NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 1 EXAMINATION 2005–2006

GEK1506 Heavenly Mathematics: Cultural Astronomy

November 2005 — Time allowed: 2 hours

1. After taking the Heavenly Mathematics class, you decide to apply for the Student Exchange Programme. Thanks to a glowing recommendation letter from me, you are now spending a year at the University of Oslo, Norway. This gives you an opportunity to study the Sun and the Moon from a completely new perspective. On your first day in Oslo you buy a copy of the Almanac published by the University of Oslo. It gives information about sunrise, sunset, moonrise and moonset for three Norwegian cities, Oslo, Trondheim and Tomsø, and a lot of other interesting astronomical and cultural information. You immediately start studying it, and even though your Norwegian is still a bit rusty, you manage to make sense of most of it. You figure out that "sol" is "sun", "måne" is "moon", "opp" is "up", "ned" is "down" and "sør" is "south".

You first gather some basic information from the Almanac. You find that Oslo is at latitude about 60° north and longitude about $10^\circ 45'$ east and that it lies in the UT $+1$ time zone and uses Daylight Saving Time from the last Sunday in March to the last Sunday in October.

Before you start exploring the Almanac seriously, you review some basic facts.

(a) *What is the latitude of the Arctic Circle?*

(b) *What is the altitude of the North Star in Oslo?*

(c) *What is the angle between the celestial equator and the horizon in Oslo?*

(d) *What are the maximal and minimal noon altitudes of the Sun in the course of the year in Oslo? Will the Sun be in the north or the south? When will they occur?*

Solution:

(a) $(90 - 23.5)^\circ = 66.5^\circ$ north.

(b) The altitude of the North Star is equal to the latitude of the observer, so it is 60° in Oslo.

(c) The angle between the celestial equator and the horizon is equal to the colatitude, so it is $(90 - 60)^\circ = 30^\circ$ in Oslo.

(d) Sine the latitude is 60°, the maximal altitude at the June solstice is $(90 - 60 + 23.5)^\circ = 53.5^\circ$ in the south. At the December solstice it is $(90 - 60 - 23.5)^\circ = 6.5^\circ$ in the south.

