The Celestial Sphere

GEK1506 Heavenly Mathematics: Cultural Astronomy

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The Earth is spherical

Since ancient time, people have known that the Earth is spherical. There is a popular myth that this was not known until the time of Columbus. However, there are several convincing arguments for why the Earth is spherical. If you observe a ship sailing out from a harbor, it will appear to sink into the sea. If you travel in a north-south direction the altitude of the stars above the horizon will change. The shadow of the Earth on the Moon during a lunar eclipse is circular, and the only object that always casts a circular shadow regardless of its orientation is a sphere. Such arguments led people in all cultures to conclude that the Earth is spherical.

Heliocentric model

The planets move around the Sun and the moons move around the planets in elliptical orbits. (Kepler's first law.) The orbits of the Earth and the Moon are almost circular. The shape of an ellipse is determined by its *eccentricity*. The eccentricity of an ellipse is between 0 and 1 and measures how stretched out the ellipse is. If the eccentricity is 0, we get a circle. The eccentricity of the Earth's orbit is 0.017, so it is very close to a circle. In fact, the variation in the distance from the Earth to the Sun is only 3%. The Earth moves fastest when it is close to the Sun, because then the force of gravity is greatest. (Kepler's second law.)
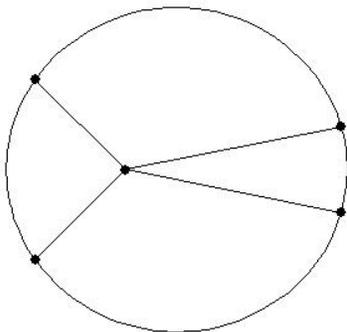
The Earth revolves around the Sun in a plane called the *ecliptic plane*. The Earth rotates around an axis that is not perpendicular to the ecliptic plane but titled 23.5° with respect to the perpendicular to the ecliptic plane. This is called the *obliquity*.
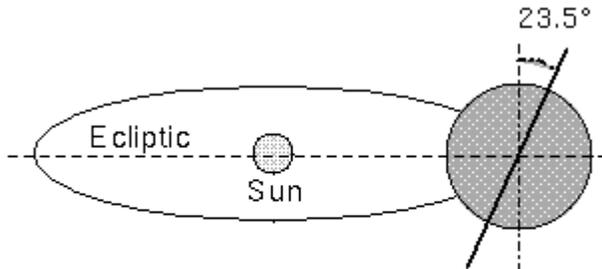
Earth's rotation axis is tilted by 23.5° with respect to the ecliptic (its orbital plane).

The solstices occur when the axis points toward the Sun, while the equinoxes occur when the axis is perpendicular to the rays of the Sun. The dates are approximately March 21, June 21, September 21 and December 21. They are usually referred to as the spring (or vernal) equinox, summer solstice, fall (or autumnal) equinox and winter solstice. However, this does not correspond to the seasons in the southern hemisphere, and makes no sense in the tropics. In order to be "hemispherically correct", some people say March equinox, June solstice, September equinox and December solstice.
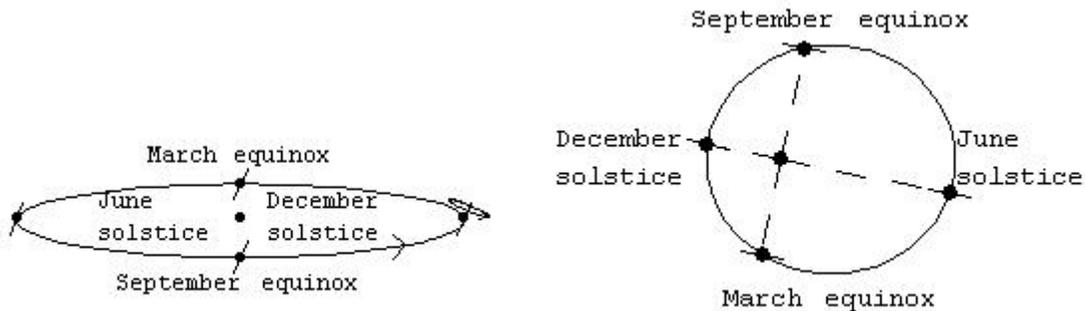
The Earth's rotation causes day and night. The Earth's revolution and the tilt of the axis cause the seasons. When the northern hemisphere tilts towards the Sun, it will be summer in the northern hemisphere and winter in the southern hemisphere and conversely when the northern hemisphere tilts away from the Sun. Notice that the seasons are not caused by the varying distance to the Sun. The point in the Earth's orbit that is closest to Sun is called *perihelion* and the point that is furthest from the Sun is called *aphelion*. The Earth is at perihelion around January 4 and at aphelion around July 4. For more details on the reasons for the seasons, please see the notes by Strobel.

