

NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 2 EXAMINATION 2008–2009

GEK1506 Heavenly Mathematics: Cultural Astronomy

April/May 2009 — Time allowed: 2 hours

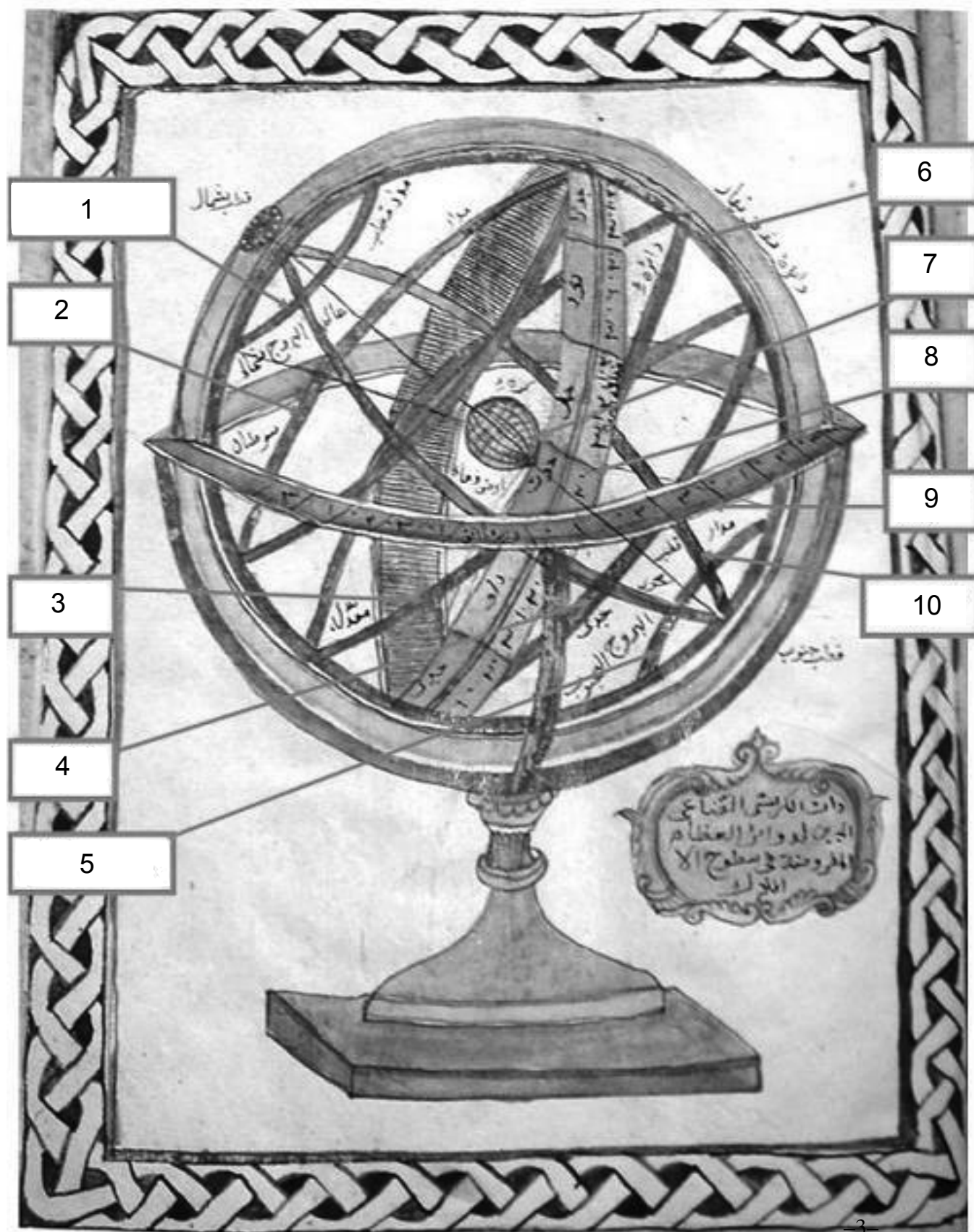


Figure 1: Armillary sphere from the book “Marifetname” (Book of Gnosis) by the Turkish and Sufi philosopher Ibrahim Hakki Erzurumi (1703 - 1780)

1. [10 marks] Figure 1 shows a picture of an armillary sphere from the book “Marifetname” (Book of Gnosis) by the Turkish and Sufi philosopher Ibrahim Hakki Erzurumi (1703 - 1780).

Match the numbers in the figure with the following objects.

- Earth
- Meridian
- Horizon
- Ecliptic
- Equinoctial colure (great circle through the equinoxes and the poles)
- Arctic circle
- Antarctic circle
- Tropic of Capricorn
- Tropic of Cancer
- Equator

Solution:

- 7 Earth
- 6 Meridian
- 9 Horizon
- 8 Ecliptic
- 10 Equinoctial colure (great circle through the equinoxes and the poles)
- 1 Arctic circle
- 5 Antarctic circle
- 4 Tropic of Capricorn
- 2 Tropic of Cancer
- 3 Equator

2. [20 marks] On the day of the December solstice, the Sun will cross the meridian in Oslo, Norway at 12:15 at an altitude of 6.5° in the South. The equation of time is about 2 minutes on the December solstice and Norway lies in the UTC + 1 time zone and does not use daylight saving time in December. *What is the latitude and longitude of Oslo?*

Solution: Since the equation of time is 2 minutes, the true Sun crossed the meridian at Greenwich at 11:58 UTC. Since Oslo is in the UTC + 1 time zone, the meridian transit in Oslo is at 11:15 UTC. The difference is 43 minutes, so the longitude of Oslo is 10.75° east. At the December solstice, the Sun would be at an altitude of 66.5° in the south. Since the observed altitude is 6.5° , it follows that the latitude of Oslo is 60° north.

