NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 1 EXAMINATION 2009–2010

GEK1506 Heavenly Mathematics: Cultural Astronomy

November 2009 — Time allowed: 2 hours

1. (i) [6 marks] What time of the year will the azimuth of the Sun at the time of sunrise on the equator be minimal? What is the minimal value?

(ii) [14 marks] What time of the year will the azimuth of the full Moon at the time of moonrise on the equator be minimal? It is hard to compute the exact minimal value for a given year, but how small can it possibly be?

Solution:

(i) The northernmost rising position of the Sun will be at the June solstice, and on the equator the azimuth will then be $66.5°$.

(ii) Since the full Moon is opposite the Sun, the northernmost rising position of the full Moon will occur at the full Moon closest to the December solstice. If the full Moon occurred near the solstice, the azimuth would be $66.5°$, but since the latitude of the Moon can be up to $5°$, the minimal azimuth can be as small as $61.5°$.

2. [20 marks] Figure 1 shows the sunrise and sunset times in Singapore in the course of the year. Explain how you can use the analemma to estimate the dates on which the sunrise and sunset are earliest or latest.

Solution: The Sun is going to rise earliest when the equation of time is maximal, which is when the rising analemma on the equator in Figure 2 is highest. This occurs in early November. This also corresponds to the earliest sunset. The latest sunrise and the latest sunset occurs when the equation of time is minimal, which is when the rising analemma on the equator is lowest. This occurs in the middle of February.

3. (i) [12 marks] In Great Britain, daylight saving time (DST) is observed from the last Sunday in March to the last Sunday in October. In 2009 this was from March 29 to October 25. In London the earliest sunrise is at 4:42 a.m., and the latest sunset is at 9:21 p.m. Draw a chart similar to the one in Figure 1 estimating the time of sunrise and sunset in London in the course of the year. Make sure you indicate whether the dates of earliest/latest surnrise/sunet will be before, on or after the days of the solstices.

(ii) [4 marks] San Francisco is in the UTC $-$ 8 time zone and observes DST from the second Sunday in March to the first Sunday in November. In 2009 this was from