The orbit of the Moon
What does the orbit of the Moon around the Sun look like? Most people think it will look like the picture on the left, but in fact, it is a 13-gon with smooth edges, like the picture on the right. There are no loops or changes in curvature.

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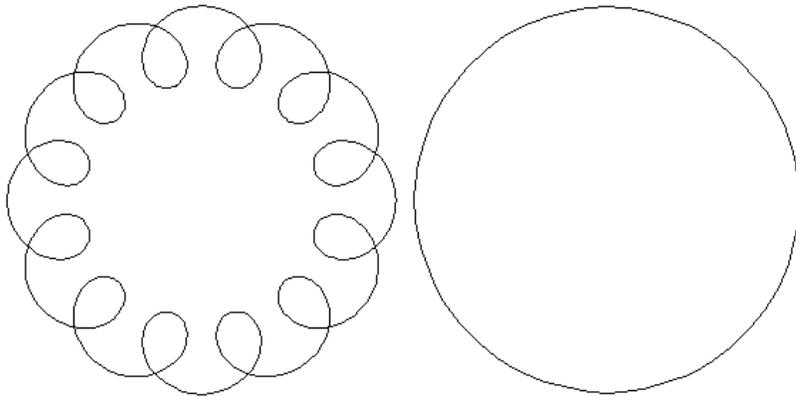
There are several ways to see this. The radius of the Earth's orbit is about 400 times the radius of the Moon's orbit. The speed of the Earth is about 30 times the speed of the Moon. The Moon makes about 13 revolutions in the course of a year.

I like to visualize this as follows. Imagine you're driving on a circular race track. You overtake a car on the right, and immediately slow down and go into the left lane. When the other car passes you, you speed up and overtake on the right again. You will then be making circles around the other car, but when seen from above, both of you are driving forward all the time and your path will be convex.

Another approach is to compute the gravitational forces involved. It can be shown that the Sun's pull on the Moon is about twice the Earth's pull on the Moon. It follows that the Moon's orbit is a perturbed ellipse.

The heliocentric model and Newton's theory of gravity explains why the heavenly bodies move the way they do. But since we live on the Earth, we need to know what it will look like when seen from the Earth. That's why this course will focus on the geocentric model.

The celestial sphere

In order to explain the geocentric model, we must first explain the *celestial sphere*. Imagine that the Earth is at the center of a huge sphere and that the stars are painted on the inside of the sphere and that the Sun, Moon and the planets are moving around on it.

If we cut a sphere in two, the cutting edges will be circles. If we cut near the surface we will get a small circle, but if we cut through the center of the sphere, we will get a circle of maximal length. Such a circle is called a *great circle*. A circle on a sphere that is not a great circle is called a *small circle*.

When describing terrestrial positions, we use latitude and longitude. *Latitude* is measured from 0 to 90 degrees north or south of the equator. Places with the same latitude lies on *circles of latitude* (or parallels of latitude). Except for the equator, which corresponds to latitude 0, the circles of parallel are small circles, and they are all parallel. Notice that at

the poles, the circles of latitude degenerate to points. The semi-great circles that run between the poles are called *meridians*. The meridian that runs through the Royal Greenwich Observatory outside London is called the *prime meridian*. The *longitude* of a place is determined by measuring the angular separation along the equator between the prime meridian and the meridian running through the point. Longitude is measured 180 degrees east or west of the Greenwich meridian. Places with the same longitude lie on meridians. The meridians all intersect at the poles, and we cannot define the longitude of the poles. Notice that the equator is defined in terms of the poles, which is defined by the rotation of the Earth, while the Greenwich meridian in a man-made invention. It was decided in 1884 at a meeting in Washington to make it the prime meridian.

Sometimes we will need the term *co-latitude*. It is equal to 90 minus the latitude.