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UKE	JUNI 2005		Oslo Sol opp	Oslo Sol sør	Oslo Sol ned	Oslo Måne opp	Oslo Måne sør	Oslo Måne ned	Juni 2005	Trondheim Sol opp	Trondheim Sol sør	Trondheim Sol ned	Trondheim Måne opp	Trondheim Måne sør	Trondheim Måne ned	Tromsø Sol opp	Tromsø Sol sør	Tromsø Sol ned	Tromsø Måne opp	Tromsø Måne sør	Tromsø Måne ned
	1 O		4 07	13 15	22 24	*2 55*	*8 54*	*15 17*	1 O	3 25	13 16	23 09	*2 58*	*8 56*	*15 20*		12 42		*2 29*	*8 20*	*14 48*
	2 T		4 06	13 15	22 26	*2 54*	*9 39*	*16 50*	2 T	3 23	13 16	23 11	*2 50*	*9 40*	*17 02*		12 42		*2 04*	*9 05*	*16 52*
	3 F	*Skoklefalldagen*	4 04	13 15	22 27	*2 54*	*10 24*	*18 24*	3 F	3 21	13 17	23 14	*2 43*	*10 26*	*18 45*		12 42		*1 36*	*9 50*	*19 05*
	4 L		4 03	13 15	22 29	*2 54*	*11 11*	*20 00*	4 L	3 19	13 17	23 16	*2 34*	*11 12*	*20 34*		12 43		*0 59 / 23 18*	*10 37*	*22 26*
	5 S	*3. søndag etter pinse*	4 02	13 16	22 30	*2 56*	*12 00*	*21 39*	5 S	3 17	13 17	23 18	*2 24*	*12 01*	*22 34*		12 43			*11 26*	
	Luk 14,16–24 Nød folk til å komme inn, så mitt hus kan bli fullt.																					
23	6 M	Korninnførsel fri 1788 *23 t. 55 m.* ●	4 01	13 16	22 32	*3 01*	*12 51*	*23 14*	6 M	3 15	13 17	23 21	*2 07*	*12 52*			12 43			*12 17*	
	7 T	Stortingsvedt. om unionsoppl. 1905	4 00	13 16	22 33	*3 14*	*13 43*	–	7 T	3 14	13 17	23 23		*13 45*			12 43			*13 09*	
		Vilhelm Aubert f. 1922																			
	8 O		3 59	13 16	22 35	*3 44*	*14 36*	*0 34*	8 O	3 12	13 17	23 24	☽	*14 38*	☽		12 43		☽	*14 02*	☽
	9 T	**Prinsesse Ragnhild** *Corpus Christi*	3 58	13 16	22 36	*4 46*	*15 28*	*1 22*	9 T	3 10	13 18	23 26		*15 29*			12 43			*14 54*	
	10 F		3 57	13 17	22 37	*6 10*	*16 18*	*1 43*	10 F	3 09	13 18	23 28	*4 47*	*16 19*	*3 10*		12 44			*15 44*	
	11 L	Alm. stemmerett for kvinner 1913	3 56	13 17	22 38	*7 42*	*17 05*	*1 52*	11 L	3 08	13 18	23 30	*6 57*	*17 06*	*2 40*		12 44			*16 31*	
	12 S	*4. søndag etter pinse*	3 55	13 17	22 39	*9 12*	*17 49*	*1 55*	12 S	3 06	13 18	23 31	*8 44*	*17 50*	*2 27*		12 44		*6 20*	*17 15*	*3 44*
		Lov om sameting 1987																			
		Rikard Nordraak f. 1842																			
	Luk 15,1–10 Gleden i himmelen.																					
24	13 M	Ingolf Schanche f. 1877	3 55	13 17	22 40	*10 39*	*18 31*	*1 56*	13 M	3 05	13 18	23 33	*10 23*	*18 32*	*2 17*		12 44		*8 54*	*17 57*	*2 40*
	14 T	Norges Bank 1816	3 54	13 17	22 41	*12 04*	*19 12*	*1 56*	14 T	3 04	13 19	23 34	*11 56*	*19 13*	*2 09*		12 44		*10 53*	*18 38*	*2 07*