. . . –2–

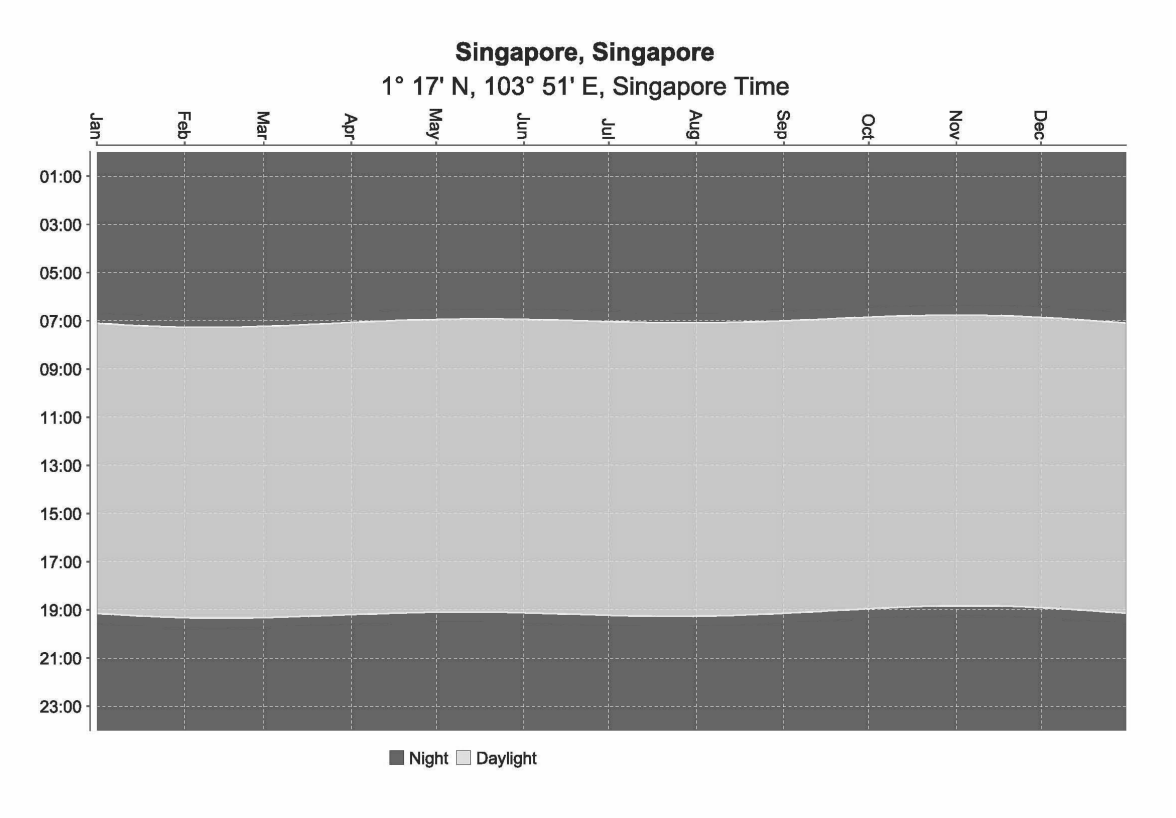

Figure 1: Sunrise and sunset times in Singapore

March 8 to November 1. Construct a table showing the time difference between Singapore and San Francisco at different times of the year in 2009.

(iii) [4 marks] Sydney is in the UTC + 10 time zone and in 2009 DST ended on April 5 and started on October 4. Construct a table showing the time difference between Singapore and Sydney at different times of the year in 2009.

Solution:

(i) Figure 3 shows how the sunrise and sunset times jump one hour later in March and one hour earlier in October. As can be seen from the shape of the rising analemma in Figure 4, the dates of the earliest sunrise will be before and the latest sunset after the June solstice. Similarly, the date of the latest sunset will be after and the date of the earliest sunset before the December solstice.

(ii)

Jan 1 to March 8	March 8 to November 1	November 1 to December 31
16 hours (or 8 hours)	15 hours (or 9 hours)	16 hours (or 8 hours)

(iii)

Jan 1 to April 5	April 5 to October 4	October 4 to December 31
3 hours	2 hours	3 hours