We need to know some reference marks on the celestial sphere. The point in the sky directly above you is called the *zenith*. The point directly below you, which you cannot see, is called the *nadir*. The great circle halfway between the zenith and the nadir is called the *horizon*. The tangent plane to the Earth at the point where you stand on the Earth will cut the celestial sphere in the horizon. Strictly speaking, you need to parallel translate the tangent plane so that it passes through the center of the Earth, but since the distance to the stars is great, we can ignore this difference.
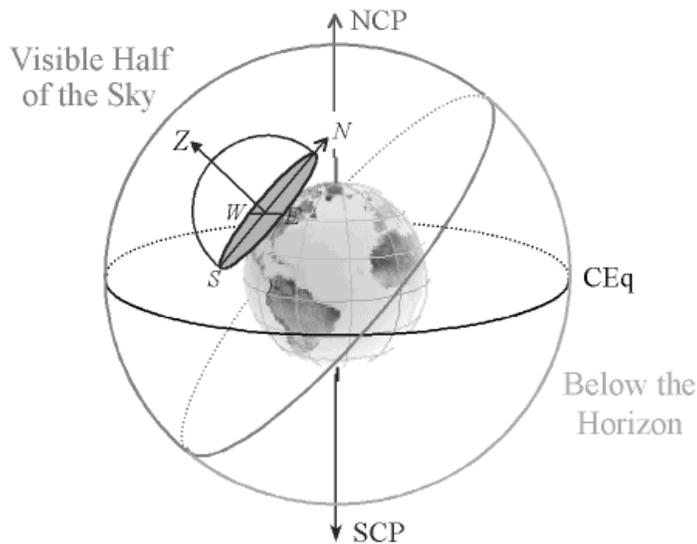

The great circle between the north point and the south point on the horizon that runs through the zenith is called the (celestial) *meridian*. The point on the celestial sphere that is directly above the (terrestrial) north pole is called the *north celestial pole* (NCP) and similarly for the *south celestial pole* (SCP). They are the points where the Earth's axis intersects the celestial sphere. The great circle halfway between the celestial poles is called the *celestial equator*.

There are several ways to describe position on the celestial sphere. For now we will only say that declination is the angular distance from the celestial equator, so it corresponds to the latitude on the Earth.

Geocentric model

The rotation of the Earth around its axis is *counter clockwise* (when seen from above the NCP). This produces and apparent *diurnal* (daily) motion of the Sky around the north and south celestial poles in the *clockwise* direction, which is opposite to the rotation of the Earth.

