defining the differences between jie qi's was the so-called **"ping qi" (平气)**. However, hundreds of years later after they discovered that the motion of the Sun is actually inconsistent (as observed from the Earth), people began to define the 24 jie qi by dividing the ecliptic into 24 equal parts, each part occupies 15 degrees. So under this definition, the years will start at the winter solstice, but a change in jie qi occurs when the Sun had actually moved 15 degrees observed from the Earth. This method of defining the jie qi by evenly dividing the ecliptic was called **"ding qi" (定气).**

Number	The Jie qi	Chinese names	English names	Approximate dates
1	Li chun	立春 (节气)	Beginning of spring	February 4
2	Yu shui	雨水 (中气)	Rain water	February 19
3	Jing zhe	惊蛰 (节气)	Waking of insects	March 6
4	Chun fen	春分 (中气)	Spring equinox	March 21
5	Qing ming	清明 (节气)	Pure brightness	April 5
6	Gu yu	谷雨 (中气)	Grain rain	April 20
7	Li xia	立夏 (节气)	Beginning of summer	May 6
8	Xiao man	小满 (中气)	Grain full	May 21
9	Mang zhong	芒种 (节气)	Grain in ear	June 6
10	Xia zhi	夏至 (中气)	Summer solstice	June 22
11	Xiao shu	小暑 (节气)	Slight hot	July 7
12	Da shu	大暑 (中气)	Great hot	July 23
13	Li qiu	立秋 (节气)	Beginning of autumn	August 8
14	Chu shu	处暑 (中气)	Limit of heat	August 23
15	Bai lu	白露 (节气)	White dew	September 8
16	Qiu fen	秋分 (中气)	Autumn equinox	September 23
17	Han lu	寒露 (节气)	Cold dew	October 8
18	Shuang jiang	霜降 (中气)	Descent of frost	October 24
19	Li dong	立冬 (节气)	Beginning of winter	November 8
20	Xiao xue	小雪 (中气)	Slight snow	November 22
21	Da xue	大雪 (节气)	Great snow	December 7
22	Dong zhi	冬至 (中气)	Winter solstice	December 22
23	Xiao han	小寒 (节气)	Slight cold	January 6
24	Da han	大寒 (中气)	Great cold	January 20

Table 4. The 24 Jie qi

Development of the Chinese calendar:

The beginning of the Chinese calendar can be traced back to the Emperor Huang Di (黄帝) in 2637 B.C.E. But officially speaking, the Chinese have used their calendars since the time of the Warring State (春秋战国时代, 770-221 B.C). Initially, the Chinese astronomers thought that the motion of the Moon was constant. In those calendars compiled at that time, they firstly defined an approximated time from one new moon to the next by simple observations and inductions; and then they used this approximated value to compute the number of days in each month. The value of this approximation differed from calendars to calendars since the editors of each calendar computed the value differently. However, all the values that were computed did not differ much from the actual value 29.5306 days. Since 29.5306 is not an integer number, but it is very close to 29.5, people compiled calendars with a short month of 29 days followed by a long month of 30 days, in this way to make the calendars in tune with the actual fact. But occasionally, a short month was followed by two successively long months, since the actual month was slightly more than 29.5 days. This phenomenon of having two successive long months was called the "lian da" (联大). However, in this so-called "ping shuo (mean Moon) theory", the calendars could have at most 2 long months together. Examples are the calendars from the Shang Dynasty (商朝, 1523-1027 B.C).

In the later period of the Eastern Han Dynasty (东汉朝, 25-220 A.D.), the Chinese astronomer Jia Kui (贾逵, ? -92 A.D) was the first to discover that the motion of the moon was inconsistent. This discovery was recorded in his "Si Fen calendar of Eastern Han Dynasty" (后汉四分历, 85A.D), but the various calculations he made for his calendar did not base on this discovery. The first time that the theory of the inconsistent motion of the Moon was brought into consideration in the calculations was the "Qian Xiang calendar" (乾象历, 223 A.D) by Kan Ze (阚泽) in the period of the Three Kingdoms (三国时代, 220-280 A.D). Although the calendar still used the "ping shuo theory" to compute the lengths of months, the inconsistent motion of the Moon was taken into account in the calculations for the eclipses. This improvement greatly increased the accuracy of predictions of the eclipses, and it also meant the initiation of the "ding shuo theory".

After the "Qian Xiang calendar", the inconsistent motion of the Moon began to be considered in the calendar computations. However, that did not mean the calendar computations were on basis of this. Years later, the Chinese astronomer He Chengtian (何承天) proposed the notion of the "ding shuo theory" in his "Yuan Jia calendar" (元嘉历, 445 A.D). But using the "ding shuo theory" would mean strings of four long months and strings of three short months possible to occur, which never happened before. This abnormal phenomenon made the authorities uncomfortable; and the authorities forbidden the theory to be introduced. Finally He Chengtian was forced to use the "ping shuo theory" in his calendar. Another calendar --- the "Wu Yin calendar" (戊寅历, 619 A.D), compiled by Fu Renjun (傅仁均) also considered the "ding shuo theory" in computations. Similarly because the occurrence of a string of four long months caused much dispute and discomfort, the basis of the computations was once again forced to revert back to the "ping shuo theory". It was only after the "Lin De calendar" (麟得历, 665 A.D) compiled in the Tang Dynasty (唐朝, 618-907 A.D), the "ding shuo theory" began to be commonly accepted.

Till the North and South Dynasty (南北朝, 386-589 A.D), the Chinese astronomers called Zhang Zixin (张子信, 6th century A.D discovered the inconsistent motion of the Sun. The astronomer Liu Zhuo (刘焯, 544-610 A.D) of the Sui Dynasty (隋朝, 589-618 A.D) agreed with this theory, and he came out with the concept of "ding qi" in his "Huang Ji calendar" (皇极历, 600 A.D). However, he did not get the actual motion of the Sun, since he thought there was a sudden change in the speed of the Sun at certain times of the year, where the speed either changed from the fastest to the slowest or vice-versa. This is incorrect. In fact the change of the speed of the Sun is gradual; its speed will continuously and slowly increase from the slowest to the fastest; then when it is at the fastest speed, it will begin to slow down until the slowest again. The process has repeated years after years. This actual movement of the Sun was finally understood by the famous astronomer Seng Yixing (僧一行, 683-727 A.D); and he used the theory as the basis for his "Da Yan calendar" (大衍历, 729 A.D). This greatly improved the accuracy of computations. However, the notion of the 24 jie qi only concerns with the changes in seasons, and the changes are normally not noticeable, so the calendars still continued to use the "ping qi theory". The notion of the "ding qi" theory only took the central stages in the calendars one thousand years later in the Qing Dynasty (清朝, 1645-1911 A.D).

To the modern China, after the 1911 Revolution, the Republican government made the Gregorian calendar the official calendar in 1912. From then on, the development of the calendars in China slowed down. And finally together with the Gregorian calendar, the Chinese calendar (the one from the 1645 calendar reform) began to be commonly used among the Chinese.

