Mathematics in Art and Architecture GEM1518K

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Polyhedra

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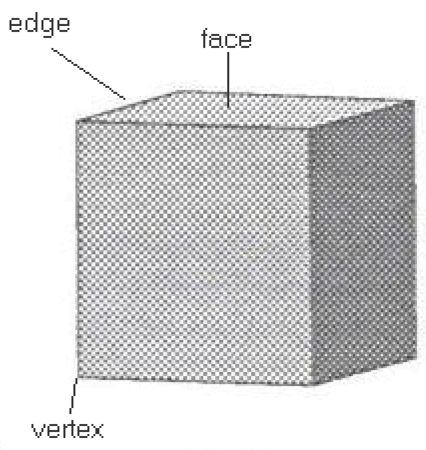
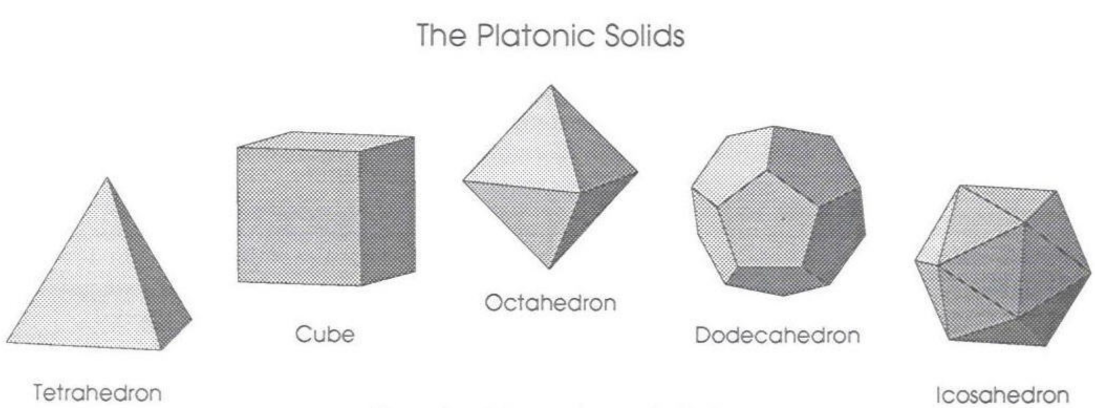
A polyhedron is a solid consisting of polygonal faces. The faces meet at edges, and the edges meet at vertices. Notice that for a polygon, we use the terms side and corner.



The most famous are the five Platonic solids. They are the only polyhedra whose faces are all identical regular polygons, and where the same number of faces meet at each vertex.



	Vertices	Edges	Faces	Sides of each face	Faces at each vertex
Tetrahedron	4	6	4	3	3
Cube	8	12	6	4	3
Octahedron	6	12	8	3	4
Dodecahedron	20	30	12	5	3
Icosahedron	12	30	20	3	5

There are several formulas that can help you. First of all, there is Euler's formula

