NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

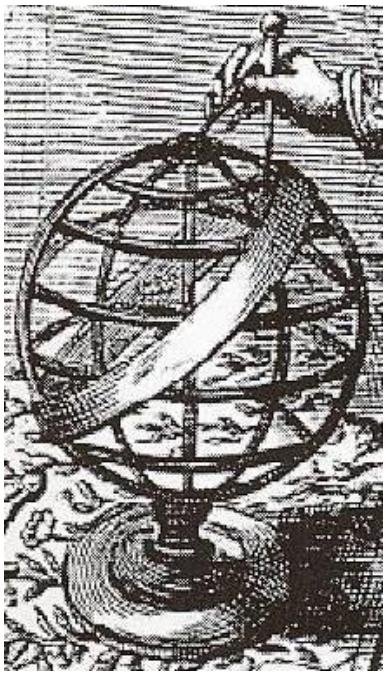
SEMESTER 2 EXAMINATION 2006–2007

**GEK1506 Heavenly Mathematics: Cultural Astronomy**

April/May 2007 — Time allowed: 2 hours

**1.** After taking the Heavenly Mathematics class, you and two friends have decided to go on a trip to China. You want to experience some of the interesting astronomical phenomena you have learned about in class and see some astronomical sights. However, your friends have not taken the class, so you will need to do a bit of explaining.

Your friends notice the picture of the Jesuit missionary Adam Schall (1591-1666) at the top of the course web page, and ask you about the armillary sphere he is pointing at.



(i) What are the five horizontal circles on the armillary sphere?

(ii) What do we call the six regions formed by the five circles?

(iii) Which astronomical phenomenon can we experience in the top and bottom regions?

(iv) Which astronomical phenomenon can we experience in the two middle regions?

(v) What is the broad, slanting band on the sphere?

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(vi) What is wrong with his armillary sphere?

Solution:

(i) Arctic Circle, Tropic of Cancer, Equator, Tropic of Capricorn, Antarctic Circle.

(ii) The Arctic, the North Temperate Zone, the northern part of the Tropics, the southern part of the Tropics, the South Temperate Zone, the Antarctic.

(iii) Midnight Sun.

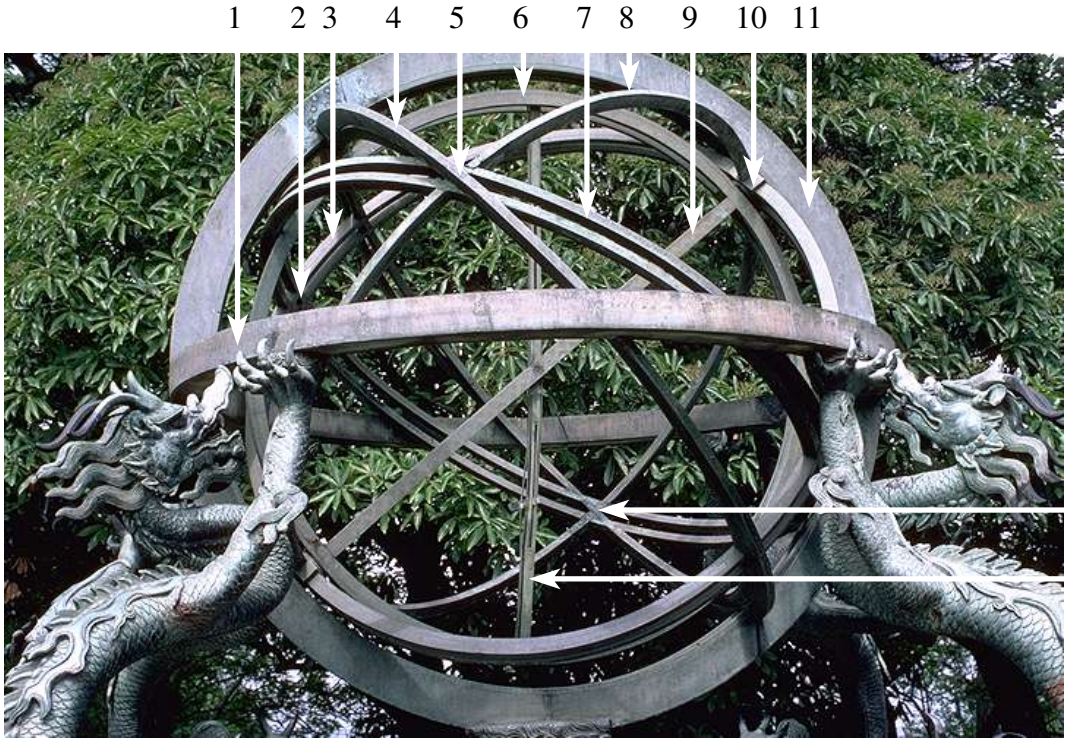
(iv) Zenith passage of the Sun.