Apart from the development of the Chinese calendars, precession (mentioned in the introduction part), which was defined as "sui cha" (岁差) also played an important role in the development of the Chinese calendars. It was firstly discovered by the Chinese astronomer Zu Chongzhi (祖冲之, 429-500 A.D) during the North and South Dynasty. He claimed that the axis of the earth was moving, and it moved a degree with 45 years and 11 months. This is a bit away off the modern value, but the discovery of the precession was in itself a great achievement. This theory was adapted later during the calendar making after the "Da Ming calendar" (大明历, 510 A.D). Later in the "Shou Shi calendar" (授时历, 1281 A.D) by Guo Shuojing (郭守敬, 1231-1316 A.D), the value was changed to a degree per 66 years and 8 months.

All the developments above concerning with the Chinese calendars can be summarized by the table below:

Theory/concept	When first introduced	When commonly accepted
Inconsistent motion of the moon	First discovered by Jia Kui (? -92 A.D) in the Eastern Han period (25-200 A.D); first discussed in the Si Fen calendar (85 A.D) of the Eastern Han Dynasty (25-200A.D)	After the Qian Xiang calendar (233 A.D) of the Three Kingdoms (220-280 A.D)
Inconsistent motion of the sun	First discovered by Zhang Zixin(6^{th} century A.D) during the North and South Dynasty(386-589 A.D); first mentioned in the Huang Ji calendar(600 A.D) of the Sui Dynasty(589-618 A.D)	After the Da Yan calendar (729 A.D) of the Tang Dynasty (618-907A.D)
Ping shuo	From the first Chinese calendars in the Shang Dynasty (1523-1027 B.C)	

Ding shuo	First proposed in the Yuan Jia calendar (445 A.D) of the North and South Dynasty (386-589A.D); first used in the Wu Yin calendar (619 A.D) of the Tang Dynasty (618-907A.D)	After the Lin De calendar (665 A.D) of the Tang Dynasty (618-907 A.D)
Ping qi	Zhuan Xu calendar (174 B.C) of the Warring States (770-221 B.C)	
Ding qi	Huang Ji calendar (600 A.D) of the Sui Dynasty (589-618 A.D)	After the Shi Xian calendar (时宪历) of the Qing Dynasty (1645-1911 A.D)
sui cha	First discovered by Zu Chongzhi(429-500 A.D) of the North and South Dynasty(386-589 A.D); first mentioned in the Da Ming calendar(510 A.D) of the North and South Dynadty(386-589 A.D)	

Table 5. The development of the Chinese calendars

4. The Last Reform of the Chinese calendar in the Qing Dynasty (清朝, 1645-1911)

--- Where the current Chinese calendar came from

In the ancient China, the astronomy was highly thought of, because of the influence by the concept that human beings and nature were interacted. But in Late Ming (明朝, 1368-1644 A.D) and Early Qing Dynasty, it turned out to be the interface of Chinese and Western cultures as the Jesuits took the astronomy as an avenue to preach Christianity in China.

4.1 The background for this calendar revolution:

In 1582, the first Jesuit missionary Matteo Ricci (1552-1610) came to China. He learned Mandarin, and then translated the classical western science of cartography and the radically improved astronomy of Galileo into Chinese. In this way he tried to impress the cultivated Chinese elite with western forms of knowledge. At the same time, he managed to convert a leading Chinese official, Xu Guangqi (徐光启, 1562-1633 A.D); together they performed an incredible task of translation and interpretation. That was at the end of the Ming Dynasty; the government was corrupted; sciences stopped advancing. And because of the ineffectiveness of the mathematical computation and the weakness of the astronomical knowledge, the Chinese calendar was no longer accurate. Positions in the Bureau of Astronomy had become hereditary; the astronomers no longer well understood the principles behind the old calendar. Mistakes began to occur frequently in the phenomenon predictions and calendar computations. On December 15, 1610, they made an error of more than half an hour in computing a solar eclipse, causing serious embarrassments. Because of this, in 1629 Xu Guangqi was asked to revise the calendar. He asked the Chinese and Muslim astronomers in the Bureau and the Jesuits outside to make predictions for an upcoming solar eclipse on June 21, 1629. In result, the Jesuits had the best predictions. So when Xu was made the director of the Astronomical Bureau, he appointed the Italian Terrentius and another Jesuit as members (this never happened in China before). In the later years, more and more Jesuits came to China in the mission of developing the Christ. And many began to be employed in the Chinese government. However, there were not commonly accepted by the Chinese until the Qing Dynasty.

After the Ming Dynasty, China began to be under the Qing Dynasty. In 1644, the German Adam Schall, Johann Adam S, von Bell (1592-1666), the leader of the Jesuits in China, went to the new rulers and presented his calculations for the upcoming eclipse on September 1. At that time, the Manchus, the new ruler of China did not trust the native Chinese --- Han Zu (汉族). As a foreigner, Schall won the trust of the rulers. Further he challenged the Chinese and the Muslim astronomers in the Bureau during some tests of predictions and computations, and won almost all the best. Finally he was appointed the director of the Astronomical Bureau. The next year, he formulated the current rules for the Chinese calendar, in which he made four main changes due to the old rules (and this will be talked about later). In addition, being knowledgeable in mathematics, astronomy, calendar making and many other areas, he easily made many friends, even the emperor Shun Zhi (顺治). Besides he was assigned a mandarin of the first grade, first division in the government. Except for Schall, other Jesuits were also well treated that time. In short, the fortunes turned for Jesuits, even though the anti-Christian officials and astronomers attacked them time-by-time.

Later in 1661, the emperor Shun Zhi died, leaving his designated heir, a child aged eight sui (岁). A committee of four ministers had been delegated by the former emperor to act as regents. But actually at that time, the real power of the country was held by one of the regents, Ao Bai (鳌拜, ?-1669 A.D). He was so powerful that he paid no attention to the young emperor. The basic policy during the regency was "rule from horseback". In other words the regents resorted to brute force. The regents also tried to restore the privileges of the Manchus, which had been undermined during the Shun Zhi era. Under these conditions, security and order in the empire became the regents' main concern. At that time, an anti-Christian Chinese official, Yang Guangxian (杨光先, 1597-1669 A.D), had as his slogan that it was "better to have a wrong calendar than to have foreigners in China". He published much of what he had written and circulated his anti-Jesuit treatises widely to attack the Christians. Yang had several complaints against the Jesuits. (a). The new calendar had two zhong qi's in the 11th month of 1661, something that was impossible under the old system. (b). Both the months after the 7th month and the 12th month had no zhong qi. And of these two, the first was a leap month, but the 12th was a fake one, which did not exist under the old system. (c). In the new calendar, the 11th month had three jie qi's in 1661, something that was not possible in the old system either. (In fact, the last jie qi occurred 39 minutes after midnight, it should have been in the following month, but the Jesuits made an error.) (d). When Schall presented to the emperor with a calendar for the next 200 years, Yang claimed that this was improper since the emperor was blessed with infinite reign. (e). Yang urged the government to beware of the treacherous, factious Jesuits who were spreading their books and teaching to attract believers in order to develop the Christian in China. (That time, the Jesuits had established many churches throughout the empire.) Yang's accusations brought to the surface the deepest fears of the regency. So, though the Jesuits made great effort for disputes and tried hard to attack back, Yang's memorials was still accepted and formal charges were to be leveled against the Jesuits.