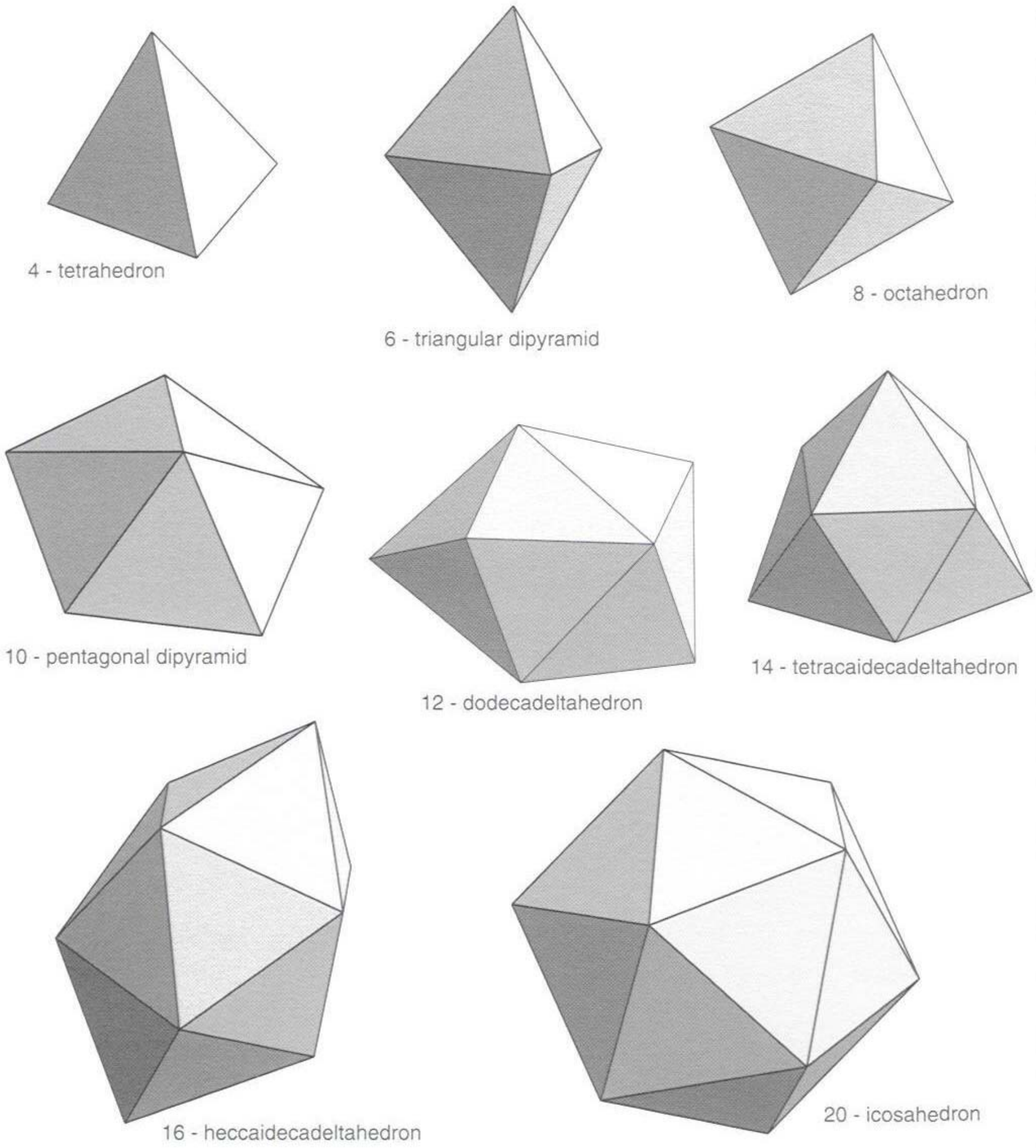
$$V - E + F = 2, \qquad \text{(A)}$$

where V denotes the number of vertices, E the number of edges and V the number of vertices. Secondly, if s denotes the number of sides of the faces and d the degree of the vertices, then