3. (a) [10 marks] The Hijra is the emigration of Muhammad and his followers to the city of Medina in 622 CE and marks the first year of the Islamic calendar. *Explain how you can use this to give an estimate for what year it is in the Islamic calendar at the moment.* (It is enough to give an estimate. You do not need to use a calculator to get an exact figure.)
- (b) [5 marks] *What time of the day does the Muslim day start?*
- (c) [15 marks] Assume for simplicity that a lunar crescent can be seen 24 hours after the new Moon. Assume that you live on the equator, that we ignore the equation of time, that your longitude equals the longitude of the centre of your time zone and that you do not use daylight saving time. *Which days of the Chinese lunar month can the first day of the Muslim month correspond to? Estimate the probability that it corresponds to the various days.*

Solution:

- (a) There are 1387 years since 622. Since the difference between the Muslim year and the Gregorian year is about 11 days, the difference adds up to one year after about $365/11 \approx 33$ years, or about 3 years each century. So after 14 centuries, the Muslim calendar has gained about 42 years, so we would expect 2009 CE to correspond to AH 1429 (AH = anno Hegirae). The correct answer is that AH 1430 runs from approximately December 28, 2008 (evening) to December 17, 2009, so our estimate is only off by one year.
- (b) At sunset.
- (c) The assumptions imply that sunset will happen at 6 p.m. If the new Moon occurs between midnight and 6 p.m. on the first day, it will be at least 24 hours old at sunset on the second day of the Chinese month. That will therefore start the Muslim month, so the first Muslim day will correspond to the third day of the Chinese month. This will happen 75% of the time. However, if the new Moon occurs between 6 p.m. and midnight on the first day, it will not be at least 24 hours old at sunset on the second day of the Chinese month, and it will only be visible at sunset on the third day. That will therefore start the Muslim month, so the first Muslim day will

correspond to the fourth day of the Chinese month. This will happen 25% of the time.

4. In the Chinese lunisolar calendar, the solar year is divided into 24 solar terms or jiéqì (节气). They are a generalization of the solstices and equinoxes. Let us consider any one of the jiéqì (节气), for instance Qīngmíng (清明).
- (i) [5 marks] *How will Qīngmíng move in the Gregorian calendar from one year to the next?*
 - (ii) [10 marks] *Estimate the probability that Qīngmíng will fall on the same date in the Gregorian calendar for four consecutive years.*
 - (iii) [5 marks] *How will Qīngmíng move in the Chinese calendar from one year to the next?*

Solution:

- (i) In a non-leap year, Qīngmíng will move about 6 hours later and in a leap year it will move about 18 hours earlier. However, since the tropical year is a bit less than 365.25, Qīngmíng will move a bit less than six hours later each year and a bit more than 18 hours earlier in a leap year. Hence, it will slowly move towards the earlier date, until a century year not divisible by 400 moves it later.
 - (ii) If Qīngmíng falls within the first six hours of the day in a leap year, it will stay on that day for the next few years. So the probability is about 25%.
 - (iii) Normally In a (Chinese) non-leap year, it will move about 11 days later. If in the first year it falls in the later part of the month, it will move into the next month. The exact date in the next month depends on whether the previous month has 29 or 30 days. If there is a Chinese leap month between the two Qīngmíng's, it will be move as described above, but go one month earlier.
5. (a) [5 marks] In India they use both solar and lunisolar calendars. The Indian calendar most commonly used in Singapore is the Tamil calendar. *How can you use the calendar in Figure 2 to determine if the Tamil calendar is a solar or lunisolar calendar?*
- (b) In the Wikipedia article about the Bengali Calendar, it says “In this calendar ... the year number ... is divided by 39. If after the division the remainder becomes zero or could be divided by 4, the year is then designated as a leap year and contains 366 days There are 10 leap years in every 37 years”
- (i) [5 marks] *What is inconsistent in this statement?*
 - (ii) [5 marks] *What is the leap year rule in the Gregorian calendar?*
 - (iii) [5 marks] *Why is there a difference between the Gregorian and Bengali leap year rules?*

正 月 初 九	雪兰莪赛马	是日肖羊相冲		宜：祭祀祈福求嗣 忌：入宅作灶
13		SELANGOR RACE DAY		30
7 Dzulkaedah				
正 月 初 十	西方情人节	是日肖猴相冲	14	宜：開市安床入殮 忌：嫁娶入宅
Valentine's Day				
8 Dzulkaedah		31		
正 月 十 一	是日肖鷄相冲		15	宜：破土開光納采 忌：開井作灶
9 Dzulkaedah		1		

Figure 2: Detail of a calendar from Singapore

Solution:

- (a) Since the length of the synodic month is 29.5 days, the months in a lunar calendar are always 29 or 30 days long. The Tamil month in Figure 2 has 31 days, so it must be a solar calendar.
- (b) (i) There will 10 leap years in 39 years, corresponding to remainders of 0, 4, 8, 12, 16, 20, 24, 28, 32 and 36.
 - (ii) Year n is a leap year if n is divisible by 4, but not by 100 or if n is divisible by 400.
 - (iii) The Indian solar calendars use the sidereal year. Since the sidereal year is about 365.2564, there must be more leap years than one every fourth year, or 10 leaps years in less than 40 years. The Gregorian calendars uses the tropical year, which is about 365.2422. Hence there must be less leap years than one every fourth year.

END OF PAPER