Later, after Schall made an error in choosing an inauspicious date for the funeral of the emperor Shun Zhi's favorite son. Which was considered to bring misfortunes to the imperial house. Yang managed to have Schall, the Belgian Ferdinand Verbiest, and two other Jesuits arrested in 1664. But "Battles" between the Jesuits and the anti-Christian group still went on. While in prison, another test was held. On January 16,1665, a solar eclipse was coming up; the Jesuits predicted it would occur at 3 pm, Yang predicted 2.15 pm, and the Muslim Wu Mingxuan (吴明炫) predicted 2.30 pm. On the day of the eclipse, the Jesuits were brought into the palace in chains, and everybody watched as the eclipse occurred at 3 pm sharp (14:59:54 according to computations by Salvo De Meis), exactly as the Jesuits predicted. Unfortunately, the regents were not impressed. On April 15 of the year, the Jesuits were sentenced to death. However, the next day a strong earthquake struck Beijing (北京, the capital of China), which was taken as a sign from the Heaven that the sentence was unjust. So the sentence of the Jesuits was first converted to flogging, and eventually to just house arrest. But the death-sentence for five of their Chinese assistants was upheld and carried out, and churches all over China were closed.

After the Jesuits were removed from the Astronomical Bureau, Yang was appointed to the position of director. The Chinese old method was again restored and Wu became the vice director. However, at that time, many officials remaining in the Bureau had come to believe in the Jesuit astronomy in which they had been trained.

In 1667, the young emperor Kang Xi (康熙) ascended to the throne. And in 1668, he took over the power from the regents. That time, Yang had failed to resolve the calendrical problems by resorting the traditional method. The emperor was uncomfortable about the Astronomical Bureau. In one of his edicts, he demanded a prediction of the Sun's position on the following day to test all the astronomers. And the winner was the Jesuit father Ferdinand Verbiest (Nan Huai-jen 南怀仁, 1623-1688 A.D) who performed a successful prediction. However, both Yang and Wu denied the accuracy of the test; so two additional tests were ordered to be set. Again the Jesuits won. By now Yang had been humiliated so thoroughly that he did not even bother to take part, and Verbiest easily beat Yang and Wu in the test. On April 17,1668, Verbiest was appointed the director of the Astronomical Bureau. In the following days, the emperor sent a copy of the 1669 calendar made by Yang and Wu to Verbiest and asked him to make comments. At the end of the twelfth month of 1668, Verbiest informed the emperor in a memorial that the calendar he had reviewed by then had many errors. And the man actually responsible for the errors, Wu Mingxuan, in order to keep his post in the Bureau, began to endorse the Verbiest's calculation of the intercalary month. Since that time, the Jesuits began to stabilize their position in the Astronomical Bureau. And the Jesuits remained at the position of the director until 1746, and other Westerners ran it until 1826.

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4.2 The Four Main Changes in the reform:

Like the inevitable triumph of "better" theories described in the scientific textbooks, the "better" Jesuit astronomy finally triumphed. However, the Jesuits were vindicated not simply because of the efficacy of their astronomy, but also because they won official approval of their astronomy. In fact, the Chinese astronomers did not easily accept the measurement of astronomical effectiveness defined by the Jesuit unless they were very reasonable. So what the Jesuits could do was to break through or reshape previous disciplinary and social boundaries in order to prevail. And finally the fact had shown that, the Jesuits had been able to demonstrate their superiority of the astronomy since the late Ming. At that time, in the Shun Zhi Era, the Jesuits were warmly treated, and one of their leaders --- Adam Schall led a revolution of the Chinese calendar, which was the last reform of the four main revolutions of the Chinese calendars; at the same time he compiled the Shi Xian calendar, in which there are four main changes due to the traditional calendars:

A. Change “觜 (zui) before 参 (can)” to “参 before 觜”

In ancient China, the astronomers divided the sky into 28 different parts (each part is a class of stars). Among these 28 districts, “参” and “觜” was the closest pair. The sequence of these two classes played an important role in the divination. And it had been a controversy which of these two is in front ever since the time they were first defined. Traditionally, people thought, “觜” is before “参”, which is so-called “觜前参后”. But after the 13th century, by the influence of “sui cha”, the traditional way “觜 before 参” began to reverse; it became “参前觜后”. But this was not discovered in China until the end of Ming Dynasty. And the Jesuit Adam Schall first indicated it in his Shi Xian calendar.

B. Defining the day into 96 quarters, and one quarter is 15 minutes

Traditionally in China, a day was divided into 100 equal intervals, and each interval is called a quarter, i.e. one day is equal to 100 quarters. What was uneasy for the traditional calendar was that the number 100 was hard to suit the actual days of 12 Shi Chen (时辰), or 24 hours. In the new calendar by Schall, a day began to be divided into 96 quarters (24 hours x 4 quarters per hour), and one hour was made up of 4 quarters (15 minutes). In other words, we can say under the new calendar, one day is consisting of 24 hours or 96 quarters. This change was easily accepted, since it simplified the calendar, but did not break the traditions.

C. Other modifications

In traditional Chinese calendars, the timings for the sunrise and sunset only respect to the capital Beijing (北京) were included. But in the new calendar, the timings for the sunrise and sunset were contained due to all different provinces through the country. However, the folks did not appreciate this modification very much, since at that time, people were only sensible to the hours in time. Actually timings for sunrise and sunset could be easily known by common sense and intuition. What is more, in the old calendars, the Chinese astronomers lost the point that the timing for jie qi was different in different time zones. In the case of China, the maximal differences were 3-4 hours. In the new calendar, the timetable for the 24 jie qi with respect to different time zones was added. But in people’s daily life, this modification was not important since the few-hour-difference could be ignorant. So far, we can find that, some of the modifications were not useful enough, but they did contribute a lot to introductions from the west to China.

D. Chang ping qi to ding qi

In the old calendar systems, the traditional method --- the so-called ping qi concept --- divided the 365.25 (approximately) days of a year into 24 equal fortnightly periods (12 months). Therefore, there were two fortnightly periods in a month. This method was reasonably simple, since it made the 24 jie qi almost fixed in the calendar. However, it was not due to the actual motion of the Sun, and this will leap one or two days' error for the time of each jie qi compared with the facts. It means that under this method, we cannot make the calendar exactly in tune with the seasons. But in the new calendar with the so-called ding qi concept (fixed fortnightly periods), we set up fortnightly periods by dividing the ecliptic into 24 equal parts. Each fortnightly period occupied 15 degrees of the ecliptic. The interval between two fortnightly periods was not fixed due to the difference of the Sun's apparent velocity in different seasons observed from the Earth. It was possible for three fortnightly periods to occur in a single lunar month. One jie qi is at the beginning of the month, another is in the middle of the month, and the third one is at the end of the month. Besides the definition of leap years and leap months was also changed. As in the new calendar, we had year n as a leap year if it had 13 months. In the leap years, the first month without zhong qi was considered as the leap month, and others are non-leap months, even the second no-zhong qi month.