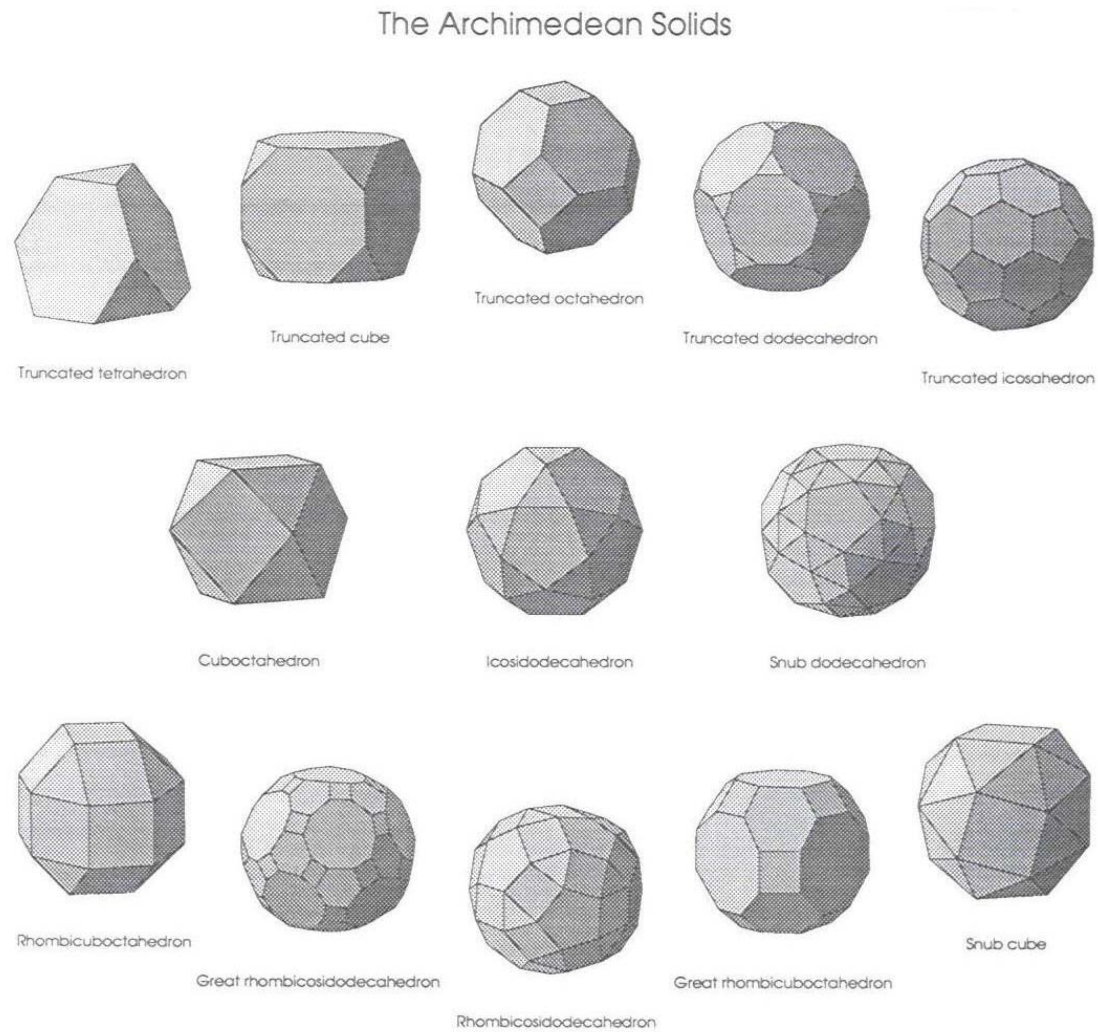
$$Fs = Vd = 2E. \qquad \text{(B)}$$

If there is more than one type of faces, then Fs denotes the sum of the products corresponding to the different types of faces.

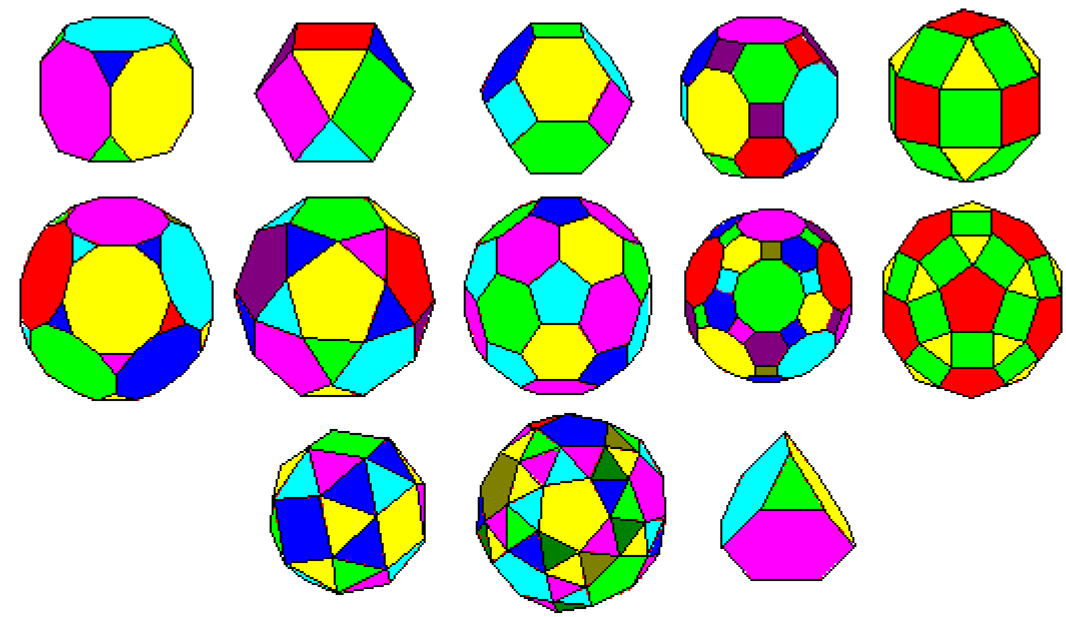
Suppose you don't want two faces that share an edge to have the same color, how many colors do you then need? (If two faces just meet at a vertex, they can have the same colour.) It turns out that one of them only need two colours, two of them need three colours, while two need four colours.