	15 O	Edvard Grieg f. 1843 ◐	3 54	13 18	22 42	*13 29*	*19 52*	*1 55*	15 O	3 04	13 19	23 35	*13 28*	*19 54*	*2 02*		12 45		*12 45*	*19 19*	*1 41*
	16 T	Joachim Holst-Jensen f. 1880	3 54	13 18	22 42	*14 55*	*20 34*	*1 55*	16 T	3 03	13 19	23 36	*15 01*	*20 36*	*1 55*	⊙	12 45	⊙	*14 38*	*20 00*	*1 18*
	17 F	Lov om folketrygd 1966	3 54	13 18	22 43	*16 26*	*21 19*	*1 54*	17 F	3 03	13 19	23 37	*16 41*	*21 20*	*1 47*		12 45		*16 40*	*20 45*	*0 53*
		Henrik Wergeland f. 1808																			
	18 L		3 53	13 18	22 43	*18 04*	*22 07*	*1 54*	18 L	3 02	13 20	23 37	*18 30*	*22 09*	*1 39*		12 45		*19 10*	*21 33*	*0 23 / 23 34*
	19 S	*5. søndag etter pinse*	3 53	13 18	22 44	*19 51*	*23 01*	*1 54*	19 S	3 02	13 20	23 38	*20 35*	*23 02*	*1 29*		12 46			*22 27*	
	Matt 7,1–5 Flisen og bjelken.																					
25	20 M	Gina Krog f. 1847	3 53	13 19	22 44	*21 45*	–	*1 58*	20 M	3 02	13 20	23 38	*23 22*	–	*1 15*		12 46			*23 26*	
	21 T	*Sommersolverv* *08 t. 46 m.*	3 54	13 19	22 44	*23 31*	*0 01*	*2 07*	21 T	3 02	13 20	23 38		*0 02*	*0 33*		12 46				
		Johan Anker f. 1871																			
	22 O	Jordlov av 1928 *6 t. 14 m.* ○	3 54	13 19	22 44	–	*1 05*	*2 37*	22 O	3 02	13 20	23 38	☽	*1 07*	☽		12 46		☽	*0 31*	☽
		Haakon 7. og dron. Maud kronet 1906																			
	23 T	*Sankthansaften (Jonsokaften)*	3 54	13 19	22 44	*0 34*	*2 12*	*3 55*	23 T	3 03	13 21	23 38		*2 14*			12 46			*1 38*	
	24 F	*Sankthans (Jonsok)*	3 55	13 20	22 44	*0 56*	*3 18*	*5 50*	24 F	3 03	13 21	23 38	*2 06*	*3 19*	*4 44*		12 47			*2 43*	
	Joh 10,40–42 Johannes vitnet sant om Jesus.																					
	25 L		3 55	13 20	22 44	*1 03*	*4 19*	*7 49*	25 L	3 04	13 21	23 38	*1 41*	*4 20*	*7 16*		12 47			*3 44*	
	26 S	*6. søndag etter pinse (Aposteldagen)*	3 56	13 20	22 44	*1 05*	*5 14*	*9 41*	26 S	3 05	13 21	23 37	*1 28*	*5 15*	*9 23*		12 47		*1 56*	*4 40*	*7 50*
	Luk 5,1–11 Legg ut på dypet!																					
26	27 M	Fabrikktilsynslov 1892. M. Tranmæl f. 1879	3 56	13 20	22 43	*1 06*	*6 05*	*11 24*	27 M	3 06	13 21	23 36	*1 19*	*6 06*	*11 17*		12 47		*1 16*	*5 31*	*10 16*
	28 T	Personalunion med Sverige 1319 ◑	3 57	13 20	22 43	*1 06*	*6 52*	*13 02*	28 T	3 07	13 22	23 35	*1 11*	*6 54*	*13 03*		12 47		*0 47*	*6 18*	*12 25*
	29 O	*Persok*	3 58	13 21	22 43	*1 05*	*7 38*	*14 36*	29 O	3 08	13 22	23 34	*1 03*	*7 39*	*14 45*		12 48		*0 22 / 23 55*	*7 04*	*14 29*
	30 T	Høyesterett åpnet 1815	3 59	13 21	22 42	*1 05*	*8 23*	*16 10*	30 T	3 10	13 22	23 33	*0 56*	*8 24*	*16 28*		12 48		*23 22*	*7 49*	*16 38*