. . . –3–

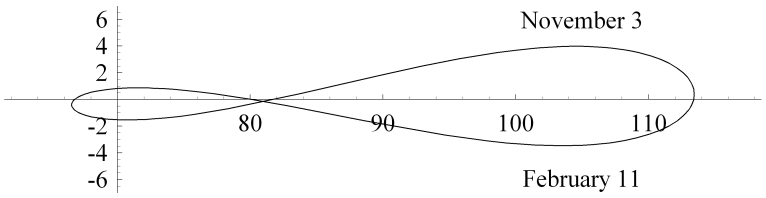

Figure 2: Rising analemma on the equator

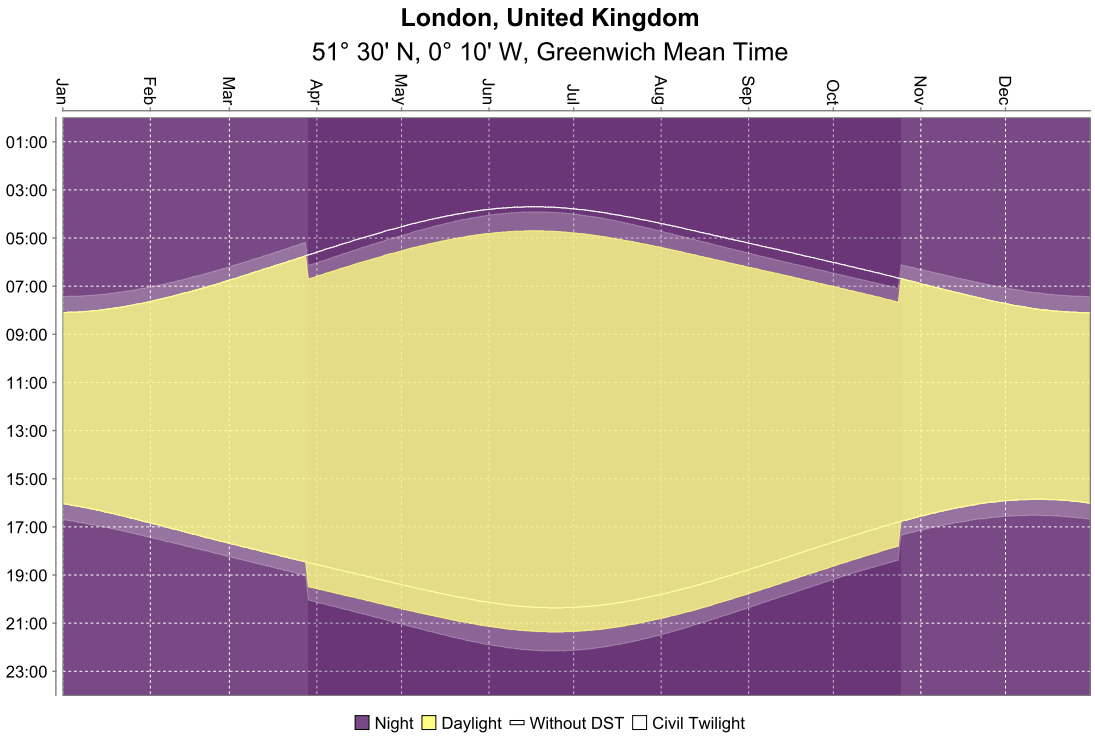

Figure 3: Sunrise and sunset times in London

. . . –4–

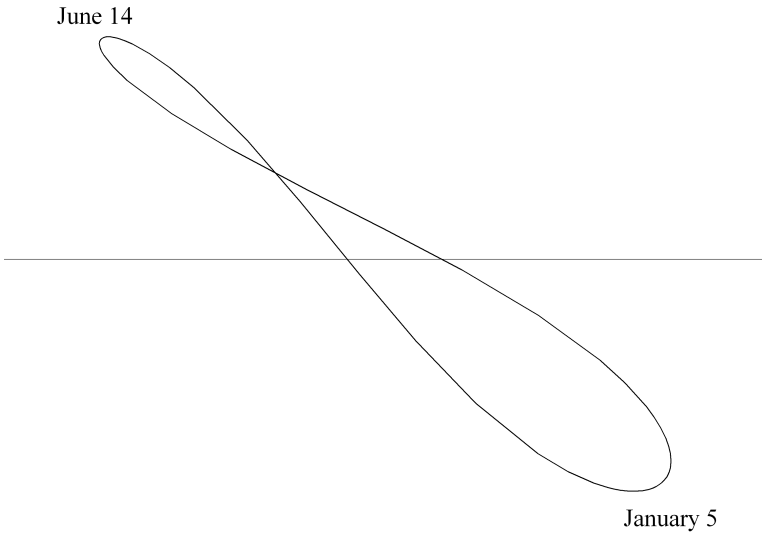

Figure 4: Rising analemma in London

4. [20 marks] The latitude of San Francisco is about 38° north and 122° west and it lies in the UTC $-$ 8 time zone. The equation of time is about 2 minutes on the December solstice. Estimate where and when the Sun will cross the meridian on the day of the December solstice.

Solution: UTC $-$ 8 corresponds to 120° west, so you would expect the Sun to cross the meridian 8 minutes later. Combined with the equation of time being 2 minutes, we see that the Sun will cross the meridian at 12:06 a.m. Since the altitude when crossing the meridian equals colatitude + declination, we get that the altitude when crossing the meridian equals $(90 - 38 - 23.5)^\circ = 28.5^\circ$ in the south.

5. [20 marks] In the Indian calendar, the lunar month is divided into 30 tithis. The day starts at sunrise and takes its number (name) from the number (name) of the tithi at the time of sunrise.

In South India the main Deepavali celebration is at sunrise on the 29th (Chaturdasi) day in a certain lunar month (Asvina). In northern India the main Diwali celebration is on the 30th (Amavasya) day of the same month during a certain time period after sunset (Pradosha). However, in northern India it is more important that the celebration takes place during the 30th tithi than during the 30th day, so it will sometimes fall on the 29th day. Give a rough estimate of how often the southern Deepavali and the northern Diwali will fall on the same day. To simplify, you can assume that both the tithi and the solar day are 24 hours and that you are located on the equator.

Solution: From Figures 5 and 6 we see that the problem boils down to whether the Amavasya tithi ends before or after the Pradosha, which starts at sunset. Since the day

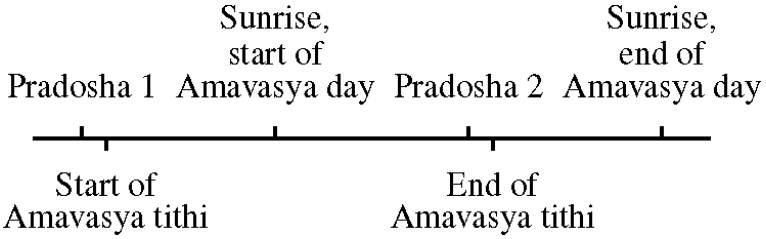

Figure 5: Tithi ends after sunset

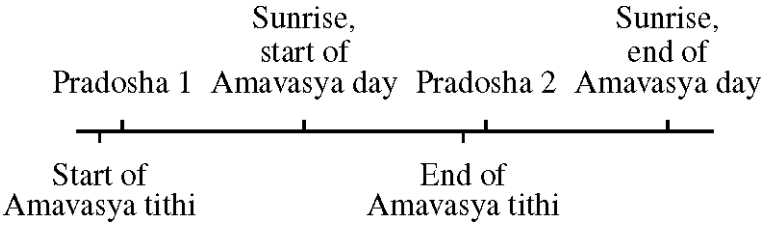

Figure 6: Tithi ends before sunset

starts at sunrise and we are assuming that we are on the equator, the two cases will be about equally likely.

END OF PAPER