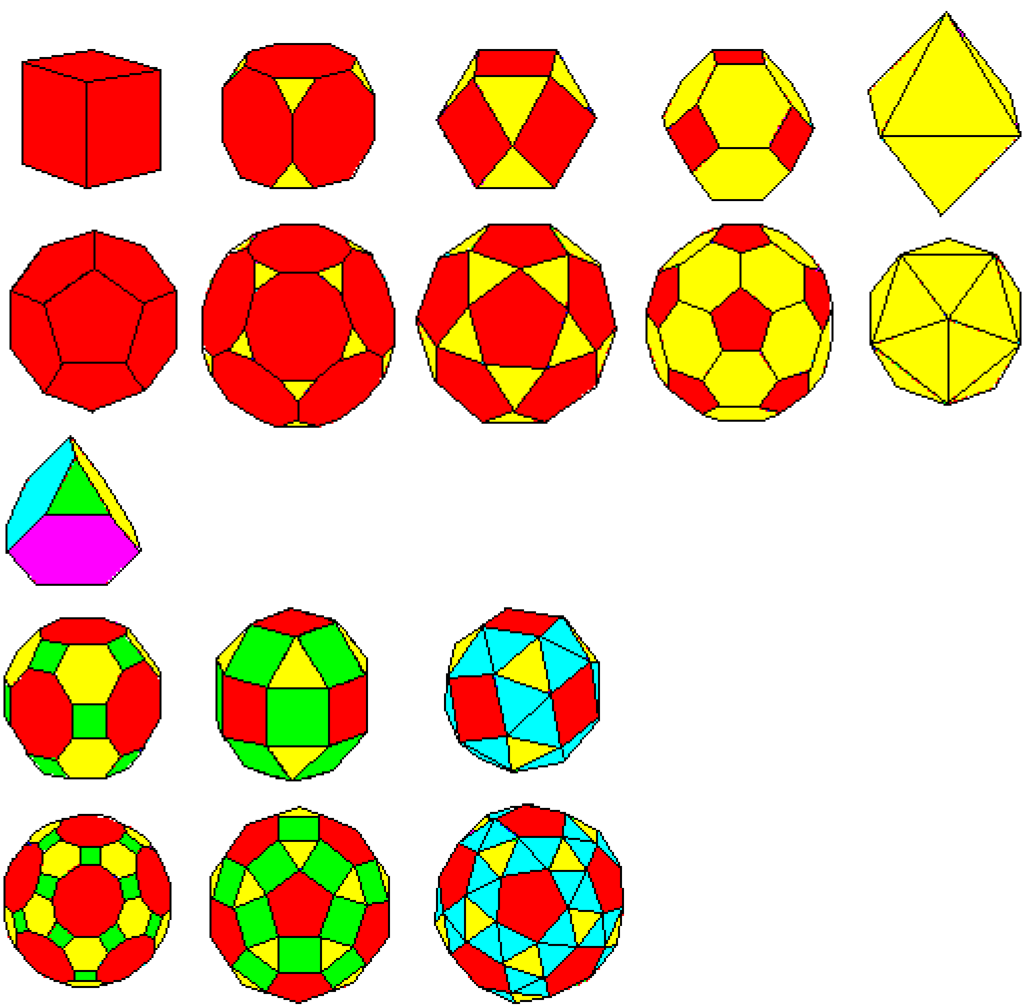


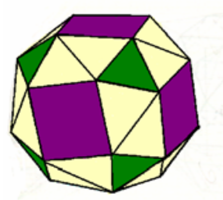
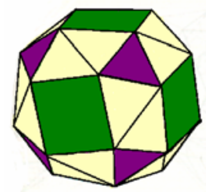
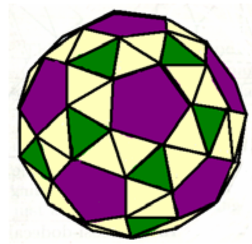
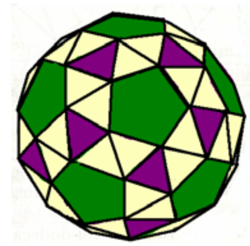
Another famous group of polyhedra are the Archimedean solids. Their faces are also regular polygons, and all the vertices are of the same type, but we are allowed to use more than one type of polygons.

