NATIONAL UNIVERSITY OF SINGAPORE

DEPARTMENT OF MATHEMATICS

SEMESTER 1 EXAMINATION 2010–2011

GEK1506   Heavenly Mathematics: Cultural Astronomy

November 2010 — Time allowed: 2 hours

1. In the Indian calendar, the lunar month from one new Moon to the next new Moon is divided into 30 *tithis*. The first tithi starts at the new Moon and ends when the difference in longitude between the Sun and the Moon is twelve degrees, and so on. Because of the complexity of the Moon's motion, the actual length of the of a tithi may change between 19 and 26 hours. The day starts at sunrise, and the number of the day in the month is determined by the number of the tithi that the Moon is in at the time of sunrise. Assume for simplicity that the Sun rises at 06:00 each morning.

   (i) [5 marks] If day x was skipped, how many sunrises were there in tithi x? If day x was repeated, how many sunrises were there in tithi x?

  (ii) [5 marks] Explain why on average the length of a tithi will be less than 24 hours, and why this sometimes leads to skipped days.

 (iii) [5 marks] How often would you expect there to be skipped days? How often would you expect the days in an Indian month to be 1 through 29 consecutively, i.e., day number 30 is skipped?

  (iv) [10 marks] In the Chinese calendar, the new Moon occurs on the first day of the month. Explain why in the Indian calendar the new Moon will occur during the last day of the month.

Solution:

   (i) [5 marks] Day x is skipped if there is no sunrise during tithi x. Day x is repeated it there were two sunrises in tithi x.

  (ii) [5 marks] The synodic month is on average 29.5 days, so each tithi will on average be about $29.5/30$ days, which is less than 24 hours. This means that sometimes you will get a tithi that does not contain any sunrise, and there will be a skipped day.

 (iii) [5 marks] On average, every second month will have a 29 days, which means that a day was skipped. So on average, 1 month every 60 months will have day 30 skipped. So once every five years.

  (iv) [10 marks] The new Moon marks the end of tithi 30 and the start of tithi 1. But day 1 only starts at the first sunrise during the first tithi, which is *after* the new Moon.

2. (a) [10 marks] At the first Conference of the World Fellowship of Buddhists held in Sri Lanka in 1950, the following resolution was adopted.

"That this Conference of the World Fellowship of Buddhists, while recording its respectful appreciation of the gracious act of His Highness the Maharaja of Nepal in making the full-moon day of Vesak a Public Holiday in Nepal, earnestly requests the Heads of Governments of all countries in which Buddhist communities are to be found, either large or small to take steps to make the full-moon day of the month of May declared as Buddha Day and observed as a Public Holiday in honour of the Lord Buddha, who is universally acclaimed as one of the greatest benefactors of Humanity"

What are the two astronomical problems with the statement "the full-moon day in the month of May"?

(b) (i) [10 marks] The apparent diameter of the Sun is about half a degree, and we will assume that refraction bends light about three quarter of a degree. How much longer do you think day will be than night at the time of the equinoxes on the equator?

(ii) [10 marks] At the time of the equinoxes, the declination changes by about half a degree in a day. It has been stated in class that at the north pole the Sun rises on the day of the equinox. If you are more careful and consider the apparent diameter of the Sun and refraction, which day do you think the Sun will actually rise?

Solution:

(a) [10 marks] There could be more than one full Moon in May. And at the moment of the full Moon, there will be two different days on Earth because of the international date line. Unless you specify a Vesak longitude, it is not clear which day you should pick.

(b) (i) [10 marks] The apparent diameter of the Sun is about half a degree, so the radius is about one quarter of a degree. Combined with the refraction that bends light about three quarter of a degree, it means that the centre of the Sun is about one degree below the horizon at the time of sunrise and sunset. So day is about 12:08 and night about 11:52. Which means that the day is actually 16 minutes longer than night.

(ii) [10 marks] As explained above, sunrise happens when the centre of the Sun is about one degree below the horizon. So the sunrise may happen two days before the day of the equinox.

3. (i) [5 marks] What is the rule of thumb for the date of Easter Sunday?

(ii) [5 marks] Why was it a problem for the Church to fix the date of the March equinox to 21 March?

    (iii) [5 marks] Explain how you can estimate the time of the equinox by looking at the location of the sunset.

    (iv) [10 marks] The decision to fix the date of the March equinox to 21 March for the purpose of computing Easter was made at a Church meeting in 325. Scholars estimate that the equinox occurred around noon on 20 March that year. Can you think of a reason why they still picked 21 March instead of 20 March as the date of the equinox.

    Hint: Imagine that they started the day at sunset.

Solution:

    (i) [5 marks] First Sunday after the first full Moon on or after the March equinox. But the Church uses a tabulated full Moon and fixes the equinox on 21 March.

    (ii) [5 marks] In 1582, the March equinox had drifted to 11 March. Since the Church used 21 March in the computation of Easter, the date of Easter was still in late March to late April. But since the equinox was moving, spring was moving earlier, too, and late April was in danger of becoming summer instead of spring. This would be noticed by people.

    (iii) [5 marks] Look at the azimuth of the Sun when it sets the evening before and after the equinox. Then estimate the exact time of the equinox by seeing how far those are from the west point. For example, if the two sunsets are equally far from the west point (but on opposite sides), the equinox probably happened near sunrise.

    (iv) [10 marks] If they counted the day from sunset, they would say that the equinox happened on 21 March if the equinox was closest to the sunset of the day they called 21 March, but which started on the evening of 20 March.

4. [20 marks] After having taken "Heavenly Mathematics", you have decided to go on holiday to Beijing to look at the Ancient Observatory. You want to test out your new skills by trying to figure out where and when the Sun will cross the meridian. You are there on the day of the June solstice, 21 June, so you know the declination of the Sun. There is an analemma carved in stone on a sundial next to the armillary sphere, and by looking at it, you estimate that the equation of time is about -1 minute. You remember from class that the latitude of Beijing is $40°$ north and the longitude is $116.5°$ east. Where and when will the Sun cross the meridian?

Solution: [20 marks] The formula colatitude + declination says that the Sun will cross the meridian at an altitude of $90 - 40 + 23.5 = 73.5°$. If you were at longitude $120°$, the mean Sun would cross the meridian at 12:00 local time, and since the equation of time is -1 minute, the true Sun is one minute late and will cross at 12:01. But Beijing is $36.5°$ west of that, so the Sun will cross at 12:15.

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