Sommertid er brukt i tabellen fra 27. mars til 30. oktober.

Dagen *øker* til solverv: 0 t. 37 m.
Dagen *avtar* fra solverv: 0 t. 7 m.

Dagen *øker* til solverv: 0 t. 58 m.
Dagen *avtar* fra solverv: 0 t. 13 m.

Figure 1: June

UKE	DESEMBER 2005	Oslo Sol opp	Oslo Sol sør	Oslo Sol ned	Oslo Måne opp	Oslo Måne sør	Oslo Måne ned
	1 T *16 t. 1 m.* ●	8 51	12 06	15 21	*9 28*	*11 54*	*14 09*
	2 F Norsk Hydro grunnlagt 1905	8 53	12 07	15 20	*11 16*	*12 54*	*14 28*
	3 L Ludvig Holberg f. 1684	8 55	12 07	15 18	*12 32*	*13 58*	*15 26*
	4 S *2. søndag i advent*	8 57	12 07	15 17	*13 01*	*15 01*	*17 09*
	Barbromesse (Barbara)						
	Luk 12,35–40 Vær også dere beredt!						
49	5 M Første hedningemisjonsforbund 1826	8 59	12 08	15 16	*13 11*	*16 01*	*19 05*
	6 T *Nilsmesse (Nicolaus biskop)*	9 01	12 08	15 16	*13 14*	*16 57*	*20 57*
	7 O Wilhelm F. K. Christie f. 1778	9 02	12 09	15 15	*13 14*	*17 48*	*22 42*
	8 T Bjørnstjerne Bjørnson f. 1832 ◐	9 04	12 09	15 14	*13 14*	*18 37*	–
	9 F Olav Engelbrektsson erkebiskop 1523	9 05	12 10	15 13	*13 13*	*19 23*	*0 22*
	10 L FNs menneskerettighetsdag	9 07	12 10	15 13	*13 12*	*20 10*	*1 59*
	Aasta Hansteen f. 1824						
	11 S *3. s. i advent* Martin Linge f. 1894	9 08	12 10	15 12	*13 12*	*20 57*	*3 35*
	Matt 11,11–19 Jesus vitner om døperen Johannes.						
50	12 M Edvard Munch f. 1863	9 10	12 11	15 12	*13 13*	*21 46*	*5 14*
	13 T *Lucinatt (Lucia)*	9 11	12 11	15 12	*13 15*	*22 38*	*6 56*
	Kristian Birkeland f. 1867						
	14 O Amundsen ved Sydpolen 1911	9 12	12 12	15 11	*13 22*	*23 32*	*8 39*
	15 T Peter A. Munch f. 1810 *17 t. 15 m.* ○	9 13	12 12	15 11	*13 40*	–	*10 15*
	16 F	9 14	12 13	15 11	*14 26*	*0 27*	*11 25*
	17 L Sophus Lie f. 1842	9 15	12 13	15 11	*15 45*	*1 22*	*11 58*
	18 S *4. søndag i advent*	9 16	12 14	15 11	*17 18*	*2 15*	*12 10*
	Joh 3,26–30 Døperen Johannes vitner om Jesus.						
51	19 M	9 17	12 14	15 12	*18 52*	*3 04*	*12 15*
	20 T Cora Sandel f. 1880	9 17	12 15	15 12	*20 21*	*3 50*	*12 16*
	21 O *Tomasmesse*	9 18	12 15	15 12	*21 46*	*4 33*	*12 16*
	Vintersolverv *19 t. 35 m.*						
	22 T Johan Sebastian Welhaven f. 1807	9 19	12 16	15 13	*23 09*	*5 13*	*12 16*
	23 F ◑	9 19	12 16	15 14	–	*5 53*	*12 14*
	24 L *Julaften* Tordis Maurstad f. 1901	9 19	12 17	15 14	*0 31*	*6 32*	*12 13*
	Luk 2,1–14 Jesus ble født i en stall.						
	25 S **Juledag**	9 19	12 17	15 15	*1 55*	*7 13*	*12 12*
	Joh 1,1–14 I begynnelsen var Ordet.						
52	26 M **2. juledag** *Stefanusdagen*	9 20	12 18	15 16	*3 24*	*7 56*	*12 12*
	Matt 10,32–39 Den som mister sitt liv for Jesu skyld, skal finne det.						
	27 T Første arbeiderforening 1848	9 20	12 18	15 17	*5 01*	*8 44*	*12 13*
	28 O Carl Gustav Fleischer f. 1883	9 20	12 19	15 18	*6 46*	*9 37*	*12 16*
	Barnedag						
	29 T Ellen Gleditsch f. 1879	9 19	12 19	15 19	*8 37*	*10 35*	*12 27*
	Thomas Angell f. 1692						
	30 F Jonas Anton Hielm f. 1782	9 19	12 20	15 20	*10 13*	*11 39*	*13 03*
	Helge Ingstad f. 1865						
	31 L *Nyttårsaften* *4 t. 12 m.* ●	9 19	12 20	15 22	*11 02*	*12 44*	*14 31*
	Joh 14,27 La ikke hjertet bli grepet av angst.						

