GEK1506 Heavenly Mathematics & Cultural Astronomy

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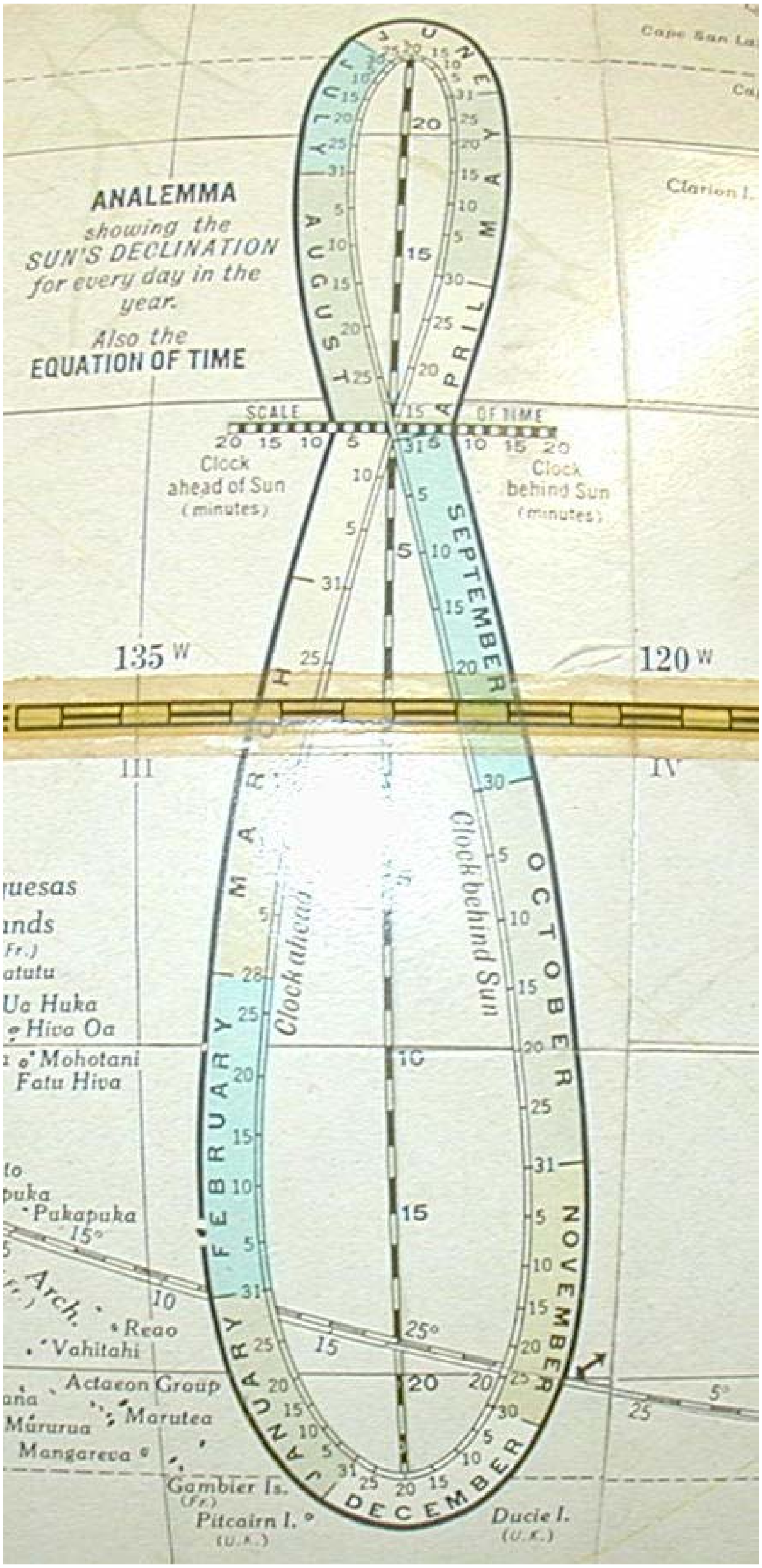
Celestial Navigation

The Equation of Time

The equation of time is defined as true time – mean time. It takes on its maximal value of about 15 minutes around November 3 and its minimum value of about -15 minutes around February 11. When the equation of time is positive, the true time is later than the mean time. That means that at (mean) noon, the (true) Sun has already crossed the meridian. Hence the true noon is before noon. In November the true Sun is 15 minutes ahead of the mean Sun, and true noon is already at 11.45am, while in February the true Sun is 15 minutes behind and true noon doesn't happen until 12.15pm.

In this picture we are looking east and November is to the right and February is to the left. In Singapore, November would be on top and February below.

3 November	Greenwich true noon = 11.45am	Local mean noon = Local true noon + 0.15
11 February	Greenwich true noon = 12.15pm	Local mean noon = Local true noon - 0.15


ANALEMMA
showing the
SUN'S DECLINATION
for every day in the
year.
Also the
EQUATION OF TIME
SCALE OF TIME
Clock
ahead of Sun
(minutes)
Clock
behind Sun
(minutes)
Clock ahead
Clock behind Sun
JUNE
JULY
AUGUST
MAY
APRIL
SEPTEMBER
MARCH
OCTOBER
FEBRUARY
NOVEMBER
JANUARY
DECEMBER
135 W
120 W
Clarion I.
Cape San La
Ua Huka
Hiva Oa
Mohotani
Fatu Hiva
Pukapuka
Reao
Vahitahi
Actaeon Group
Marutea
Mururoa
Mangareva
Gambier Is.
(Fr.)
Pitcairn I.
(U.K.)
Ducie I.
(U.K.)

Finding your longitude

Let us first ignore the equation of time. You use a sextant to determine the time (measured in UTC) when the Sun culminates, i.e., reaches its maximum altitude. If that happens before UTC noon, you are east of Greenwich, if happens after UTC noon, you are west of Greenwich. Each hour of time difference, corresponds to 15°, and four minutes correspond to one degree.

Let us now consider the equation of time. You must know the equation of time on that date. Suppose we make our observation on 3 November and find true noon to be at 6am. In order to compute the time difference, we must compare local true noon to Greenwich true noonon. From the above table, we know that the true noon at Greenwich is at 11.45am (the Sun is 15 minutes early). So the computation is 11.45 - 6 = 5.45. The longitude is therefore 90° - 3°45' = 86°15' degrees east (minutes, not decimals!).

Finding your latitude

When we computed the longitude, we only used the time of true noon. Now we will use the altitude of the Sun at true noon. This time we must know the declination on that date. Suppose we make our observation on 3 November and find the maximal altitude of the Sun to be 70° in the south. From a table we see that the declination on that date is about -15°. So for an observer on the equator, the Sun will culminate at altitude 75° in the south. Since our observed altitude is only 70°, the globe has been tilted 5° south, so we are at 5° north. Think of the globe with the equator up. When you tilt it south, latitude 5° north will be on top.

You can also use the formula we derived in tutorials saying

$$\text{altitude} = \text{declination} + \text{colatitude}.$$

It follows that

$$\text{altitude} = \text{declination} + 90 - \text{latitude}$$

or

$$\text{latitude} = 90 + \text{declination} - \text{altitude}.$$

In this case we get 90 + (-15) -70 = 5.