NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION 2007–2008

GEK1506   Heavenly Mathematics: Cultural Astronomy

April/May 2008 — Time allowed: 2 hours

Matriculation Number:

INSTRUCTIONS TO CANDIDATES

 (i) This examination paper contains 4 questions and comprises 7 pages including this page.

 (ii) Answers and your matriculation number should be written on this question paper. You may also write on the back of the pages. There is a blank page at the end if you need more space. Please indicate clearly if you use this for your answers.

(iii) **Explain and justify your answers carefully.**

(iv) Candidates may use calculators.

1. You've learned in class that if the date of Chinese New Year in a certain year is X, then the following year it will either be 10, 11 or 12 days earlier or 18, 19 or 20 days later. In other words, it will fall on $X - 12$, $X - 11$, $X - 10$, $X + 18$, $X + 19$ or $X + 20$. We will now try to derive a similar statement for Easter Sunday.

   (i) [6 marks] *If the date of Easter Sunday is X in a certain year, what day of the week is X in the following year?*

  (ii) [24 marks] Easter Sunday falls on the first Sunday after the first ecclesiastical full Moon on or after March 21. The ecclesiastical full Moon is an approximation to the real full Moon used by the Church and the first one after March 21 is called the Paschal Moon. Each year it will move either 10 or 11 days earlier, or 18 or 19 days later. *If the date of Easter Sunday is X in a certain year, what are the possible dates that it can fall on in the following year?*

2.   (i) [10 marks] *Derive a formula for the altitude of the Sun when it crosses the meridian on a given day.*

   (ii) [10 marks] Assume that the latitude of Singapore is 1°N and the longitude 104°E. Assume for simplicity that the equation of time on the June solstice is zero. *Where and when will the Sun cross the meridian in Singapore on the June solstice?*

3. In order to use equatorial coordinates, you need to know where the vernal equinox is (in terms of horizontal coordinates). For simplicity you can assume in this question that the Sun crosses the meridian at noon and that the latitude of Singapore is $0°$.

   (i) [10 marks] *Where is the vernal equinox at noon on the day of the vernal equinox for an observer in Singapore?*

  (ii) [10 marks] *Where is the vernal equinox at sunrise on the day of the summer solstice for an observer in Singapore?*

4. Cable TV uses geostationary satellites. They are satellites that move around the Earth in an orbit directly above the equator with an orbital velocity that matches the rotation speed of the Earth. That means that they will stay fixed over the same place on Earth. At certain times on certain days of the year, the satellite will come directly between the Sun and the receiver on Earth, as in Figure 1. This will cause disruption to TV reception and is called Sun outage.

We will consider a satellite that stays over a point with latitude $0°$ and longitude $95°$E. Figure 2 shows where and when Sun outage will occur from this satellite.
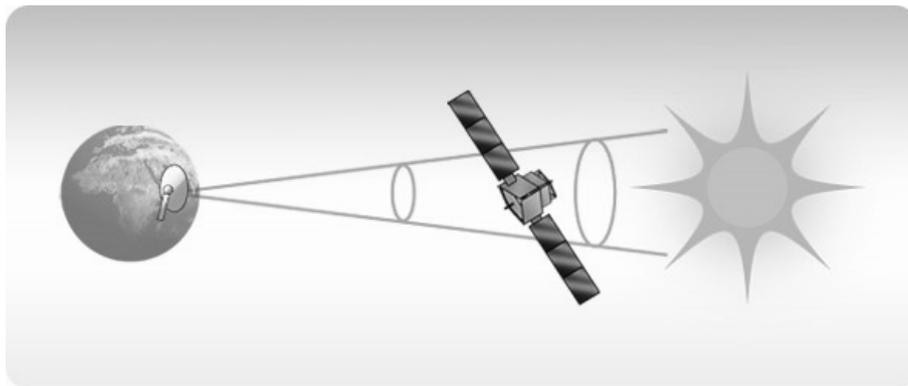

Figure 1: Sun outage

(i) [7 marks] *Why do places north of the equator experience Sun outage before the vernal equinox?*

(ii) [7 marks] *Why does Singapore experience Sun outage from this satellite in the afternoon?*

(iii) [8 marks] Look at the curves that indicate the time of day of the Sun outage. In the middle they look almost straight, but towards the right they curve. *What do those curves look like on the surface of the Earth?* (For simplicity you can assume that the equation of time is zero for this part.)

