NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION 2005–2006

GEK1506 Heavenly Mathematics: Cultural Astronomy

April/May 2006 — Time allowed: 2 hours

1. After taking the Heavenly Mathematics class, you and your friend have decided to go on a round the world tour. You want to experience some of the interesting astronomical phenomena you have learned about in the class. However, your friend has not taken the class, so you will need to explain a bit. You and your friend have seen the Jackie Chan movie "Round the World in 80 days", based on the book by Jules Verne. Your friend enjoyed the movie, but did not understand why the travelers had gained one day by traveling eastward. You want to explain this carefully, so you start by explaining the principle behind time zones. Fortunately you have a world time zone map. Your friend is really excited about your map and asks you a lot of interesting questions.

(i) *Why does Singapore seem to be in the "wrong" time zone?*

(ii) *Why is it not a problem that Singapore is in the "wrong" time zone?*

(iii) Judging from the map, you note that China spans four time zones. However, all of China uses the time zone of the capital, Beijing.
Is this a problem for people in the western part of China?

(iv) *If the capital of China had been in the western part of China, would it have been a problem for people in the eastern part of China to use the same time zone as the capital?*

Solution:

(i) The longitude of Singapore is 104°, so we should have been in the UTC $+ 7$ time zone. However, West Malaysia follows East Malaysia and we follow West Malaysia.

(ii) In effect, we are on permanent Daylight Saving Time. Since the day has the same length all year round, it is not a problem to have DST all year round, too.

(iii) The western part of China uses "triple DST". That means that the Sun will rise very late in the winter. However, the western part of China does not have high latitude, so the Sun will not rise extremely late, even in the winter.

(iv) If the capital was in the west, people in the east would have used "triple anti-DST". In that case, the Sun would have set very early in the winter. The eastern part of China also has higher latitude, which will aggravate the problem.

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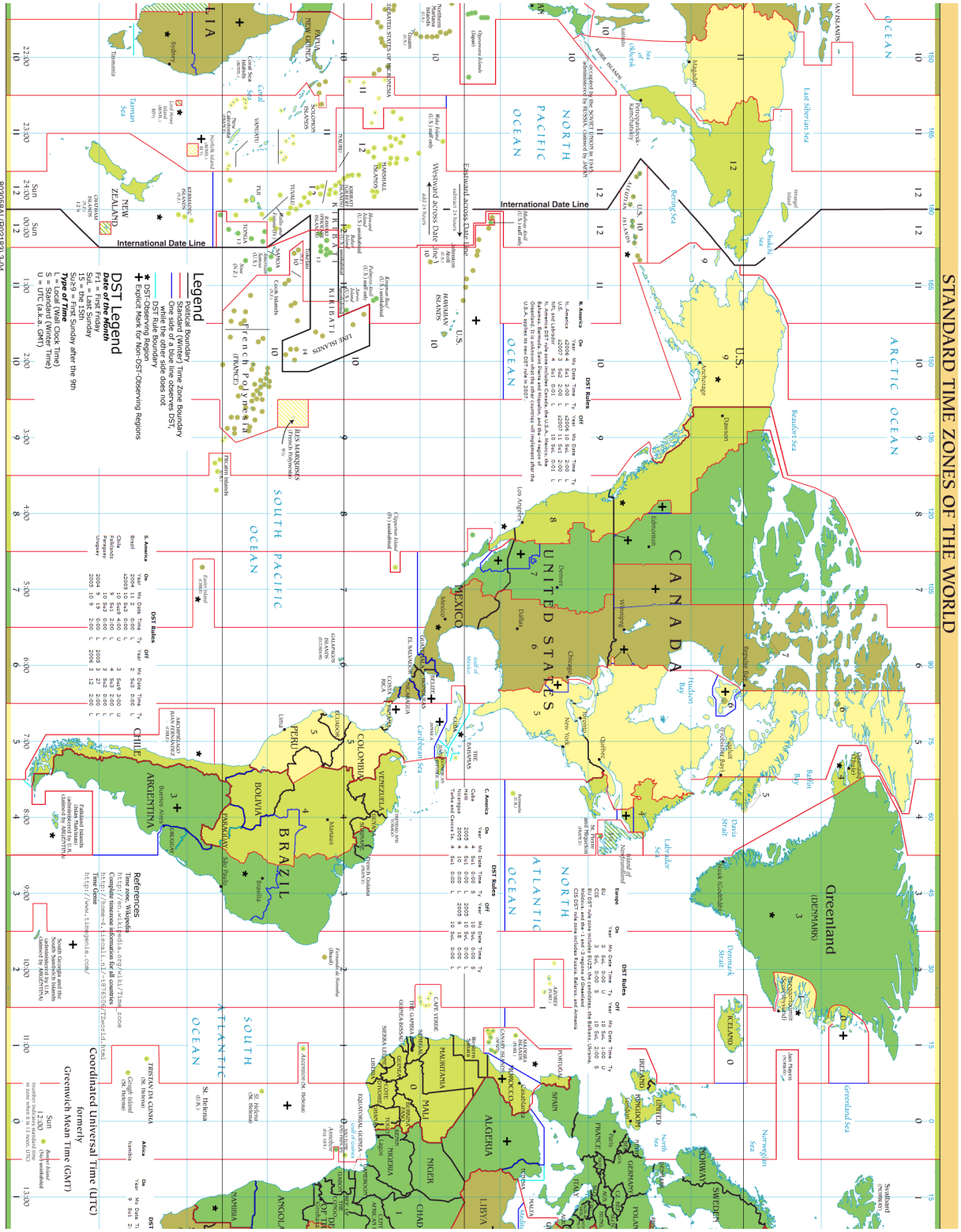

