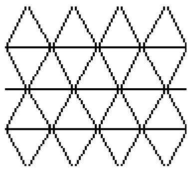
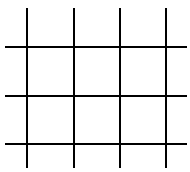
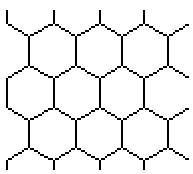
GEM1518 Mathematics in Art and Architecture, 2003/04 II, Tutorial 1 Solutions

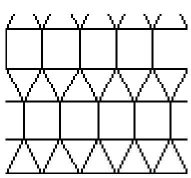
1. Draw pictures of all the Platonic and Archimedean tilings.

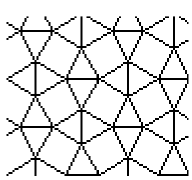
Solution:

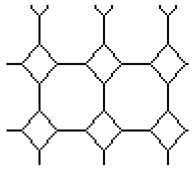
a. (3^6) 

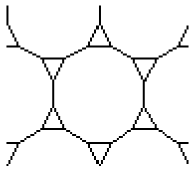
b. (4^4) 

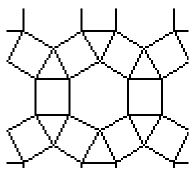
c. (6^3) 

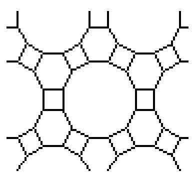
d. $(3^3 . 4^2)$ 

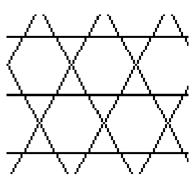
e. $(3^2.4.3.4)$ 

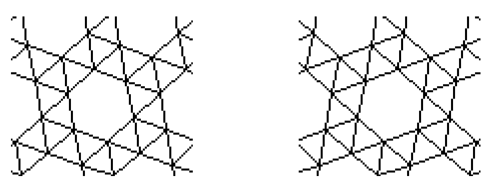
f. $(4.\ 8^2)$ 

g. $(3.\ 12^2)$ 

h. (3.4.6.4) 

i. (4.6.12) 

j. (3.6.3.6) 

k. $(3^4.6)$ in both left and right versions. 

2. Explain why there are only three Platonic tilings of the plane.

Solution: I will outline three arguments. If you look carefully, you will see that they really are the same; we just look at it from different points of view. (Please note that all n-gons are assumed to be regular.)

Then interior angel of a regular n-gon is Π (n-2)/n. In a Platonic tiling where d n-gons meet at each vertex, we have

$d\,\Pi\,(n-2)/n = 2\,\Pi,$
$(n-2)(d-2) = 4.$

The only possible solutions are:

n	d	Tile
3	6	Triangles
4	4	Squares
6	3	Hexagons

Alternatively, you can start by observing that triangles, squares and hexagons can be used, while pentagons are not possible since the vertex angle is 108, which does not divide 360. Since (n-2)/n = 1 – 2/n, it follows that the vertex angle of an n-gon increases with n. We need three tiles to form a vertex, but if n is bigger than 6, then the angle would be bigger than 120, and we can't fit 3 tiles around to form a vertex.

Finally we can focus on the number of n-gons meeting at the vertex. We know that we can have 3, 4 or 6 at the vertex. We cannot have more than 3, since otherwise it wouldn't be a vertex. 5 is not possible, since then the angle would be 72, and that doesn't correspond to an n-gon, and if there were more than 6, the angle would be less than 60, which again doesn't correspond to an n-gon.

Helmer Aslaksen
Department of Mathematics
National University of Singapore
aslaksen@math.nus.edu.sg
www.math.nus.edu.sg/aslaksen/