

NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 1 EXAMINATION 2005–2006

GEK1506 Heavenly Mathematics: Cultural Astronomy

November 2005 — Time allowed: 2 hours

Matriculation Number:

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains 8 questions and comprises 11 pages including this page.
2. Explain and justify your answers carefully.
3. Answers and your matriculation number should be written on this question paper.
4. Candidates may use calculators.

1. After taking the Heavenly Mathematics class, you decide to apply for the Student Exchange Programme. Thanks to a glowing recommendation letter from me, you are now spending a year at the University of Oslo, Norway. This gives you an opportunity to study the Sun and the Moon from a completely new perspective. On your first day in Oslo you buy a copy of the Almanac published by the University of Oslo. It gives information about sunrise, sunset, moonrise and moonset for three Norwegian cities, Oslo, Trondheim and Tomsø, and a lot of other interesting astronomical and cultural information. You immediately start studying it, and even though your Norwegian is still a bit rusty, you manage to make sense of most of it. You figure out that “sol” is “sun”, “måne” is “moon”, “opp” is “up”, “ned” is “down” and “sør” is “south”.

You first gather some basic information from the Almanac. You find that Oslo is at latitude about 60° north and longitude about $10^\circ 45'$ east and that it lies in the UT + 1 time zone and uses Daylight Saving Time from the last Sunday in March to the last Sunday in October.

Before you start exploring the Almanac seriously, you review some basic facts.

- (a) *What is the latitude of the Arctic Circle?*
- (b) *What is the altitude of the North Star in Oslo?*
- (c) *What is the angle between the celestial equator and the horizon in Oslo?*
- (d) *What are the maximal and minimal noon altitudes of the Sun in the course of the year in Oslo? Will the Sun be in the north or the south? When will they occur?*

U K E	Almanakk 2005																		
	Oslo							Trondheim							Tromsø				
	Sol			Måne				Sol			Måne				Sol		Måne		
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26	27 M 28 T 29 O 30 T	3 56 3 57 3 58 3 59	13 20 13 20 13 21 13 21	22 43 22 43 22 43 22 42	1 06 1 06 1 05 1 05	6 05 6 52 7 38 8 23	11 24 13 02 14 36 16 10	27 M 28 T 29 O 30 T	3 06 3 07 3 08 3 10	13 21 13 22 13 22 13 22	23 36 23 35 23 34 23 33	1 19 1 11 1 03 0 56	6 06 6 54 7 39 8 24	12 47 12 47 12 48 12 48					

Sommeretid er brukt i tabellen fra 27. mars til 30. oktober.

Dagen øker til solværv: 0 t. 37 m.
Dagen avtar fra solværv: 0 t. 7 m.

Dagen øker til solværv: 0 t. 58 m.
Dagen avtar fra solværv: 0 t. 13 m.

Figure 1: June

2. The column “sør” (south) gives the time of the meridian passage, i.e., when the Sun is in the south. From what you have learned about the equation of time, you are expecting the time of meridian passage to be no more than about 15 minutes away from noon, and to be about 11h58m15s on December 21 and about 12h01m45s on June 21, since the values of the equation of time is about $-1m45s$ on June 21 and about $+1m45s$ on December 21. But you see that on the December solstice (vintersolverv) on December 21 the Sun crosses the meridian at 12:15. *How can you use the longitude of Oslo to explain this?*

3. Next you look at the table of sunrise (opp) and sunset (ned). You see that the earliest sunrise is at 03:53 on June 19 and the latest sunrise is at 09:20 on December 27. The June solstice (sommersolverv) is on June 21 and the December solstice (vintersolverv) is on December 21. *Use the analemma to explain why the earliest and latest sunrises do not occur on the two solstices?*

4. You have a great time in Oslo and make a lot of new friends. In particular you get to know a friend from Tromsø, a city north of the Arctic Circle. You are immediately excited and he/she invites you to come and visit to see the midnight Sun on the June solstice! You bring your sextant along, and at noon on June 21 you see the Sun crossing the meridian at 12:46 local time (which includes both the UT + 1 time zone and the Daylight Saving Time), at an altitude of 44° in the south. You know that the equation of time is $-1\text{m}45\text{s}$ on June 21. *What is the latitude and longitude of Tromsø?*

5. *Why do you expect the full Moon in Singapore to rise around the time of sunset and set around the time of sunrise?*

6. From Singapore you are used to seeing the full Moon rise and set around the time of sunset and sunrise. However, when you look at the full Moon in Oslo on December 15, you see that the Sun rises at 9:13, while the full Moon doesn't set until 10:15, and then it rises again at 13:40 while the Sun does not set until 15:11. *How can you explain this difference between Oslo and Singapore?* (Hint: How does the Moon's latitude relate to its altitude at different latitudes?)

7. In December, your friend again invites you to Tromsø. You are of course looking forward to seeing the darkness at noon, but there is also something else you are looking forward to seeing. You have noticed some strange symbols in the Moon columns for Tromsø. You realize that the circle in the second column of December 15 means that it is a full Moon. *What do the symbols in the Moon columns for Tromsø on December 15 mean?*

8. You know that Chinese New Year was on January 29 in 2006. You want to treat your Norwegian friends to a nice Chinese New Year dinner in 2007. *When do you think Chinese New Year will be in 2007?*

END OF PAPER