(iv) [8 marks] The 05:54 curve slants down to the right. On a corresponding chart for the autumnal equinox (not included), the corresponding curve would slant down to the left. *How can you explain that using the analemma?*
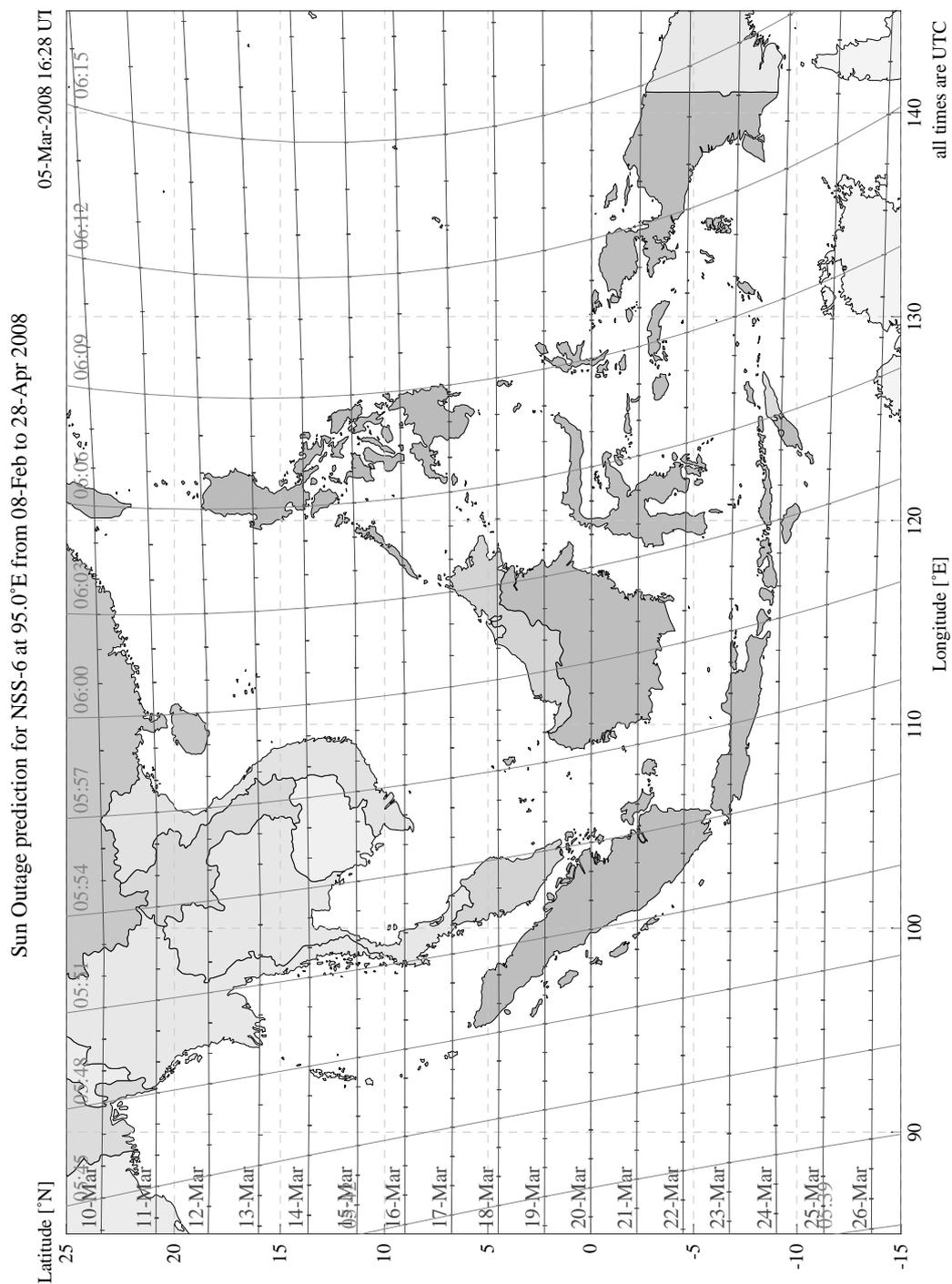
Figure 2: Sun outage map for a satellite at $95°$E

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