We cannot see the Sun and the stars at the same time, but by noticing which stars rise just after the Sun sets or set just before the Sun has risen, we can trace the motion of the Sun among the stars in the course of the year. This apparent *annual* (yearly) motion is *counterclockwise*, just like the revolution of the Earth around the Sun (when seen from above the NCP).
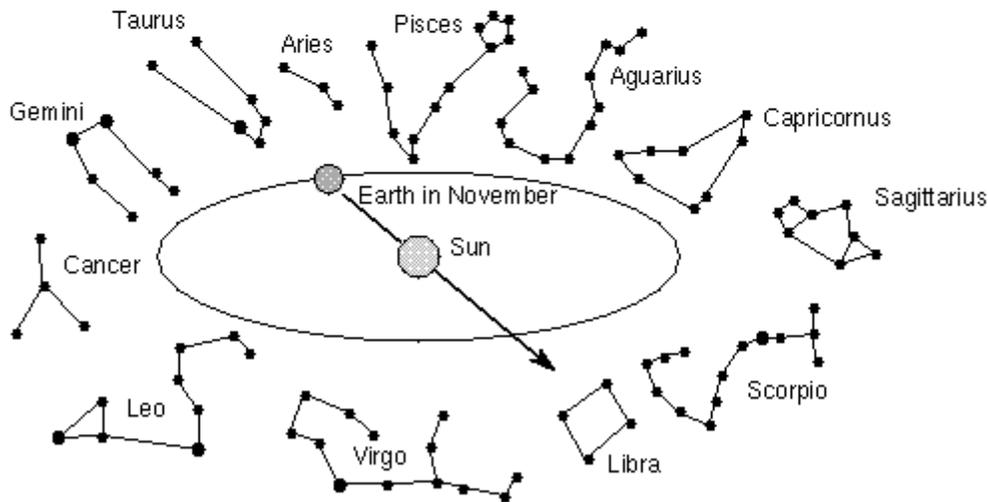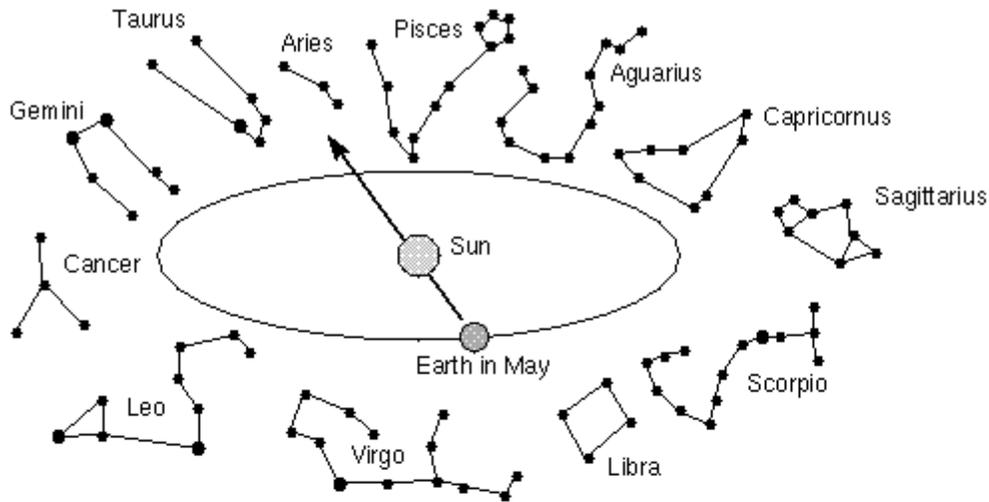
As the Earth moves around the Sun, the Sun **appears** to drift among the zodiac constellations along the path called the **ecliptic.** The ecliptic is the projection of the Earth's orbit onto the sky.
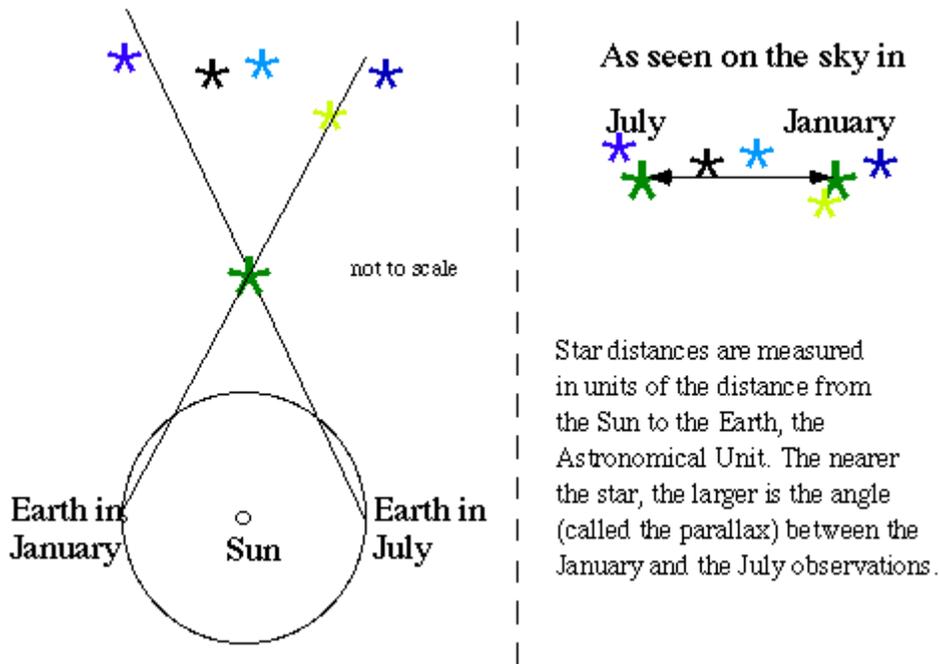
Remember, when seen from above the NCP, both the rotation and the revolution are CCW, but the apparent daily motion is CW, while the apparent yearly motion is CCW.

The daily motion goes all the way around the sky every day, while the yearly motion takes one year to go all the way around, so the net effect is to just marginally slow down the daily motion of the Sun compared to the daily motion of the stars. In particular, you will never see the Sun move in a different direction than the stars, all you will notice is that it lags behind them.

The daily motion is relative to any static frame of reference on Earth, while the yearly motion is only apparent if you use the (moving) stars as your frame of reference.

The yearly CCW motion of the Sun is an example of *parallax*. Parallax is the apparent change in the position of an object when you change your viewing position. The simplest example is to use your hands to cover one eye at the time when viewing objects. You will notice that the effect of parallax decreases with distance.

Now consider the way we observe the stars at different times of the year. A moment's consideration should convince you that there should be a change in position.

As seen on the sky in

July   January

not to scale

Star distances are measured in units of the distance from the Sun to the Earth, the Astronomical Unit. The nearer the star, the larger is the angle (called the parallax) between the January and the July observations.

Earth in January   Sun   Earth in July

But we don't observe that! Throughout history, the lack of observed stellar parallax was one of the key arguments against any theory that involved a moving Earth. The famous Danish astronomer Tycho Brahe rejected the Copernican system for precisely this reason. He was the greatest naked eye observer in history and had a clear idea about the maximal error in his observations. Based on that, he could compute how far away the stars had to be in order to *not* show any parallax. This distance was "astronomical" and made no sense to most people at the time, so he concluded that the Earth did not move. It was not until 1838 that the German astronomer Friedrich Wilhelm Bessel was able to measure stellar parallax with a telescope.