Some of the changes are very important, but others seem not that necessary. This caused a lot of controversy over the contributions of the Jesuits. It was really a complex issue. Some Chinese critics argued that this was an attempt at keeping China backwards by introducing too much from the west in. But it is important to realize that when it comes to calendar making, the accurate observations and computational skills are more important. At this point, this calendar reform by the Jesuit missionaries really made a positive contribution to the development in China, since it emphasized more on the actual observations and computations --- more related to modern sciences and technologies.

Ever since the sixth century CE they had again discovered that the motion of the Sun was also irregular. Then why did the astronomers not adopt this in the calendars until the Qing Dynasty? This is because the discovery was really a great achievement; however, using the results of the discoveries meant bringing a lot of troubles in. To avoid these troubles they forbade the new discoveries to be introduced. And this avoidance would last until they found that the inefficient traditional method had caused much more troubles and mistakes. In the early Qing Dynasty (nearly 1000 years after the first discovery), the true Sun had started to be used in the calendars. This change gave the Jesuits chances to demonstrate their superiority in calendars. And making changes of the calendar by using the true Sun was a great way of making themselves essential as well. But in spite of this, the Jesuits really made a positive contribution to China, which can serve as an example of a successful Sino-Western exchange.

5. Strings of three Short Months and four Long Months in the Chinese calendar:

Before we get to know the strings of short months and long months, we should have some background knowledge about the basic astronomical facts.

As explained above in the introduction part, the Earth revolves counterclockwise around the Sun in an elliptical orbit with the Sun at one of its foci. (See Figure 4 below)

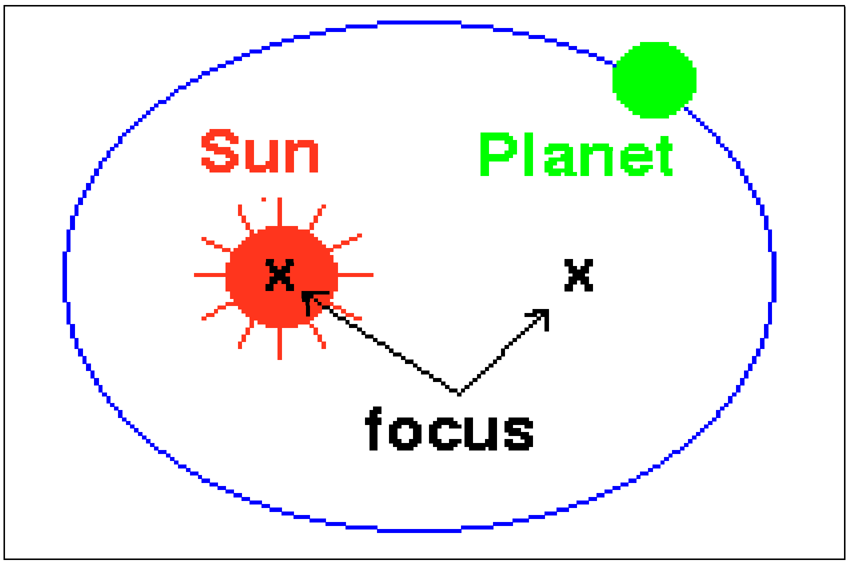


Figure 4. The orbit of the Earth is an ellipse, with the Sun at one focus of the ellipse

Then how does the Earth perform the movement of revolving? Early astronomers already realized that the motion of the Earth along the ecliptic was not uniform. Due to the consequence of Kepler's Second Law, it is said that the line joining the Earth to the Sun sweeps out equal areas in equal times as the Earth travels around the ellipse. (See Figure 5)

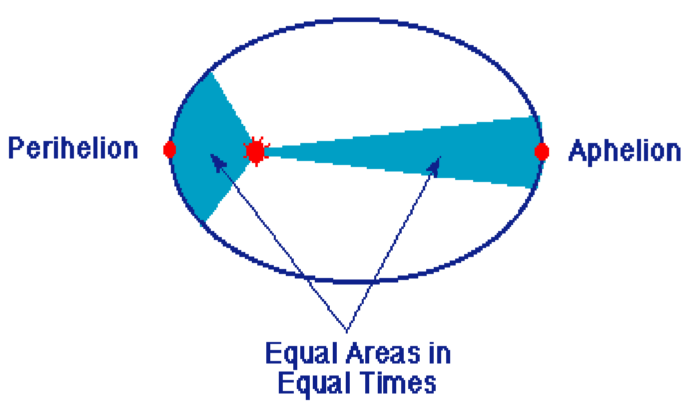


Figure5. Kepler's Second Law

This means that the Earth moves faster along the orbit near perihelion --- the point on the orbit where the Earth is closest to the Sun, while it moves slower along the orbit near aphelion --- the farthest point on the orbit from the Sun. And further, the speed of the Earth will change gradually from the fastest to the slowest, or vice versa. (See the Figure below)

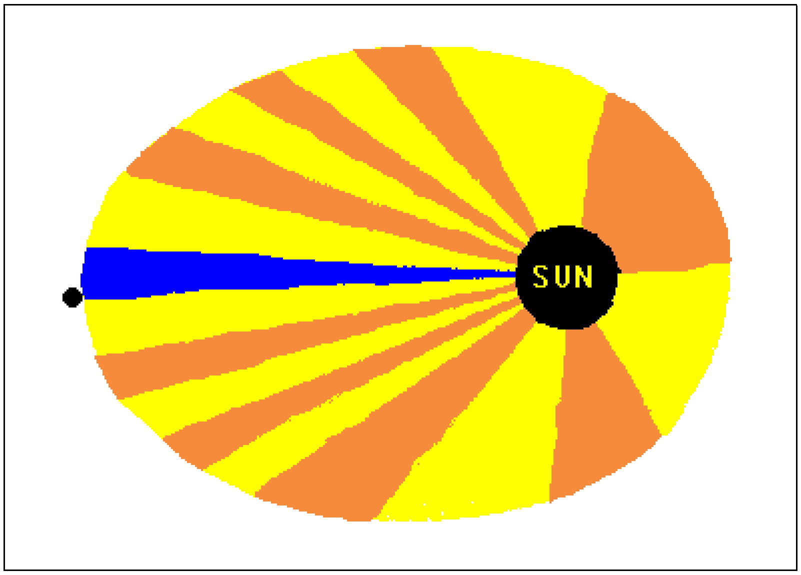


Figure 6. The motion of the Earth around the Sun

Due to the fact that the Chinese calendar focuses more on the motion of the Moon, apart from the motion of the Earth, we should have a look at the motion of the Moon as well. Indeed the motion of the Moon is very complex; it is highly irregular. However, we find it moves around the Earth counterclockwise in an elliptical orbit. (The same direction as the Earth moving around the Sun) In this way, when the Earth moves faster, the eccentricity of the Moon's orbit is larger, and the Moon should take longer time to finish one revolution, since at that case, it is harder for the Moon to catch up with the Earth. But when the Earth moves slower, the eccentricity is smaller, and it will take less time for the Moon to complete one revolution.