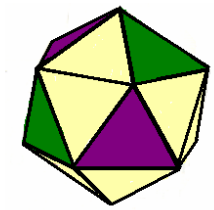
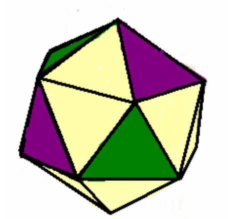


The names of the Archimedean solids can be confusing. First there are the 5 truncated Platonics. Then there are the two "mid-point truncations", namely the cuboctahedron and the icosidodecahedron. If we try to truncate these two, we get rectangles instead of squares, but if we stretch them a bit to make the rectangles squares, we get the "truncated cuboctahedron" and the "truncated icosidodecahedron", which are called the great rhombicuboctahedron and the great rhombicosidodecahedron. We can also make the "midpoint truncated cuboctahedron", called the rhombicuboctahedron and the "midpoint truncated icosidodecahedron", called the rhombicosidodecahedron.

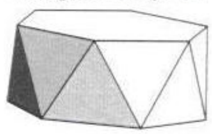




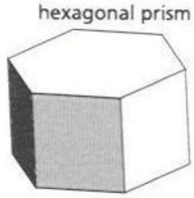





hexagonal antiprism



hexagonal prism

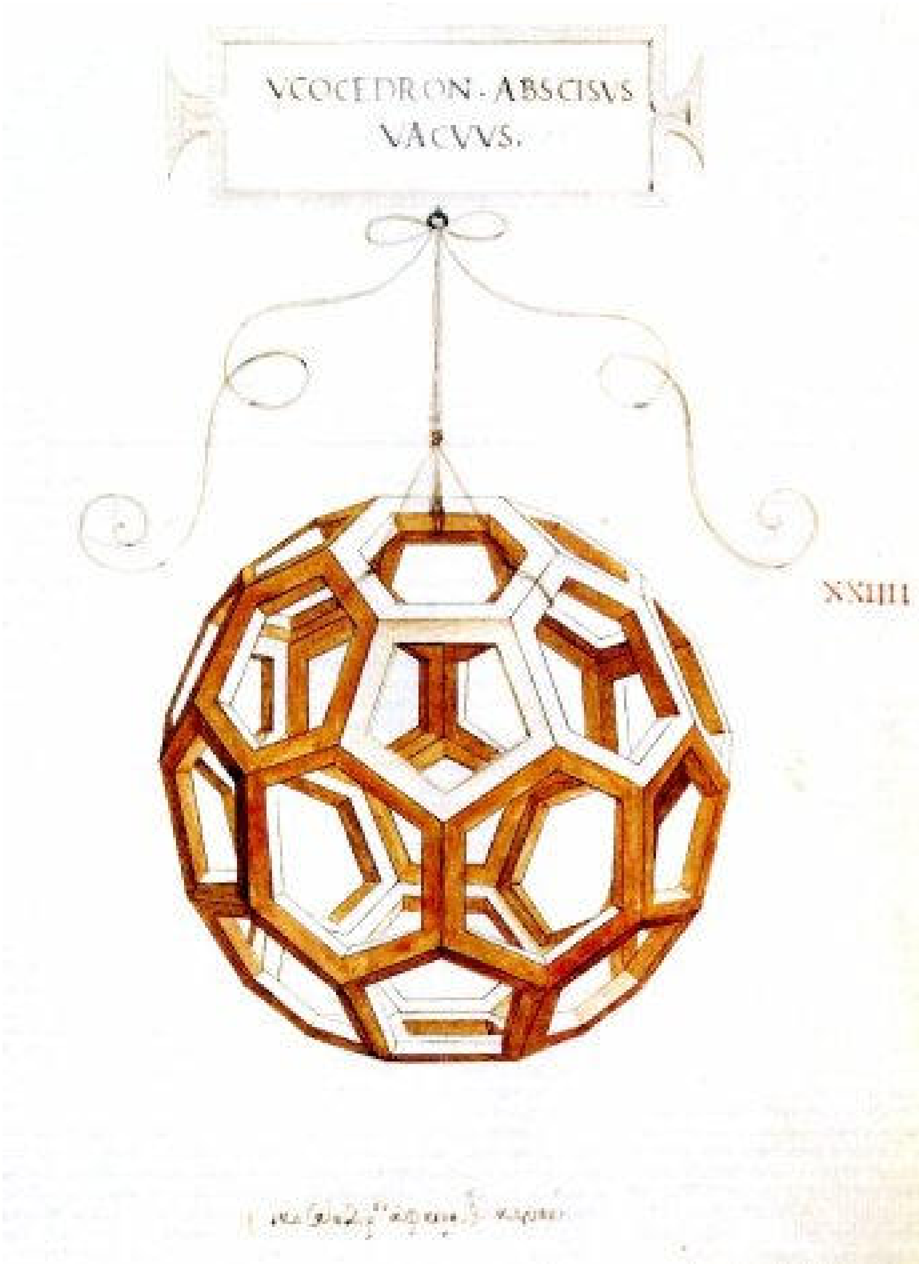


A prism is obtained by taking two equal polygons, and joining corresponding sides with square. Look at the hexagonal prism above. Do you know another name for the square prism?

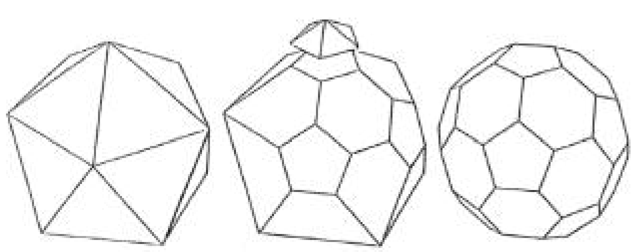