The stars seen from the Earth

The rotation of the Earth makes the star move along circular paths in the course of the day. The only exceptions are stars at the celestial poles. The star Polaris is very close to the NCP, so it will seem to stand still, while the rest of the stars rotate around it. It is therefore also known as the Pole Star or the North Star. Some stars will rise and set in the course of the day, while some will rise and set. Stars that never set are called *circumpolar*.

Equator

45° N Latitude

North Pole
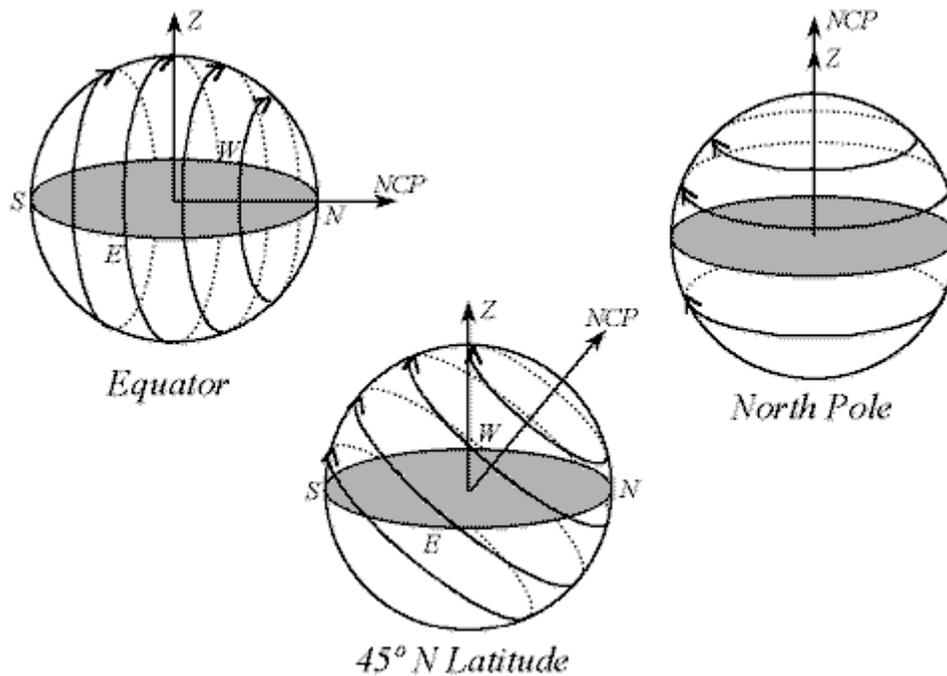
The *diurnal path* or the daily path of a star is a circle parallel to the celestial equator. Whether you can see a star or not depends on two things. First of all, the star must be above the horizon, and secondly it must be night. At any given place on the Earth, the celestial sphere consists of three portions. The circumpolar stars that never set and are visible throughout the year (except during periods of midnight Sun), stars that rise and set and are visible for part of the year, and stars that are never visible.

Imagine that you're moving from the north pole to the equator. The area of the sky consisting of the circumpolar stars will change. At the north pole, all stars are circumpolar, while at the equator, there are no circumpolar stars.

The stars that rise and set will be above the horizon during the day for part of the year, and below the horizon during the night for part of the year. So the appearance of the sky will change in the course of the year. At the north pole, half of the stars will never be visible. As you move south towards the equator, the portion of the sky that you never see will get smaller. At the north pole, you will only see half of the stars, but you see them for 24 hours for half a year (somewhat simplified). At the equator, you will see all the stars, but any given star will be below the horizon during the night for half the year.

As you walk south towards the equator, the altitude of the pole star above the horizon will decrease. It is equal to the latitude. At the same time, the altitude of the celestial equator will also change. It is equal to the colatitude.

The Sun seen from the Earth

The tilt of the Earth's axis makes the declination of the Sun change from 23.5 to -23.5 degrees and back again in the course of a year. In the course of the day, the Sun will trace out a diurnal (daily) path that is approximately a circle parallel to the celestial equator and with center along the Earth's axis. Since the declination changes in the course of the year, the yearly path of the Sun will be a "spiral of circles".

In 349, during the Eastern Jìn dynasty (东晋), the Chinese general Gùan Sùi (灌邃) fought a war in what is now Vietnam and noticed that in May the Sun was in the north.

Strobel's notes discuss the discovery of the precession of the equinoxes by Hipparchos around 150 B.C.E. This was discovered independently in China around 320 C.E. by Yú Xǐ (虞喜). These are the only known discoveries of precession.