Figure 1: Time Zones

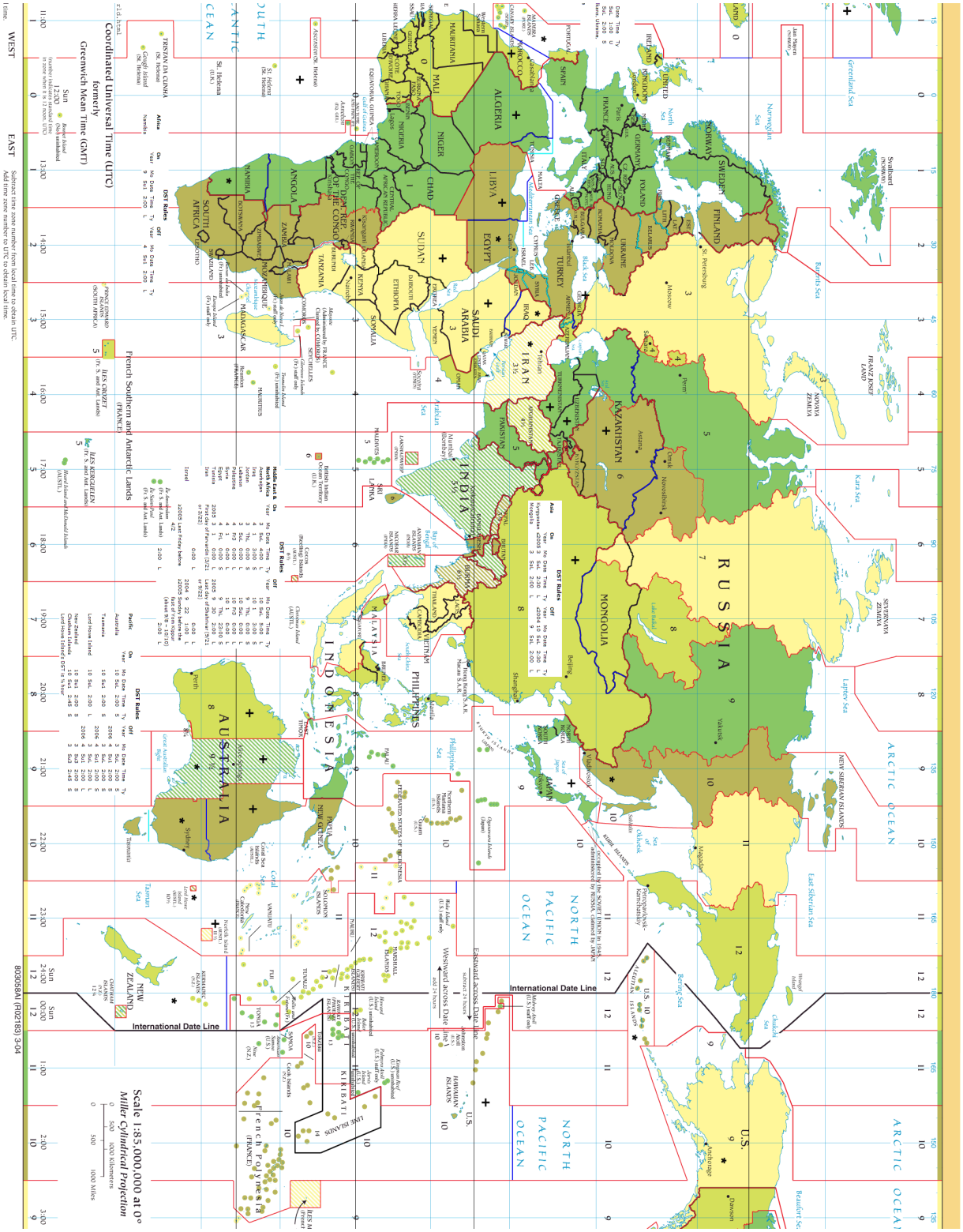

Figure 2: Time Zones

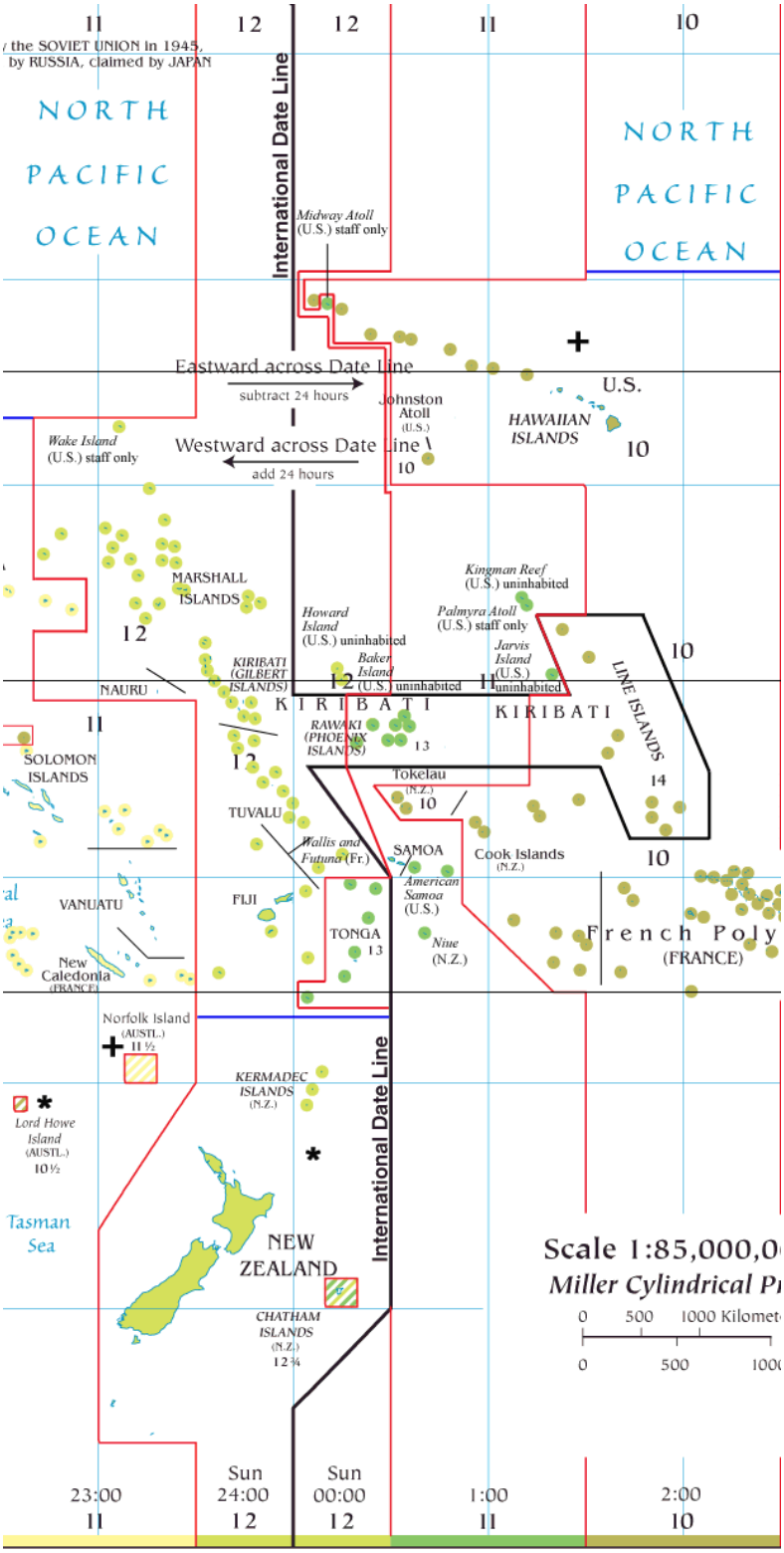

Figure 3: Time Zones

2. (i) Your friend notices that in the Pacific Ocean there are time zones called UTC$(GTM)+12$, UTC $+ 13$, UTC $+ 14$ and UTC $- 12$.

Why are these time zones confusing?

(ii) At first you don't know how to explain this, but then you notice that the islands in UTC $+ 13$ and UTC $+ 14$ are parts of a country whose capital lies in UTC $+ 12$ and most of the islands in UTC $- 12$ are part of the US.

How can you use this to explain the "extra" time zones?

Solution:

(i) You would expect there to be 24 time zones, UTC $- 11, ...,$ UTC $+ 11$ and either UTC12 or UTC $- 12$. There seems to be three redundant time zones.

(ii) It is more convenient for different parts of a country to have time zones that differ by two hours than by 22 hours. Hawaii is in UTC $- 10$, so again it is better for US possessions to be two time zones away in UTC $- 12$ than 22 hours away in UTC $+ 12$.

