NATIONAL UNIVERSITY OF SINGAPORE

FACULTY OF SCIENCE

SEMESTER 1 EXAMINATION 2003–2004

GEM1506K   Heavenly Mathematics: Highlights of Cultural Astronomy

November 2003 — Time allowed: 2 hours

1. You have already started planning for Valentine's Day. After you took that amazing class on Heavenly Mathematics, there is only one place you could possibly go, namely the restaurant Equinox at the top of the Raffles City Complex. Unfortunately, your date was not in the class, and you are not sure how much he/she knows about astronomy, so you will need to explain what the equinox is. I described the equinoxes in many ways in class. You are eager to impress your date, so you sit down and try to remember as many of them as possible. *Give as many definitions as you can of the equinoxes.*

   Solution:

   1. Earth's axis is perpendicular to the rays from the Sun.
   2. Day and night are equally long.
   3. Intersection between the celestial equator and the ecliptic, so declination is 0.
   4. Sun rises (sets) due east (west).
   5. The daily path of the shadow of a stick will be a straight line.

2. You decide to make an on-line reservation on the restaurant's web page. On the web page it says: "Equinox takes its name from the time when day and night are of perfectly equal length. It is an unusual phenomenon, which occurs only two days in a year." What do you think of this statement? *Are day and night of perfectly equal length at the equinoxes? How unusual is that in Singapore?*

   Solution: The path of the Sun on the day of the equinox is almost the celestial equator, so half of it is above the horizon. Day runs from sunrise to sunset. Sunrise happens when the upper limb of the Sun crosses the horizon, while sunset happens when the upper limb crosses the horizon. At the equinox, it is 12 hours between the center of the Sun crosses the two horizons.

   Refraction also makes the day longer.

   You also need to worry about the exact time of the equinox.

   On the equator, day and night will be (almost) equally long every day.

3. Equinox is located at level 69, giving you a magnificent view of Singapore. You want to be able to see the city both by day and night. So you need to know when the Sun will

set in order to make the reservation for the right time. *What is the equation of time? Use the equation of time to predict when the Sun will set on Valentine's Day, February 14. Illustrate it graphically using the analemma.*

Solution: Our longitude is $103°$, so our local time is almost equal to the UTC $+7$ time zone standard time. Since we are almost on the equator, you would therefore expect the Sun to set at 6pm. However, we are actually in the UTC $+8$ time zone, so you would expect the Sun to set at 7pm. The minimum value of the equation of time is about -15 minutes on February 11. That means that the sunset will be around 7.15pm. The analemma is horizontal in Singapore.

4. While waiting for the elevators, your date asks when the March equinox will be. *You know that in 2003, the March equinox occurred around 1am on 21 March. How can you use this to explain to your date when you expect the March equinox to occur in 2004, 2005, 2006, 2007 and 2008.*

Solution: Since the tropical year is about 365.25 days, the equinoxes will occur about 6 hours later each year, except when there is a leap year.

2003	1am 21/3
2004	7am 20/3
2005	1pm 20/3
2006	7pm 20/3
2007	1am 21/3
2008	7am 20/3

5. (i) The waitress at the restaurant gives you a table that faces west, saying that the Sun will set due west on the day of an equinox. *Why will the Sun set due west on the day of an equinox?*

(ii) Your date is by now convinced that you are an expert astronomer, so he/she asks you whether that is really true. *Explain why is it not always true that the Sun will set exactly due west on the day of an equinox.*

Solution: The intersection between the celestial equator and the horizon consists of the east and west points.

The Sun is on the celestial equator at the time of the equinox, but you don't know that the equinox will occur at either sunrise or sunset.

6. After you took Heavenly Mathematics last year, you have started paying attention to the Moon, and you never leave home without first checking which day it is in the lunar or lunisolar calendar. So you know that there will be a full Moon on Valentine's Day. (OK, I'm cheating here. In 2004 Valentine's Day will fall on the 24th day of the first Chinese month, but it's not that easy to write a whole exam paper about a Valentine's Day dinner!) As soon as the Sun has set, you ask your date to look in the other direction, and voila, a great full Moon appears in the east! Your date is really impressed and asks why. *Explain*

why on the equator the full Moon rises at sunset, and why on the day of an equinox, a full Moon would rise almost due east.

Solution: The full Moon is $180°$ away from the Sun. At the equator, the rising and setting points on the Sun's daily path are always $180°$ degrees apart.

If there was a full Moon at the equinox, the Sun would set due west and the Moon would rise due east. However, the Moon may be up to $5°$ off the ecliptic.

7. Your date is having a great time and is enjoying your passion for astronomy, so he/she immediately asks the obvious question: What if you don't live on the equator? This was something you didn't learn in class, but you remember that I told you that if you really understood the Java applets, you could answer almost all questions about the Moon and the Sun, so you quickly come up with a way to answer the question. *Explain why off the equator the full Moon does not necessarily rise at sunset.*

Solution: The latitude of the Moon can be up to $5°$. On the equator, the angle between the horizon and the ecliptic is between $66.5°$ and $90°$, so most of the those five degrees will be translated into azimuth and very little into altitude. However, at higher latitudes, the angle between the ecliptic and the horizon is smaller, and more of the latitude will translate into altitude.

8. To impress your date, you have brought along your plastic celestial sphere. You pull it out and show your date the two equinoxes. Your date is having a hard time remembering which equinox is which. *How do you tell the difference between the two equinoxes on the celestial sphere?*

Solution: Since the Sum revolves counterclockwise, the March equinox will be the one where the ecliptic points up to the right, when looking at the celestial sphere from the outside with the north celestial pole pointing up.

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