Des. 2005	Trondheim Sol opp	Trondheim Sol sør	Trondheim Sol ned	Trondheim Måne opp	Trondheim Måne sør	Trondheim Måne ned	Tromsø Sol opp	Tromsø Sol sør	Tromsø Sol ned	Tromsø Måne opp	Tromsø Måne sør	Tromsø Måne ned
1 T	9 27	12 07	14 47	*10 45*	*11 55*	*12 55*		11 33			*11 19*	
2 F	9 30	12 08	14 46		*12 56*			11 34			*12 20*	
3 L	9 32	12 08	14 44	☽	*13 59*	☽		11 34		☽	*13 23*	☽
4 S	9 35	12 09	14 42		*15 02*			11 34			*14 27*	
5 M	9 37	12 09	14 41	*13 59*	*16 02*	*18 21*		11 35			*15 27*	
6 T	9 39	12 09	14 39	*13 42*	*16 58*	*20 33*		11 35		*14 39*	*16 23*	*18 30*
7 O	9 41	12 10	14 38	*13 32*	*17 50*	*22 30*		11 36		*13 41*	*17 14*	*21 15*
8 T	9 44	12 10	14 37	*13 23*	*18 38*	–		11 36		*13 08*	*18 03*	*23 30*
9 F	9 46	12 11	14 36	*13 15*	*19 25*	*0 19*		11 37		*12 41*	*18 49*	–
10 L	9 47	12 11	14 35	*13 07*	*20 11*	*2 04*		11 37		*12 15*	*19 36*	*1 37*
11 S	9 49	12 12	14 34	*12 58*	*20 58*	*3 49*		11 38		*11 44*	*20 23*	*3 46*
12 M	9 51	12 12	14 33	*12 48*	*21 47*	*5 39*		11 38		*10 58*	*21 12*	*6 14*
13 T	9 53	12 13	14 32	*12 36*	*22 39*	*7 36*		11 38			*22 04*	
14 O	9 54	12 13	14 32	*12 10*	*23 33*	*9 53*		11 39			*22 58*	
15 T	9 55	12 14	14 32					11 39		☽	*23 53*	☽
16 F	9 57	12 14	14 31	☽	*0 29*	☽	☉	11 40	☉			
17 L	9 58	12 15	14 31		*1 23*			11 40			*0 48*	
18 S	9 59	12 15	14 31	*16 20*	*2 16*	*13 12*		11 41			*1 41*	
19 M	10 00	12 16	14 31	*18 17*	*3 05*	*12 53*		11 41			*2 30*	
20 T	10 00	12 16	14 32	*20 00*	*3 51*	*12 42*		11 42		*18 13*	*3 16*	*13 22*
21 O	10 01	12 17	14 32	*21 34*	*4 34*	*12 33*		11 42		*20 21*	*3 59*	*12 41*
22 T	10 02	12 17	14 33	*23 04*	*5 14*	*12 25*		11 43		*22 13*	*4 39*	*12 13*
23 F	10 02	12 18	14 33	–	*5 54*	*12 18*		11 43			*5 19*	*11 49*
24 L	10 02	12 18	14 34	*0 34*	*6 33*	*12 10*		11 44		*0 01*	*5 58*	*11 25*
25 S	10 02	12 19	14 35	*2 05*	*7 14*	*12 02*		11 44		*1 52*	*6 39*	*10 59*
26 M	10 02	12 19	14 36	*3 43*	*7 58*	*11 53*		11 45		*3 56*	*7 22*	*10 25*
27 T	10 02	12 20	14 37	*5 32*	*8 45*	*11 42*		11 45		*6 47*	*8 10*	*9 13*
28 O	10 02	12 20	14 38	*7 40*	*9 38*	*11 24*		11 46			*9 03*	
29 T	10 01	12 21	14 40	☽	*10 37*	☽		11 46		☽	*10 01*	☽
30 F	10 01	12 21	14 41		*11 40*			11 47			*11 04*	
31 L	10 00	12 21	14 43		*12 45*			11 47			*12 09*	

Sommertid er brukt i tabellen fra 27. mars til 30. oktober.

Dagen *avtar* til solverv: 0 t. 39 m.
Dagen *øker* fra solverv: 0 t. 9 m.

Dagen *avtar* til solverv: 0 t. 53 m.
Dagen *øker* fra solverv: 0 t. 12 m.

Figure 2: December

2. The column "sør" (south) gives the time of the meridian passage, i.e., when the Sun is in the south. From what you have learned about the equation of time, you are expecting the time of meridian passage to be no more than about 15 minutes away from noon, and to be about 11h58m15s on December 21 and about 12h01m45s on June 21, since the values of the equation of time is about -1m45s on June 21 and about $+1$m45s on December 21. But you see that on the December solstice (vintersolverv) on December 21 the Sun crosses the meridian at 12:15. *How can you use the longitude of Oslo to explain this?*

Solution: You know that Oslo is $4°15'$ west of the center of the time zone at $15°$, so the Sun will cross the meridian 17 minutes later than at the center of the time zone. This gives you 12h15m15s, which rounds to 12:15.