Now, after have some knowledge about the movement of the Moon, we should have a look at the lunar month, which is closely related to the motion of the Moon. Astronomically speaking, a lunar month is actually the interval between two lunar phenomena or phases (for example, the new Moon), rather than a fixed period of time. There are several different kinds of months defined in astronomy textbooks (anomalistic, sidereal, synodic, and so on), but the one linked directly to the Moon's phase --- the synodic month of about 29.25 days --- is most familiar in everyday life.

Further, the period between new Moons (the time for the Moon to complete one revolution) is also influenced by the eccentricity of the Moon's orbit around the Earth and the eccentricity of the Earth's orbit around the Sun. In fact, the Moon makes more than one complete revolution between two new Moons, since it must make up the additional angular distance that the Earth has moved during the month. If the Moon is near perigee, the Moon moves faster, the extra distance can be made up quicker than average, the eccentricity of the Moon's orbit is higher and the synodic month is shorter. While if the Moon is near apogee, the Moon is moving slower at this point, the extra distance is made up slower than average, the eccentricity of the Moon's orbit is higher and the synodic month is longer. (See Figure 7) Additionally, if the Earth is near perihelion, it will move faster, increasing the amount of distance that the Moon must make up. In this case, the synodic month will be longer. But if the Earth is near aphelion, it will move slower, reducing the amount of distance that the Moon must make up; then the synodic month will be relatively shorter.

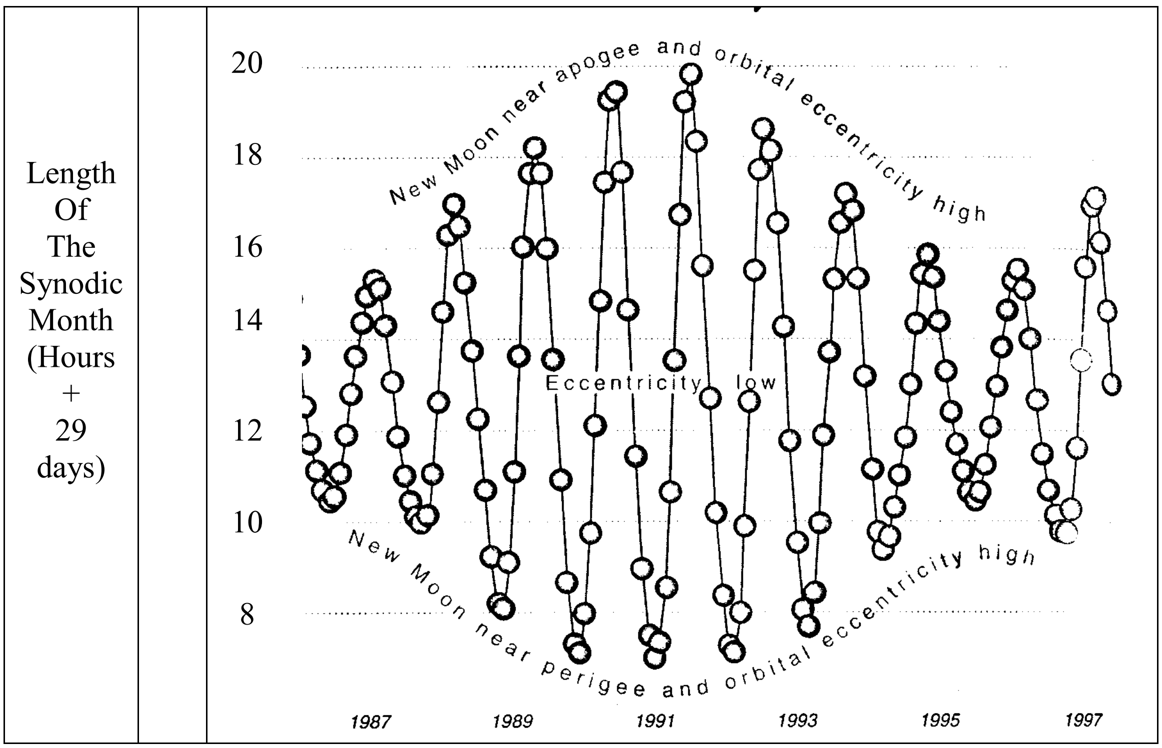


Figure 7. The length of the synodic months

From above, we know, the points perigee and aphelion have the same effect, reducing the length of the synodic month. So if perigee, aphelion, and the new Moon are coincident, the effects will reinforce each other. For example, in the very case, the period between the new Moons of 15 May and 13 June in 1999 was only 29 days, 6 hours, 58 minutes. Similarly if apogee, perihelion and the new Moon are coincident, the effects will also reinforce each other. The example is the period between the new Moons of December 7, 1999 and January 6, 2000, whose length was 29 days, 19 hours, 42 minutes.

Furthermore, considering the exact length of the synodic month (or lunation), from Figure 7, we find its average value is $29^{d}\,12^{h}\,44^{m}$. The shortest --- at the trough of the figure --- is $29^{d}\,6^{h}\,31^{m}$. It happened with the new Moon on June 18, 1708. While the longest --- at the peak of the figure --- is $29^{d}\,19^{h}\,59^{m}$, and it happened with the new Moon on December 15, 1610. In other words, a lunation can be as much as $6^{h}\,13^{m}$ shorter, or $7^{h}\,15^{m}$ longer, than its averages. In fact, if the lunar month is measured between consecutive full Moon rather than new Moon, exactly the same range of values will be found.

Besides, due to the fact that the period between perigees (the anomalistic month) on average is 27.554 days, and the synodic month is 29.531, it happens that the time for 14 synodic months (average moon phase cycle) is very close to 15 anomalistic months, which is about 413 days. This is considered as the period of the short cycle in Figure 7, which means that every 14 new Moons, there is a good coincidence between perigee/apogee and the new Moon. However, this period of 14 new months is a bit longer than a year, which loses the reinforcing effects with aphelion/perihelion.

In addition, we find that the time for 112 synodic months is very close to 120 anamalistic months. This is considered as the period of the longer cycle of Figure 7 (we name it the Jawad Cycle), which means that every 112 new moons, there is a good coincidence between perigee/apogee and the new Moon, causing periods between new moons that are less/more than the average. What is more interesting is that this period is pretty close to nine years. That means every Jawad period, there is a very good coincidence between perigee/apogee, aphelion/perihelion and the new Moon, causing the occurrence of the shortest or longest synodic months.

14 synodic months	approximately equal to	413.434 days.
15 anomalistic months	approximately equal to	413.310 days.
One year	approximately equals to	365.242 days.

112 synodic months	approximately equal to	3307.472 days.
120 anomalistic months	approximately equal to	3306.546 days.
Nine years	approximately equals to	3287.178 days.

However, on the research of the Chinese calendar, these periods are of no interest to us, since both are much away from integers. Calendrically speaking, to define the months, we are only interested in the days when the new Moon appears. Based on the meridian 120^{o} east, the day on which a new Moon occurs is defined as the first day of the new month. So even if the new Moon takes places a bit before the midnight, the whole day is still considered as the first day of the new month. This differs a lot from the purely astronomical definitions. Under this definition, a month can have 30 days even though the length between two new moons is only 29 day plus a few hours. For instance, if the first new Moon occurs at 23:59 on May 1, and the next new Moon will occur at 06:00 on May 30, then this lunar month will be of 30 days though its actual length is only 29 days and 6 hours.

