NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION 2007–2008

GEK1506 Heavenly Mathematics: Cultural Astronomy

April/May 2008 — Time allowed: 2 hours

1. You've learned in class that if the date of Chinese New Year in a certain year is X, then the following year it will either be 10, 11 or 12 days earlier or 18, 19 or 20 days later. In other words, it will fall on $X-12$, $X-11$, $X-10$, $X+18$, $X+19$ or $X+20$. We will now try to derive a similar statement for Easter Sunday.

(i) [6 marks] *If the date of Easter Sunday is X in a certain year, what day of the week is X in the following year?*

(ii) [24 marks] Easter Sunday falls on the first Sunday after the first ecclesiastical full Moon on or after March 21. The ecclesiastical full Moon is an approximation to the real full Moon used by the Church and the first one after March 21 is called the Paschal Moon. Each year it will move either 10 or 11 days earlier, or 18 or 19 days later. *If the date of Easter Sunday is X in a certain year, what are the possible dates that it can fall on in the following year?*

Solution:

(i) $365 = 52 * 7 + 1$, so X will be a Monday. If the following year is a leap year, X will be a Tuesday.

(ii) The full Moon is between $X-7$ and $X-1$, so the following year there will be a full Moon some time between $X-18$ and $X-11$. If that is on or after March 21, then that will be the Paschal Moon. If the following year is a normal year, then $X-1$ is a Sunday, in which case Easter Sunday will be $X-15$ or $X-8$. If the following year is a leap year, then $X-2$ is a Sunday, in which case Easter Sunday will be $X-16$ or $X-9$.

If the full Moon between $X-18$ and $X-11$ is before March 21, we must instead add 18 or 19 days to the full Moon between $X-7$ and $X-1$, which takes us to some time between $X+11$ and $X+18$. If the following year is a normal year, then $X-1$ is a Sunday, in which case Easter Sunday will be $X+13$ or $X+20$. If the following year is a leap year, then $X-2$ is a Sunday, in which case Easter Sunday will be $X+12$ or $X+19$.

2. (i) [10 marks] *Derive a formula for the altitude of the Sun when it crosses the meridian on a given day.*

(ii) [10 marks] Assume that the latitude of Singapore is $1°$N and the longitude $104°$E. Assume for simplicity that the equation of time on the June solstice is zero. *Where and when will the Sun cross the meridian in Singapore on the June solstice?*

Solution:

(i) If you are at latitude l, the celestial equator will cross the meridian at a point corresponding to the colatitude. Here I measure altitude between $0°$ and $180°$ from the south point on the horizon, and latitude is measured from $-90°$ to $90°$. In order to account for the time of the year, I simply add the declination. So the formula is colatitude + declination.

(ii) If the equation of time is 0, you are in UTC $+ 8$ and live at $120°$E, the Sun will cross the meridian at noon. If you live at $104°$E, it will cross $16 * 4 = 64$ minutes later, at 13:04UTC $+ 8$.

Since the declination is $23.5°$, the Sun will cross the meridian at an altitude of $(90 - 1 + 23.5)° = 112.5°$, which is the same as altitude $67.5°$ in the north.

3. In order to use equatorial coordinates, you need to know where the vernal equinox is (in terms of horizontal coordinates). For simplicity you can assume in this question that the Sun crosses the meridian at noon and that the latitude of Singapore is $0°$.

(i) [10 marks] *Where is the vernal equinox at noon on the day of the vernal equinox for an observer in Singapore?*

(ii) [10 marks] *Where is the vernal equinox at sunrise on the day of the summer solstice for an observer in Singapore?*

Solution:

(i) At noon on the day of the vernal equinox, the Sun will be at the vernal equinox, and the Sun will be at the meridian. In Singapore, the celestial equator goes through zenith, so the vernal equinox will be in zenith.

(ii) At sunrise on the day of the summer solstice, the Sun will be at the eastern horizon, and the vernal equinox will be in zenith.

4. Cable TV uses geostationary satellites. They are satellites that move around the Earth in an orbit directly above the equator with an orbital velocity that matches the rotation speed of the Earth. That means that they will stay fixed over the same place on Earth. At certain times on certain days of the year, the satellite will come directly between the Sun and the receiver on Earth, as in Figure 1. This will cause disruption to TV reception and is called Sun outage.

We will consider a satellite that stays over a point with latitude $0°$ and longitude $95°$E. Figure 2 shows where and when Sun outage will occur from this satellite.

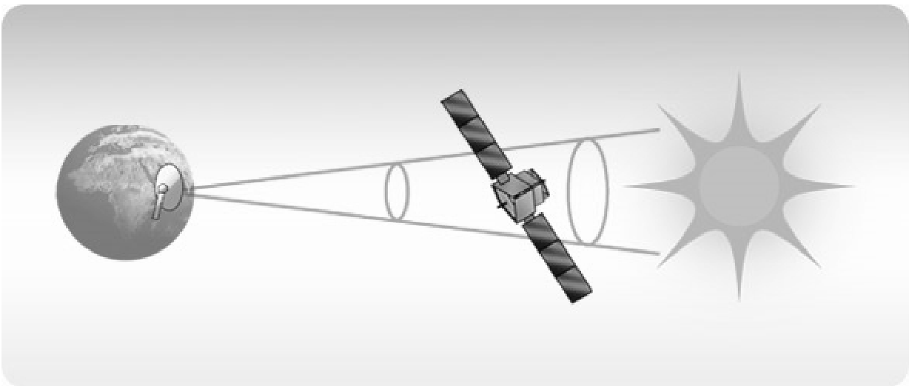

Figure 1: Sun outage

(i) [7 marks] *Why do places north of the equator experience Sun outage before the vernal equinox?*

(ii) [7 marks] *Why does Singapore experience Sun outage from this satellite in the afternoon?*

(iii) [8 marks] Look at the curves that indicate the time of day of the Sun outage. In the middle they look almost straight, but towards the right they curve. *What do those curves look like on the surface of the Earth?* (For simplicity you can assume that the equation of time is zero for this part.)

