GEM1518 Mathematics in Art and Architecture, 2003/04 II, Tutorial 2 Solutions

1. Complete the table for the Platonic solids.

Solution:

	Vertices	Edges	Faces	Shape of face	Degree of vertex
Tetrahedron	4	6	4	3	3
Cube	8	12	6	4	3
Octahedron	6	12	8	3	4
Dodecahedron	20	30	12	5	3
Icosahedron	12	30	20	3	5

There are several formulas that can help you. First of all, there is Euler's formula

$$V - E + F = 2, \qquad \text{(A)}$$

where *V* denotes the number of vertices, *E* the number of edges and *V* the number of vertices. Secondly, if *s* denotes the number of sides of the faces and *d* the degree of the vertices, then

$$Fs = Vd = 2E. \qquad \text{(B)}$$

If there is more than one type of faces, then *Fs* denotes the sum of the products corresponding to the different types of faces.

I assume that you know enough about the Platonic solids to fill in *F*, *s* and *d*.

	Vertices	Edges	Faces	Shape	Degree
Tetrahedron			4	3	3
Cube			6	4	3
Octahedron			8	3	4
Dodecahedron			12	5	3
Icosahedron			20	3	5

Then you use duality to fill in *V*

	Vertices	Edges	Faces	Shape	Degree
Tetrahedron	4		4	3	3
Cube	8		6	4	3
Octahedron	6		8	3	4
Dodecahedron	20		12	5	3
Icosahedron	12		20	3	5

Now you can use (A) or (B) or a geometric counting argument to fill in the edges. Note that duality tells you that you only need to determine *E* for the tetrahedron and for one from each of the two pairs.

2. How many colors do you need to color the different Platonic solids? Two faces that have a common edge are not allowed to have the same color.

Solution: It is easy to see that the tetrahedron requires 4, the cube 3, the octahedron 2, and the dodecahedron 3. A bit of geometrical shows that you only need 3 for the icosahedron.

Helmer Aslaksen
Department of Mathematics
National University of Singapore
aslaksen@math.nus.edu.sg
www.math.nus.edu.sg/aslaksen/