3. Next you look at the table of sunrise (opp) and sunset (ned). You see that the earliest sunrise is at 03:53 on June 19 and the latest sunrise is at 09:20 on December 27. The June solstice (sommersolverv) is on June 21 and the December solstice (vintersolverv) is on December 21. *Use the analemma to explain why the earliest and latest sunrises do not occur on the two solstices?*

Solution: Follows from Figures 3 and 4.

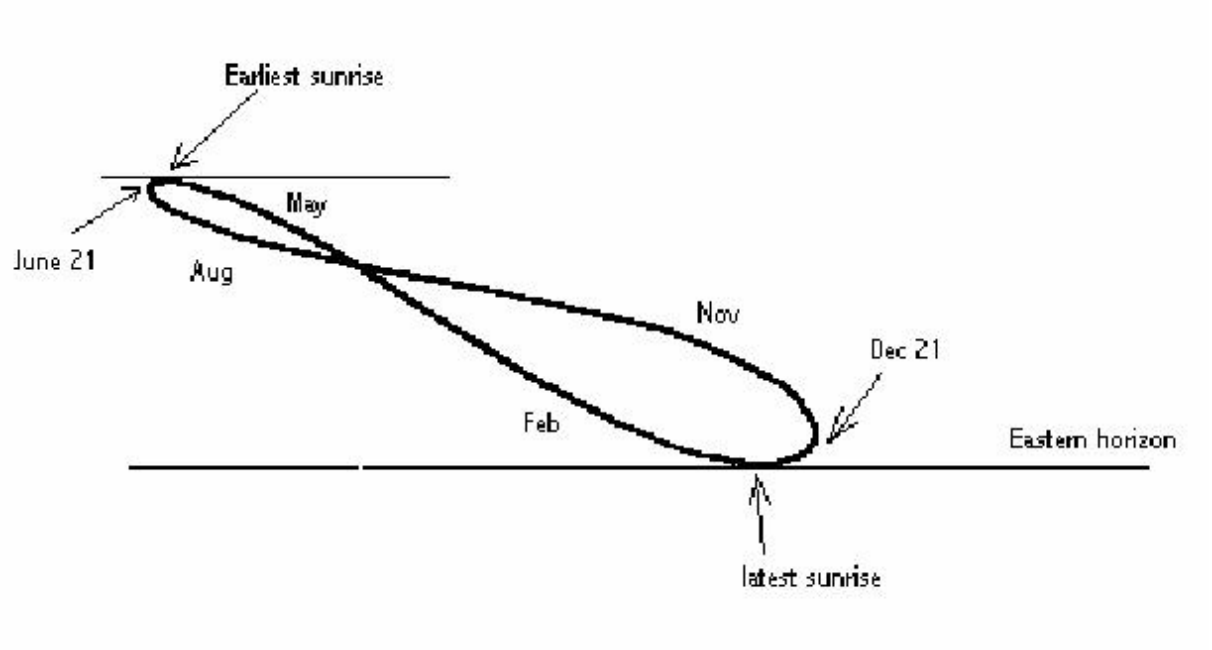

Figure 3: Rising analemma

4. You have a great time in Oslo and make a lot of new friends. In particular you get to know a friend from Tromsø, a city north of the Arctic Circle. You are immediately excited and he/she invites you to come and visit to see the midnight Sun on the June solstice! You bring your sextant along, and at noon on June 21 you see the Sun crossing the meridian

