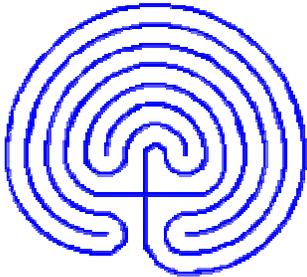


## Mazes and Labyrinths

Here you see a typical labyrinth. You start at the bottom and walk towards the centre. This kind of labyrinth is called unicursal, meaning that there is only one possible path to take.

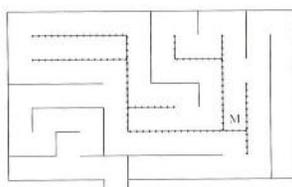


Here is a more complex path. If you look carefully, you will see that there are branch points where you have to choose where to go, and dead ends. This is called multicursal. This means that there's more than one possible path, you have to make choices, and you can get lost. Some people call this a maze, and reserve the name labyrinth for unicursal labyrinths.



How can you find your way in such a maze? One method is to go in, put your hand on the right side of the wall, and just follow that wall. This method will work for the maze above, but not always.

Try the hand on the wall method for the maze below. Can you figure out why it doesn't work?



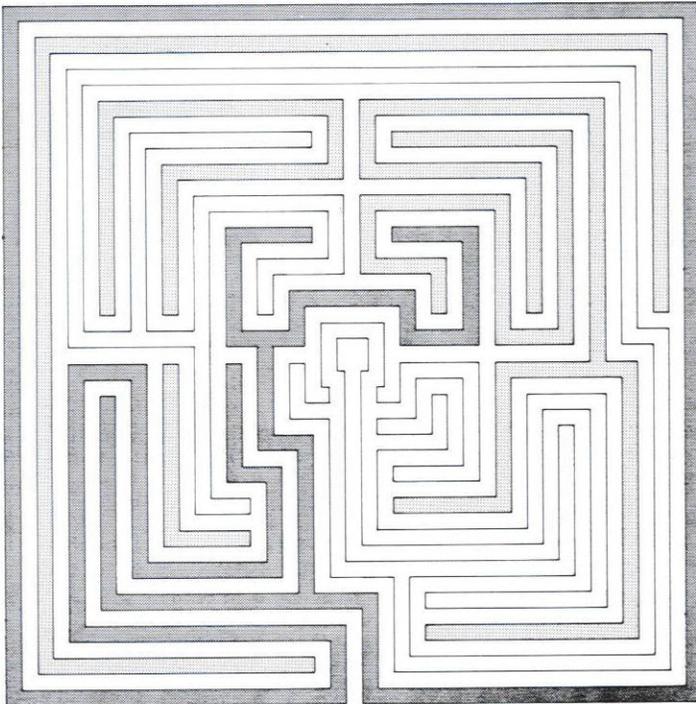
Hint: The M in the middle is surrounded by "dotted" walls, while the entrance is part of the plain walls. So the walls are not connected. If you put your hand on the plain wall, you will never get to the dotted wall.

Can you come up with a method for solving a maze that will always work?

Hint: You may assume that you have a marker with you, so that you can mark where you have been. Every time you come to a fork, mark which path you take.

Here is another maze you can practice on. It's a very famous maze in Chevening in Kent. It was built in the 1820's and is the earliest example of a maze that cannot be solved by the hand on the wall method. Why does the wall have three colours?

Hint: The wall consists of three parts. The goal is surrounded by white walls, while the entrance is part of the dark gray part of the wall. That's why the hand on the wall method will fail.



There are several general algorithms for solving a maze. We will discuss one called Tremaux's algorithm. It is designed to be used by a human inside of the Maze and will find a solution for all Mazes: As you walk down a passage, draw a line behind you to mark your path. When you hit a dead end turn around and go back the way you came. When you encounter a junction you haven't visited before, pick a new passage at random. If you're walking down a new passage and encounter a junction you have visited before, treat it like a dead end and go back the way you came. (That last step is the key which prevents you from going around in circles or missing passages.) If walking down a

passage you have visited before (i.e. marked once) and you encounter a junction, take any new passage if one is available, otherwise take an old passage (i.e. one you've marked once).

All passages will either be empty, meaning you haven't visited it yet, marked once, meaning you've gone down it exactly once, or marked twice, meaning you've gone down it and were forced to backtrack in the opposite direction. When you finally reach the solution, paths marked exactly once will indicate a direct way back to the start. If the Maze has no solution, you'll find yourself back at the start with all passages marked twice.