3. Suppose you plan to leave Singapore at noon on 1 December 2006 and you travel 15° of longitude each day.

(i) *If you travel eastward, when will you be back in Singapore?*

(ii) *If you travel westward, when will you be back in Singapore?*

Solution:

(i) If you travel eastward, you are traveling towards the Sun, so your day becomes shorter. If you travel 15° of longitude each day, you will normally move one 1-hour time zone each day, so you will travel 24 23-hour days. However, this is equal to 23 days, so you will be back on 24 December. Alteratively, you can say that you travel for 24 days, but you gain one day when you cross the International Date Line.

(ii) If you travel westward, you are traveling away from the Sun, so your day becomes longer. If you travel 15° of longitude each day, you will normally move one 1-hour time zone each day, so you will travel 24 25-hour days. However, this is equal to 25 days, so you will be back on 26 December. Alteratively, you can say that you travel for 24 days, but you lose one day when you cross the International Date Line.

4. When you suggest leaving on 1 December 2006, your friend becomes worried about making it back to Singapore for Chinese New Year. You know that in 2006, Chinese New Year was on 29 January.

When do you expect Chinese New Year to be in 2007?

Solution: Subtracting 11 days would take you to 18 January, which is earlier than 21 January. So instead you add 19 days, giving you 48 January, which is the same as 17

February. It turns out that it will be on 18 February, but the rule of thumb often has an error of one day.

5. You decide to travel eastward. Since you will leave in December, you want to first head to Wellington, New Zealand, at latitude 41° S to explore the southern summer and the southern sky.

(i) *What is the right ascension of the December solstice?*

(ii) *Will you be able to see a star with right ascension 18h and declination -70° in Singapore in late December?*

(iii) *Will you be able to see a star with right ascension 18h and declination -70° in Wellington in late December?*

Solution:

(i) 18h.