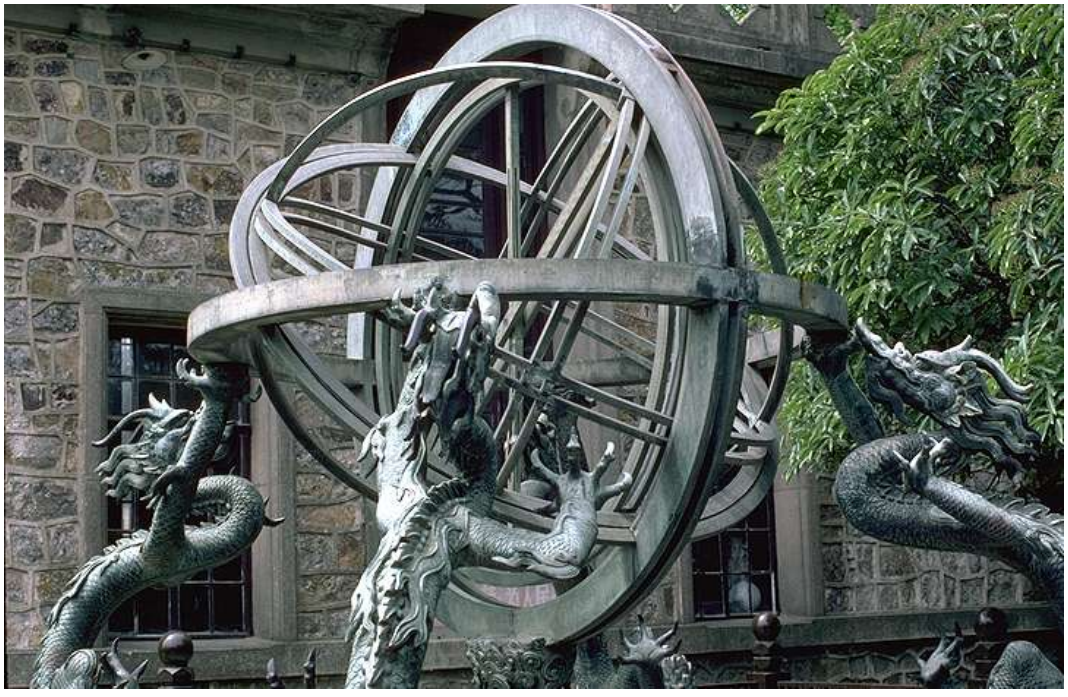
(v) The ecliptic.

(vi) The ecliptic should go between the two Tropics and not from the Arctic Circle to the Antarctic Circle.

**2.** Your first stop is at the Ancient Observatory in Beijing (古观象台 [古觀象台]), where you see this lovely armillary sphere designed by Guō Shǒujìng (郭守敬, 1231–1316). On the next page there are two pictures of it, from slightly different angles. Please note that 2, 5, 10 and 12 are “points”, 9 and 13 are “lines", and the rest are circles. Match the following items with the numbers in the picture.

- ☐ Horizon
- ☐ North Celestial Pole
- ☐ Meridian
- ☐ Polar axis
- ☐ Celestial equator
- ☐ Ecliptic
- ☐ Vernal equinox
- ☐ Autumnal equinox
- ☐ Equinoctial colure (great circle through the equinoxes and the poles)
- ☐ Solstitial colure (great circle through the solstices and the poles)
- ☐ Winter solstice
- ☐ Sighting tube
- ☐ Movable right ascension circle



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Solution:

- $\boxed{1}$ Horizon
- $\boxed{10}$ North Celestial Pole
- $\boxed{11}$ Meridian
- $\boxed{9}$ Polar axis
- $\boxed{4}$ Celestial equator
- $\boxed{7}$ Ecliptic
- $\boxed{5}$ Vernal equinox
- $\boxed{12}$ Autumnal equinox
- $\boxed{8}$ Equinoctial colure (great circle through the equinoxes and the poles)
- $\boxed{3}$ Solstitial colure (great circle through the solstices and the poles)
- $\boxed{2}$ Winter solstice
- $\boxed{13}$ Sighting tube
- $\boxed{6}$ Movable right ascension circle

**3.** Your friends wonder what the armillary sphere can be used for. Give estimates for the horizon, equatorial and ecliptic coordinates of the object that the sighting tube is pointing at in the pictures. (I'm only expecting rough estimates. You may need to look at both pictures.)

Solution:

Horizontal: altitude $= 85°$, azimth $= 85°$.
Equatorial: declination $= 40°$, right ascension $= 20$h.
Ecliptic: latitude $= 60°$, longitude $= 310°$.

**4.** Your friends are very interested in the Chinese calendar and ask you these questions.

(i) Why is the Mid-Autumn Festival celebrated on the 15th day of the eight Chinese month?

(ii) Why was the Mid-Autumn Festival celebrated on different days in China and Hong Kong in 1978?

Solution:

(i) If the first day of the first month marks the beginning of spring, the autumn will start on the first day of the seventh month, so the middle of the eight month is the middle of autumn.

(ii) Before 1978, many calendars in Hong Kong and Taiwan were still based on the old imperial calendar from 1908, the year in which the last Qīng emperor ascended the throne.

The new Moon that marked the start of the 8th month in 1978 would occur just before midnight at 23h 53m on September 2, 1978, making the 7th month a short month. The astronomers at the Purple Mountain Observatory in Nanjing had computed that the new Moon would occur after midnight at 0h 07m on September 3, 1978, making the 7th month a long month. The Mid-Autumn Festival would therefore be one day later in China.

**5.** One of your friends is an exchange student from Norway. He knows that Norway changed from the Julian to the Gregorian calendar at the end of February 18, 1700, but he doesn't know which date of the Gregorian calendar the following day was. Help him figure out that date.

Solution: Since they did it in February 1700, they would only have to skip 10 days, so the next day would be March 1, 1700.

**6.** One of your friends has always wondered about the date of Qīngmíng (清明). She says that it usually falls on April 5, but sometimes it can fall on April 4. You know that it is one of the 24 Chinese solar terms, or jiéqì (节气 [節氣]), so you can easily explain this to her.

(i) Why does Qīngmíng move in the Chinese calendar?

(ii) Why can Qīngmíng move by one day in the Gregorian calendar?

(iii) Explain how Qīngmíng will move in the Gregorian calendar in the course of 100 years.

Solution:

(i) The solar terms are determined by the Sun and will move in the lunar calendar.

(ii) The Gregorian calendar uses leap days to approximate the Sun. Qīngmíng may therefore move by a day.

(iii) Normally Qīngmíng will fall six hours later each year, but in a leap year it will jump 18 hours earlier. However, since the tropical year is a bit less than 365.25, Qīngmíng will move less than six hours later each year, so the leap year day will make Qīngmíng drift earlier over the years. However, the skipped leap year in most century years will restore balance.

**7.** You have spent a wonderful morning at the Ancient Observatory in Beijing, but before you go for lunch, you decide to try to determine the coordinates of the observatory by taking a reading of the Sun's meridian transit. You are there on June 21, the day of the June solstice, so you know the declination of the Sun. There is an analemma carved in stone on a sundial next to the armillary sphere, and by looking at it, you estimate that the equation of time is about $-1$ minute. At 12:15 p.m. local time (UTC $+ 8$) you observe the Sun crossing the meridian at an altitude of $73.5^\circ$ in the south. Use this to compute the coordinates of Beijing.

Solution: The declination at the June solstice is $23.5^\circ$. At the equator, the Sun would therefore be at an altitude of $66.5^\circ$ in the north. Since we see the Sun $40^\circ$ degrees to the south of that, the latitude of Beijing is $40^\circ$ north. If the equation of time is $-1$ minute, the Sun will cross the meridian at longitude $120$ east at 12:01. Since it crosses 14 minutes later in Beijing, the longitude of Beijing must be $14/4 = 3.5^\circ$ west, at $116.5^\circ$ east.

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