(iv) [8 marks] The 05:54 curve slants down to the right. On a corresponding chart for the autumnal equinox (not included), the corresponding curve would slant down to the left. *How can you explain that using the analemma?*

Solution:

(i) Since the Sun is over the equator, it will line up between the Sun and places on the northern hemisphere when the Sun is in the southern part of the sky, i.e., before the vernal equinox.

(ii) The satellite is to the west of us, so it will line up with the Sun in the afternoon.

(iii) If you look at the Sun at the same time of the day over several days, the path of the Sun will be a straight line parallel to the meridian. (Here I am assuming that the equation of time is zero.) Together with the position of the satellite, this line generates a plane. The places that experience Sun outage at the same time of day must therefore lie on the circle that is the intersection of this plane and the Earth.

(iv) If we consider the analemma, we no longer get a straight line parallel to the meridian, but a curve that slants down to the left in the sky around the time of the vernal equinox. But when we map this to the surface of the Earth, it gets turned around, so it will slant down to the right.

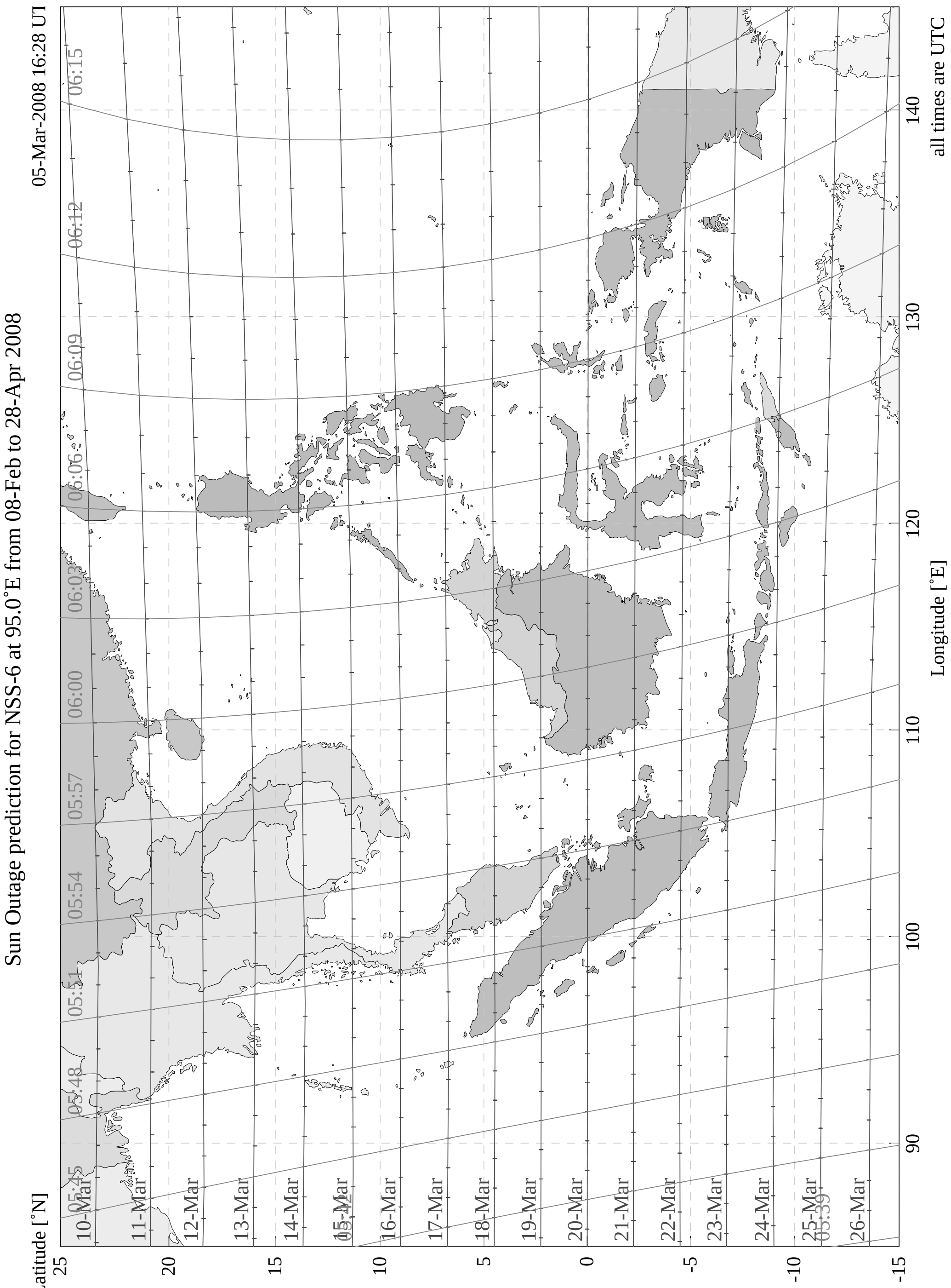

Figure 2: Sun outage map for a satellite at 95°E

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