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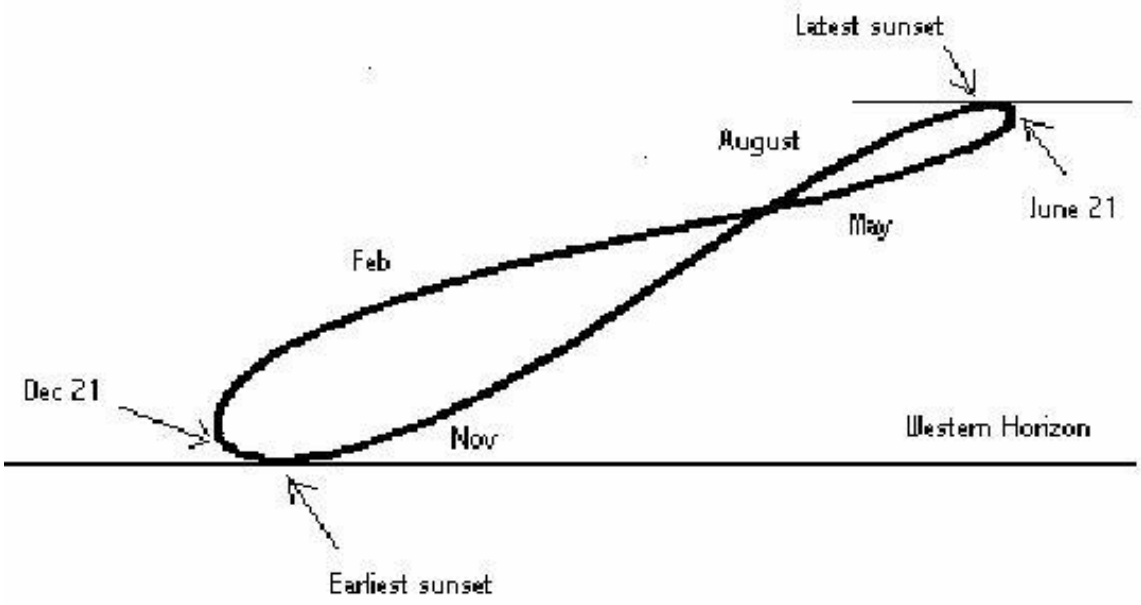

Figure 4: Setting analemma

at 12:46 local time (which includes both the UT + 1 time zone and the Daylight Saving Time), at an altitude of $44°$ in the south. You know that the equation of time is -1m45s on June 21. *What is the latitude and longitude of Tromsø?*

Solution: The latitude is $(90 + 23.5 - 44)° = 69.5°$. True noon at Greenwich was at 12h01m45s UT and true noon at Tromsø was at 10:46, so the difference is 1h15m45s and it follows that the longitude of Tromsø is about $19°$ east.

5. *Why do you expect the full Moon in Singapore to rise around the time of sunset and set around the time of sunrise?*

Solution: At the time of the full Moon, the Moon is opposite the Sun.

6. From Singapore you are used to seeing the full Moon rise and set around the time of sunset and sunrise. However, when you look at the full Moon in Oslo on December 15, you see that the Sun rises at 9:13, while the full Moon doesn't set until 10:15, and then it rises again at 13:40 while the Sun does not set until 15:11. *How can you explain this difference between Oslo and Singapore?* (Hint: How does the Moon's latitude relate to its altitude at different latitudes?)

Solution: At the equator the angle between the ecliptic and the horizon is always more than $66.5°$. Therefore the Moon's latitude mostly changes the azimuth, and not the altitude. However, in Oslo, the angle between the ecliptic and the horizon changes between $(90 - 60 - 23.5)° = 6.5°$ and $(90 - 60 + 23.5)° = 53.5°$, so if the latitude of the Moon is $\pm 5°$, that may translate into several degrees of altitude of the Moon. Since the daily path of the Moon is parallel to the celestial equator, the Moon rises at an angle of $30°$ in

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Oslo, so even a few degrees of altitude may have a large impact on the time of moonrise or moonset.

7. In December, your friend again invites you to Tromsø. You are of course looking forward to seeing the darkness at noon, but there is also something else you are looking forward to seeing. You have noticed some strange symbols in the Moon columns for Tromsø. You realize that the circle in the second column of December 15 means that it is a full Moon. *What do the symbols in the Moon columns for Tromsø on December 15 mean?*

Solution: Since the full Moon is opposite the Sun, the Moon will be above the horizon all day when the Sun is below the horizon all day.

8. You know that Chinese New Year was on January 29 in 2006. You want to treat your Norwegian friends to a nice Chinese New Year dinner in 2007. *When do you think Chinese New Year will be in 2007?*

Solution: You first subtract 11 days, but that would take you to January 18, which is before January 21, so instead you add 19 days, giving you February 17.

END OF PAPER