After the adoption of the "ding shuo" theory, under the definition above, we find if there is a good coincidence of apogee/perigee and perihelion/aphelion at the new Moon, it will be possible to have a sequence of three short months or four long months. However, besides of this, to have 3 short months in a row also requires that the period between four consecutive new Moons must be less than 88 days and the first new Moon should occur a bit after the midnight. And to have 4 long months in a row requires that the length between five consecutive new Moons be greater than 119 days and the first new Moon should occur a bit before the midnight. About every 9 years, these conditions are fulfilled. For strings of short months, it is when there is a good reinforcement between perigee and aphelion, and for strings of long months, it is when there is a good reinforcement between apogee and perihelion. For example, the period between the new Moon of April 16, 1999 and the new Moon of July 13 that year was 87 days, 22 hours, 2 minutes (just slightly less than 88 days), so there happened to be a string of three short months that year. The period between the new Moon of October 9,1999 and the new Moon of February 5, 2000 was 119 days, 0 hour and 29 minutes (just slightly greater than 119 days), and that year there happened to be a string of four long months.

In the above paragraph, we mentioned the specific time slot for the first new Moon as another condition for strings of short months or long months --- a bit before or after the midnight. Then how to define this "a bit"? For the three short months, it requires that the length of three consecutive short months is slightly less than 88 days; that means the average length of the three months is slightly less than 29 days and 8 hours. Since the new Moon that marks the start of the string must be slightly after midnight, what if the new Moon last April 16 had occurred at 00:01, then on average after 29 days and 8 hours, the next new Moon would have occurred on May 15 at 07:59, marking the end of the first short month. The following new Moon would have occurred another 29 days and 8 hours later on June 13 at 15:59, marking the end of the second short month. And finally the new Moon happened last 29 days and 8 hours later on July 12 at 23:58, marking the end of the third short month. This is an extreme example taking the length of three consecutive short months slightly less than 88 days. But if the length of three consecutive months is "M" hours less than 88 days, then the first new Moon must occur less than M hours after the midnight, in order to make the occurrence of a string of three short months. While for the strings of four long months, it requires the length of four consecutive months is a bit larger than 119 days. That means on average each of the four long months should be a little greater than 29 days and 18 hours. So similarly, if the length of the four long months is "N" hours greater than 119 days, then the initial new Moon must occur less than N hours before the midnight. Otherwise the string of four long months is impossible to happen.

However, the time of the new Moon is not very likely to happen exactly as what we want. So now we know how to get a string of short months or long months. But if we know the circumstance of the first string (including the timing of the initial new Moon and the length of the string), then how can we predict the timing for the occurrence of the next string? In other words, we are interested in the requirement for the difference between the initial new moons of the two consecutive strings. We find that in order to get two consecutive strings of short months or long months, we should have the difference being pretty close to an integer number of days. And this integer number of days should be the resonance of X synodic months, Y anomalistic months and Z years, where X, Y and Z are integers. We name this difference as the resonance period. Under the definition of the resonance periods, the time of the first new Moon of the strings will be about the same time. If of the strings of short months, it is at 06:10 one time, and 06:00 the next time, and 05:50 the next (and these are about nine years apart), then maybe there will be long periods without the sequence of three short months, followed by periods in which the first new Moon will regularly occur close enough to midnight to cause three short months in a row on a regular basis.

Following, we will have a look at how the strings of short months or long months occurred in the 1000 years from the last calendar reform in 1645. (See Table 6 & 7)

Year	Month	Day	Year	Month	Day
1646	2	16	2107	3	25
1700	3	21	2328	3	13
1708	2	22	2336	2	14
1921	3	10	2382	4	15
1929	2	10	2398	2	18
1983	3	15	2444	4	19
1991	2	15	2452	3	22
2037	4	16	2673	3	11
2045	3	19	2727	4	14
2053	2	19	2735	3	17

Table 6. The ending dates of strings of long months

From these two tables (Table 6 & 7), we find that strings of long months always occur near the winter solstices at the perihelion (covering November, December, January and February; sometimes March being included), while the strings of short months always occur near the summer solstices at the aphelion (covering May, June and July; sometimes August and September are included). Further, we find there are more strings of short months than strings of long months. This will be explained later.

Year	Month	Day	Year	Month	Day
1735	9	16	2151	9	10
1743	8	19	2158	7	25
1744	9	6	2167	7	16
1752	8	9	2487	9	17
1753	8	28	2488	10	5
1760	7	12	2495	8	20
1761	7	31	2496	9	7
1762	8	19	2497	9	26
1769	7	3	2504	8	11
1770	7	22	2505	8	30
1788	9	29	2512	7	14
1797	9	20	2513	8	2
1805	8	24	2514	8	21
1806	9	12	2521	7	5
1814	8	15	2522	7	24
1822	7	18	2541	10	20
2089	9	4	2549	9	22
2097	8	7	2557	8	25
2098	8	26	2558	9	13
2133	9	28	2566	8	16
2143	10	8	2575	8	7
2150	8	22			

Table 7. The ending dates of strings of short months

After considering the dates of the strings to happen, let us have a look at the differences between the strings of short or long months --- the resonance periods.

3 short				1735	1743	1744	1752	1753	1760
4 long	1646	1700	1708						
Differences	19756	2894	10068	2894	384	2894	384	2510	384

3 short	1761	1762	1769	1770	1788	1797	1805	1806	1814
4 long									
Differences	384	2510	384	6644	3278	2894	384	2894	2894

3 short	1822								2089
4 long		1921	1929	1983	1991	2037	2045	2053	
Differences	36029	2894	19756	2894	16862	2894	2894	13346	2894

3 short	2097	2098		2133	2142	2143	2150	2151	2152

4 long			2107						
Differences	384	3132	9684	3278	384	2510	384	384	2126

3 short	2158	2159	2160	2167	2168	2176	2177		
4 long								2328	2336
Differences	384	384	2510	384	2894	384	55017	2894	16862

3 short						2487	2488	2495	2496
4 long	2382	2390	2398	2444	2452				
Differences	2894	2894	16862	2894	12962	384	2510	384	384

Table 8. The strings of 3 short months or 4 long months & the differences in days

From Table 8, we find that calendrically speaking, there are in total 8 different resonance periods, which are (1) 384 days; (2) 2126 days; (3) 2510 days; (4) 2894 days; (5) 3278 days; (6) 6644 days; (7) 16862 days; (8) 19756 days. (See Table 10) Based on definition of the resonance period, we propose the theory as follows:

Some known definitions:

1 syn = 1 synodic month = 29.53059 days
1 ano = 1 anomalistic month = 27.55455 days
1 year = 365.25964 days

Proposed theory:

A resonance period is a triple of three integers X, Y, Z, so that

1. X synodic months are approximately equal to integer number of days.
2. X synodic months are approximately equal to Y * anomalistic months
3. X synodic months are approximately equal to Z years.