(ii) No, because it will be washed out by the Sun. At the equator, all objects with the same RA will be above or below the horizon at the same time.

(iii) Yes, because at latitude 41° S, everything will declination less than -49° will be circumpolar, so it will be visible once the Sun has set.

6. After a quick stop in Mexico studying Mayan pyramids, you fly on to visit a friend in Trondheim, Norway, who used to be an exchange student at the NUS. You would like to determine the geographical coordinates of Trondheim, so on the day of the December solstice on 22 December 2006 you observe the Sun crossing the meridian at 12:17 at an altitude of 3° in the south. Trondheim is in the UTC $+ 1$ time zone, and the equation of time on 22 December is about 1 minute.

What is the latitude and longitude of Trondheim?

Solution:

(i) If you were in Singapore, the Sun would cross the meridian on the day of the December solstice at an altitude of 66.5° in the South. The latitude of Trondheim is therefore 63.5° N.

Since the equation of time is 1 minute, the Sun is 1 minute fast, so true noon at Greenwich was at 11:59. Your watch is using UTC $+ 1$, so true noon at Trondheim was at 11:17 UTC. Hence the difference is 42 minutes so the longitude of Trondheim is 10.5° E.

7. You're having a great time in Trondheim, and your friend invites you to stay until Easter. Unfortunately, your friend doesn't know when Easter will fall in 2007 and doesn't have an almanac for 2007. Fortunately, you manage to find an almanac for 2006, and you see that the spring equinox occurred at 18:26 UTC on Monday 20 March 2006. You also see that there was a full Moon at 16:40 UTC on Thursday 13 April 2006.

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(i) *What is the rule of thumb for computing the date of Easter?*

(ii) *Why is the rule difficult to implement unless you know a lot about mathematics and astronomy?*

(iii) *When do you think the spring equinox will occur in 2007?*

(iv) *When do you think the first full Moon after the spring equinox will be in 2007?*

(v) *When do you think Easter Sunday will be in 2007?*

Solution:

(i) Easter Sunday is the first Sunday after the first full Moon on or after the vernal equinox.

(ii) How do you determine the vernal equinox? How do you determine the full Moon? Which meridian do you use for determining Sunday?

(iii) Since the tropical year is about 365.25 days, you would guess that the March equinox will occur at around 00:26 UTC on 21 March 2007. (The actual time will be around 00:07 UTC).

(iv) A lunar month is about 29.53 days, so 12 lunar months will be about 354.36 days, or about 354 1/3 days. I would therefore guess that there will be a full Moon around 00:40 UTC on 3 April 2007. (The actual time will be 17:15 UTC on 2 April.)

(v) If 13 April was a Thursday in 2006, then it will be a Friday in 2007, and it follows that 3 April will be Tuesday. Easter Sunday is therefore likely to be 8 April 2007.

8. Having sorted out Easter, you start looking at how the Muslim calendar is affected by the high latitude of Trondheim.

(i) *What is the angle between the celestial equator and the horizon in Singapore? What is the angle between the ecliptic and the horizon in Singapore?*

(ii) *What is the angle between the celestial equator and the horizon in Trondheim? What is the angle between the ecliptic and the horizon in Trondheim?*

(iii) *How will the ecliptic lie with respect to the horizon in Trondheim at the time of sunset near the September equinox?*

(iv) *Why is this likely to delay the first visibility of the waxing crescent in Trondheim?*

Solution:

(i) In Singapore the angle between the celestial equator and the horizon is equal to 90° and the angle between the ecliptic and the horizon changes between 66.5° and 90°.

(ii) Since we know the latitude of Trondheim is 63.5° the angle between the celestial equator and the horizon is equal to 26.5° and the angle between the ecliptic and the horizon changes between 50° and 3°.

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(iii) At the time of the September equinox, the ecliptic will point down, so the angle between the ecliptic and the horizon will be about 3°

(iv) This means that as the Moon moves away from the Sun after the new Moon, the altitude of the Moon at the time of sunset will increase slowly from day to day. Since the angle between the celestial equator and the horizon is relatively small, too, the twilight will last for a long time. It will therefore be very hard to see the waxing crescent for several days after the new Moon.

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