An antiprism is obtained by taking two equal polygons, and using triangles to join the sides of one with the vertices of the other. Look at the hexagonal antiprism above. Do you know another name for the triangular antiprism?

A dipyramid is obtained by putting two pyramids base to base. Look at the pentagonal dipyramid above. Can you make a triangular dipyramid? Do you know another name for the square dipyramid?

Polyhedra have fascinated many artists. Leonardo Da Vinci illustrated a book called “The Divine Proportion”, written by Luca Pacioli in 1509. It contains a lot of information about polyhedra. Here is one Leonardo's illustrations.



Do you see what solid this is? It's a truncated icosahdron. It's obtained from an icosahedron by cutting away a slice at each vertex. An icosahedron consists of 20 triangles and five triangles meet at each of the 12 vertices.



When we truncate we get 12 pentagons corresponding to the 12 vertices, and the 20 triangles become 20 hexagons.

Do you recognize that a football is a truncated icosahedron?

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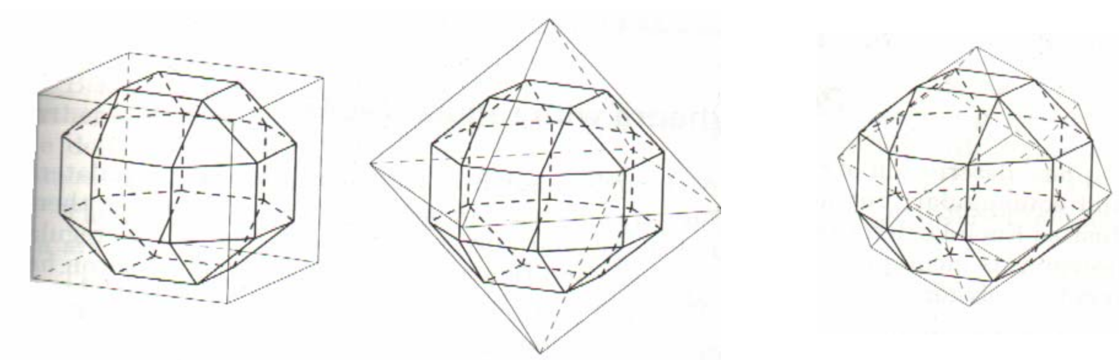
Why rhombicuboctahedron?



Miller's solid or Sommerville's solid. The vertices are congruent, but not equivalent!