NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 1 EXAMINATION 2010–2011

GEK1506   Heavenly Mathematics: Cultural Astronomy

November 2010 — Time allowed: 2 hours

Matriculation Number:

INSTRUCTIONS TO CANDIDATES

(i) This examination paper contains four questions and comprises six pages including this page and a blank page at the end.

(ii) Write your matriculation number on the front page.

(iii) Write your answers on this question paper. You may also write on the back of the pages. There is a blank page at the end if you need more space. Please indicate clearly if you continue a question on the last page.

(iv) This is a closed book exam.

(v) **Explain and justify your answers carefully.**

(vi) Answer all questions.

(vii) You may use a calculator.

1. In the Indian calendar, the lunar month from one new Moon to the next new Moon is divided into 30 *tithis*. The first tithi starts at the new Moon and ends when the difference in longitude between the Sun and the Moon is twelve degrees, and so on. Because of the complexity of the Moon's motion, the actual length of the of a tithi may change between 19 and 26 hours. The day starts at sunrise, and the number of the day in the month is determined by the number of the tithi that the Moon is in at the time of sunrise. Assume for simplicity that the Sun rises at 06:00 each morning.

   (i) [5 marks] If day x was skipped, how many sunrises were there in tithi x? If day x was repeated, how many sunrises were there in tithi x?

  (ii) [5 marks] Explain why on average the length of a tithi will be less than 24 hours, and why this sometimes leads to skipped days.

 (iii) [5 marks] How often would you expect there to be skipped days? How often would you expect the days in an Indian month to be 1 through 29 consecutively, i.e., day number 30 is skipped?

 (iv) [10 marks] In the Chinese calendar, the new Moon occurs on the first day of the month. Explain why in the Indian calendar the new Moon will occur during the last day of the month.

2. (a) [10 marks] At the first Conference of the World Fellowship of Buddhists held in Sri Lanka in 1950, the following resolution was adopted.

"That this Conference of the World Fellowship of Buddhists, while recording its respectful appreciation of the gracious act of His Highness the Maharaja of Nepal in making the full-moon day of Vesak a Public Holiday in Nepal, earnestly requests the Heads of Governments of all countries in which Buddhist communities are to be found, either large or small to take steps to make the full-moon day of the month of May declared as Buddha Day and observed as a Public Holiday in honour of the Lord Buddha, who is universally acclaimed as one of the greatest benefactors of Humanity"

What are the two astronomical problems with the statement "the full-moon day in the month of May"?

(b) (i) [10 marks] The apparent diameter of the Sun is about half a degree, and we will assume that refraction bends light about three quarter of a degree. How much longer do you think day will be than night at the time of the equinoxes on the equator?

(ii) [10 marks] At the time of the equinoxes, the declination changes by about half a degree in a day. It has been stated in class that at the north pole the Sun rises on the day of the equinox. If you are more careful and consider the apparent diameter of the Sun and refraction, which day do you think the Sun will actually rise?

3. (i) [5 marks] What is the rule of thumb for the date of Easter Sunday?

  (ii) [5 marks] Why was it a problem for the Church to fix the date of the March equinox to 21 March?

  (iii) [5 marks] Explain how you can estimate the time of the equinox by looking at the location of the sunset.

  (iv) [10 marks] The decision to fix the date of the March equinox to 21 March for the purpose of computing Easter was made at a Church meeting in 325. Scholars estimate that the equinox occurred around noon on 20 March that year. Can you think of a reason why they still picked 21 March instead of 20 March as the date of the equinox.

     Hint: Imagine that they started the day at sunset.

4. [20 marks] After having taken "Heavenly Mathematics", you have decided to go on holiday to Beijing to look at the Ancient Observatory. You want to test out your new skills by trying to figure out where and when the Sun will cross the meridian. You are there on the day of the June solstice, 21 June, so you know the declination of the Sun. There is an analemma carved in stone on a sundial next to the armillary sphere, and by looking at it, you estimate that the equation of time is about -1 minute. You remember from class that the latitude of Beijing is $40°$ north and the longitude is $116.5°$ east. Where and when will the Sun cross the meridian?

END OF PAPER