Define:

Error 1 = X * syn – the integer days (the period)
Error 2 = X * syn – Y * ano
Error 3 = X * syn – Z * years

Table 9. The hypothesis

Period	Days	X*Syn Approximately == y * Ano Approximately == z * Years			X * Syn	Y * Ano	Z * Years	Error 1	Error 2	Error 3
		x	y	z						
1	384	13	14	1	383.89767	385.7637	365.25964	-0.10233	-1.86603	18.63803
2	2126	72	77	6	2126.20248	2121.70035	2191.55784	0.20248	4.50213	-65.35536
3	2510	85	91	7	2510.10015	2507.46405	2556.81748	0.10015	2.63610	-46.71733
4	2894	98	105	8	2893.99782	2893.22775	2922.07712	-0.00218	0.77007	-28.07930
5	3278	111	119	9	3277.89549	3278.99145	3287.33676	-0.10451	-1.09596	-9.44127
6	6644	225	241	18	6644.38275	6640.64655	6574.67352	0.38275	3.35345	0.932648
7	16862	571	612	46	16861.96689	16863.3846	16801.94344	-0.03311	-1.41771	60.02345
8	19756	669	717	54	19755.96471	19756.61235	19724.02056	-0.03529	-0.64764	31.94415

Table 10. Analysis of the eight periods I

Periods	Days	Frequency of periods for strings of short months	Frequency of periods for strings of Long months
1	384	17 (most frequently)	
2	2126	1	
3	2510	5	
4	2894	7	9 (frequently for both strings)
5	3278	2	
6	6644	1	
7	16862		3
8	19756		2

Table 11. Analysis of the eight periods II

From Tables 10 and 11, we find:

(1). The periods are surprisingly related to each other.
For strings of short months:
2510 = 2126 + 384;
2894 = 2510 + 384 = 2126 + (2 * 384);
3278 = 2894 + 384 = 2126 + (3 * 384);
For strings of long months:
19756 = 16862 + 2894.

(2). The period of 2894 days happens frequently for strings of both short and long months.

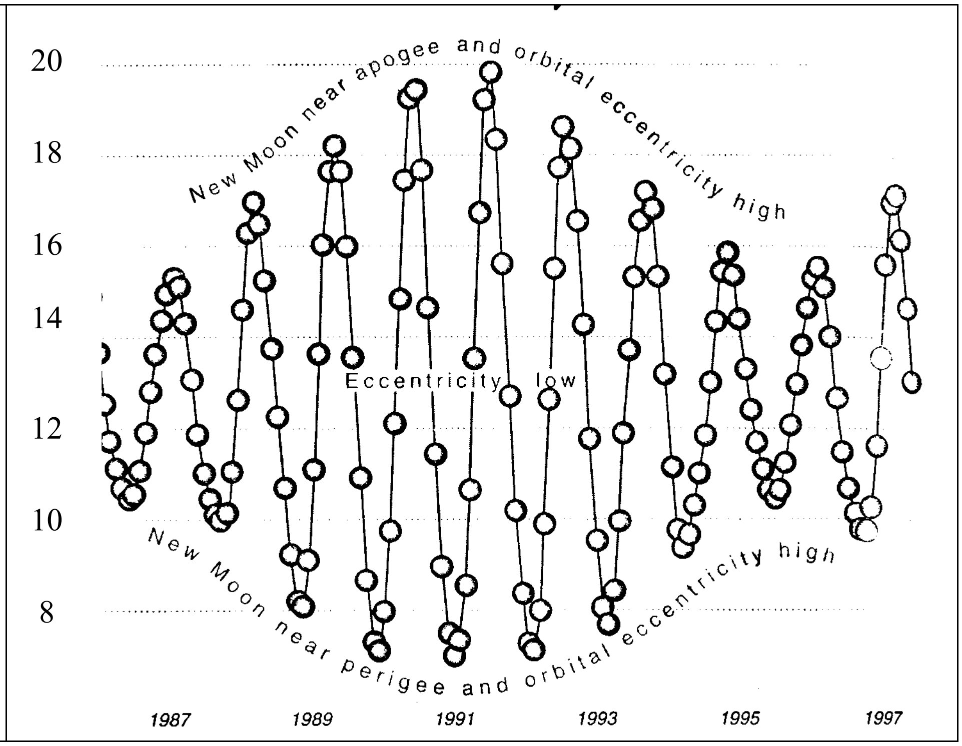
(3). Both for the strings of short months and long months, the shortest period happens the most frequently.

(4). The periods of 16862 days and 19756 days only happen for strings of long months.

(5). The periods of 384 days, 2126 days, 2510 days, 3278 days and 6644 days only happen for strings of short months.

Before explaining these five phenomenons, we should get familiar with the requirements for the strings of short months or long months --- the first new Moon of the strings should happen when there is a coincidence of apogee/perigee and perihelion /aphelion; the length of three short months should be less than 88 days, and for strings of long months it should be greater than 119 days.

Length Of The Synodic Month (Hours + 29 days)



Explanations of why only Periods 4, 7 & 8 happen for strings of long months and why the frequencies of different periods are different:

Referring to the figure above, two features are obvious: a short cycle of slightly more than one year (about 413 days), and a longer cycle (the Jawad Cycle) of about nine years (about 3307 days). However these two periods are much away from integers (explained above), they are not the resonance periods. But the two closest resonance periods --- 384 days and 3278 days --- are more interesting; both are one month shorter than the cycles of the figure. In principle, these two periods should be the most likely to happen among all the eight resonance periods. However, this is incorrect for strings of long months. We know that to have two consecutive strings, we must have the timing for the first new moons being consistent. That means we should have Error 1 as small as possible. The smaller Error 1 is, the more consistent the first new moons of the strings will be, and the more possible consecutive strings will happen. Among the eight periods, except Periods 4, 7 and 8, the other five all have Error 1 more than two hours. For strings of long months, the length of the string is hardly two hours greater than 119 days, which means the first new Moon of strings of long months must occur less than two hours before the midnight. It also means Error 1 of the periods must be less than two hours. So only Periods 4, 7 & 8 are possible for strings of long months. This is the reason why Periods 1, 2, 3, 5, 6 never happen for strings of long months. Additionally, the amount of Error 1 also plays an important role determining the frequencies of the possible resonance periods. For strings of short months, except Period 1, Period 4 of 2894 days has the least Error 1, i.e. the timing for the first new Moon of the strings is almost consistent, and this period happens the most frequently. Period 3 has a bit more Error 1 than Period 4, and it occurs less frequently. Period 5, Period 2 and Period 6 have more Error 1, so they happen farther less frequently. For strings of long months, the same case happens: Period 4 has the least Error, and it happens the most frequently; Period 7 and 8 have bigger errors, and they occur less frequently.

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8
Error 1	-0.1023	0.2025	0.1002	-0.0022	-0.1045	0.3828	-0.0331	-0.0353
In Hours	-2.4559	4.8595	2.4036	-0.0523	-2.5082	9.286	-0.7946	-0.8570

Table 12. Error 1 in hours

Explanations of how the resonance periods happen:

We know that, it is required that the length of the strings should be less than 88 days or longer than 119 days, which means that the synodic months should be at its extreme values, either very long or very short. In Figure 7 it means the strings can only happen in the middle of the Jawad Cycle. Besides we find that in each Jawad Cycle, there are at most three strings of long months named as SL1, SL2 and SL3, or at most four strings of short months named as SS1, SS2, SS3 and SS4 respectively. For strings of long months, there are in total three possible periods --- Periods 4, 7 and 8. Period 4 = 2894 = 3307 – 413 = 3278 – 384; Period 7 = 16862 = 5 x 3307 + 384 – about 2 months; Period 8 = 19756 = 16862 + 2894. For Period 7 & 8, the error of three months is acceptable, because (1) the strings are not starting exactly from the peak of the Jawad cycles, instead a bit before that; (2) we are only considering the cluster of the peaks or the troughs; (3) the errors of Periods 7 and 8 are extremely small, which ensures the consistency of the timings for the first new moons of the consecutive strings. Now we can analyze Periods 4, 7 and 8 as follows --- the positions in the Jawad cycle:

	Days	First String	The Next String in later JC
Period 4	2894	SL2 / SL3	SL1 / SL2
Period 7	16862	SL1 / SL2	SL2 / SL3
Period 8	19756	SL1 / SL2	SL1 / SL2

Table 13. Analysis of the periods for strings of long months

To verify the results above, we take the string of 1991 A.D as an example, where Period 7 occurred: the string of long months of 1991 started on October 18 in 1990 at 23:36, just a bit before the midnight, and ended on February 15 at 01:32, just a bit after the midnight; after 384 days (13 synodic months – Period 1 with Error 1 more than 2 hours), the new Moon occurred at 19:10 on November 6 in 1991, which was much away from the midnight, and this was not qualified for occurrence of strings of long months. 2126 days (Period 2 with Error 1 about 5 hours) after the first string, the new Moon happened at 15:34 on August 14 in 1996; similarly this was not qualified for the occurrence of strings of long months either. Due to the same reason, Periods 3 (the new Moon was at 07:52), Period 4 (the new Moon was at 01:01), Period 5 (the new Moon was at 19:34) and Period 6 (the new Moon was at 15:11) failed producing another string of long months. Until 16862 days later, in 2036 the new Moon occurred at 23:33 on December 17, which was pretty good, beside of this the synodic months that time were long enough (four consecutive months was longer than 119 days); so that year, another string of long months happened --- the string of 2037.

Considering the positions of the strings of long months in the Jawad cycle, we take the strings starting from 2328 A.D as an example. If the string of long months in 2328 was a SL3 in the Jawad cycle, then after 2894 days, by checking Table 13, the new string would be SL2 in 2336. After another 16862 days, the string became SL3 in 2382; the following strings would be SL2 in 2390, SL1 in 2398, SL2 in 2444 and SL1 in 2452, which ended the sequence of the strings.

For strings of short months, the possible periods are Periods 1, 2, 3, 4, 5 and 6. Among them, Period 1 is the shortest and the most likely to be encountered. Period 3 = 2126 + 384; Period 4 = 2126 + 2 x 384; Period 5 = 2126 + 3 x 384 and 6644 = 2 x 3307 + one month. For Period 6, the error of one month is acceptable. (The same reason as for analysis of Period 7) We further analyze six periods as following --- the positions in the Jawad cycle:

	Days	First String	The Next String in the following JC
Period 1	384	SS1 / SS2 / SS3	SS2 / SS3 / SS4
Period 2	2126	SS4	SS1
Period 3	2510	SS3 / SS4	SS1 / SS2
Period 4	2894	SS2/ SS3 / SS4	SS1 / SS2 / SS3
Period 5	3278	SS1 / SS2 / SS3 / SS4	SS1 / SS2 / SS3 / SS4
Period 6	6644	SS1 / SS2 / SS3 / SS4	SS1 / SS2 / SS3 / SS4

Table 14. Analysis of the periods for strings of short months

The same as the way illustrating the stings of long months, we take the string of1788 as an example, where Period 5 happened: the string of short months of 1788 started at 00:20 on July 4 in 1788, just slightly after the midnight, and ended at 23:35 on September 29 in the same year, just a bit before the midnight. After 384 days, which is Period 1 with Error of more than 2 hours, the new Moon occurred at 00:08, which was quite ok, however, it was pitied that the synodic months were too long to make another string to happen. 2126 days (Period 2) after the first string, the new Moon occurred at 23:56, which was a bit before the midnight, not qualified for the strings of short months to happen. It was of the same reason for Period 3. But for Period 4, i.e. 2894 days after last string of short months, the new Moon occurred at 00:03, slightly after the midnight, however, that that month was 30 days, which was once again too long for the string to happen. Until 1797, 3298 days after the last string, the new Moon happened at 00:28 on June 25, which is just a very little bit after the midnight, meanwhile the following three months was less than 88 days, so another string of short months happened that year.

Furthermore, considering the positions of the strings of short months in the Jawad cycle, we take the strings starting from 2133 A.D as an example. If the string of short months in 2133 was a SS3 in the Jawad cycle, then after 3278 days, by checking Table 14, the new string would be SS3 in 2142. After another 384 days, the string became SS4 in 2143; the following strings would be SS2 in 2150, SS3 in 2151, SS4 in 2152, SS1 in 2158, SS2 in 2159, SS3 in 2160, SS1 in 2167, SS2 in 2168, SS1 in 2176 and SS2 in 2177, which ended the sequence of the strings.

From Tables 13 & 14, we further find, for strings of long months, there are only a few possible connections between SL1, SL2 and SL3. For example, if the first string of long months is a SL1, then there are only two possibilities for the next string of long months --- SL1 & SL2; both will occupy a very large time gap. If the first string is a SL2, then the next string can only be SL2 or SL3. If the first string is SL3, the next string must be SL2; otherwise there will be a long time without strings of long months. This means it is very hard to have many consecutive strings of long months. On the other hand, for strings of short months, there are a lot of possible connections between SS1, SS2, SS3 and SS4. This means for instance if the first string of short months is a SS1, then next string can be either SS1 or SS2, and all the involved periods --- Periods 1, 5 and 6 --- have high frequencies. In short it is more likely to have more strings of short months in one row. This is the reason why we have more strings of short months than strings of long months.

Explanations of why Periods 7 & 8 never happened for strings of short months:

For the strings of short months, in principle, all the eight resonance periods are possible to occur. However, Periods 7 and 8 are so large that we hardly have chances to encounter these two periods with missing all the six small periods. The underground reason is almost same as the one for the game of rolling a ball to the holes. Now assuming that we have eight holes on a sloping line with each representing one of the eight resonance periods. The bigger the period is, the far behind the respective hole will be. So the holes representing Periods 7 and 8 are at the last. If we roll the ball from the start point, it will be the most likely for the ball to get into the first hole representing Period 1. And so forth for other holes. But it is almost impossible that the ball will miss all the holes in front and get to the last two holes. This is the reason why Periods 7 and 8 never happen for strings of short months.

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