GEM1518 Mathematics in Art and Architecture, 2003/04 II, Tutorial 3 Solutions

1. Complete the table for the Platonic and Archimedean solids.

Solution:

	Vertices	Edges	Faces	Shape of face	Degree of vertex
Tetrahedron	4	6	4	3	3
Cube	8	12	6	4	3
Octahedron	6	12	8	3	4
Dodecahedron	20	30	12	5	3
Icosahedron	12	30	20	3	5
Truncated tetrahedron	12	18	4	6	3
			4	3	
Truncated cube	24	36	6	8	3
			8	3	
Truncated octahedron	24	36	8	6	3
			6	4	
Truncated dodecahedron	60	90	12	10	3
			20	3	
Truncated icosahedron	60	90	20	6	3
			12	5	
Cuboctahedron	12	24	6	4	4
			8	3	
Great rhombicuboctahedron	48	72	6	8	3
			8	6	
			12	4	
Rhombicuboctahedron	24	48	8	3	4
			18	4	
Icosidodecahedron	30	60	12	5	4
			20	3	
Great rhombicosidodecahedron	120	180	12	10	3
			20	6	
			30	4	
Rhombicosidodecahedron	60	120	12	5	4
			20	3	
			30	4	
Snub cube	24	60	6	4	5
			32	3	
Snub dodecahedron	60	150	12	5	5
			80	3	

There are several formulas that can help you. First of all, there is Euler’s formula

$$V - E + F = 2, \quad \text{(A)}$$

where *V* denotes the number of vertices, *E* the number of edges and *V* the number of vertices. Secondly, if *s* denotes the number of sides of the faces and *d* the degree of the vertices, then

$$Fs = Vd = 2E. \quad \text{(B)}$$

If there is more than one type of faces, then *Fs* denotes the sum of the products corresponding to the different types of faces.

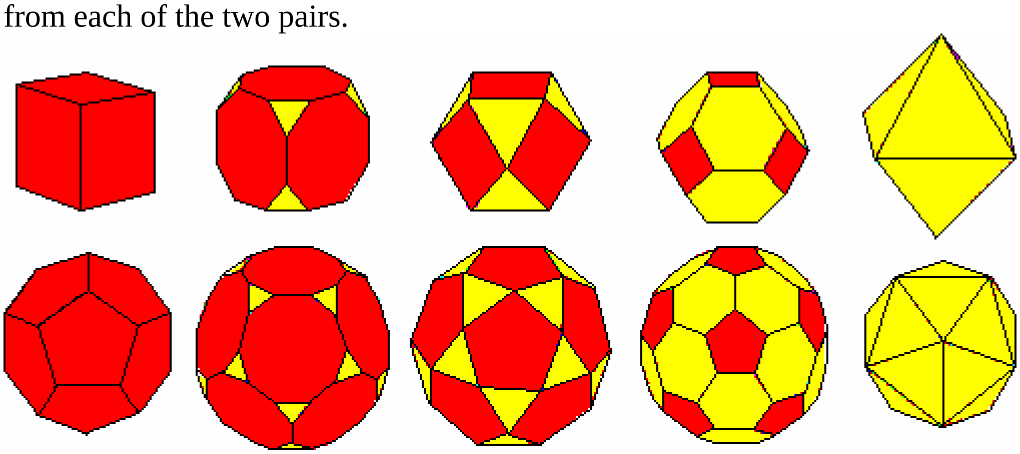
I assume that you know enough about the Platonic solids to fill in *F*, *s* and *d*.

	Vertices	Edges	Faces	Shape	Degree
Tetrahedron			4	3	3
Cube			6	4	3
Octahedron			8	3	4
Dodecahedron			12	5	3
Icosahedron			20	3	5

Then you use duality to fill in *V*

	Vertices	Edges	Faces	Shape	Degree
Tetrahedron	4		4	3	3
Cube	8		6	4	3
Octahedron	6		8	3	4
Dodecahedron	20		12	5	3
Icosahedron	12		20	3	5

Now you can use (A) or (B) or a geometric counting argument to fill in the edges. Note that duality tells you that you only need to determine *E* for the tetrahedron and for one from each of the two pairs.

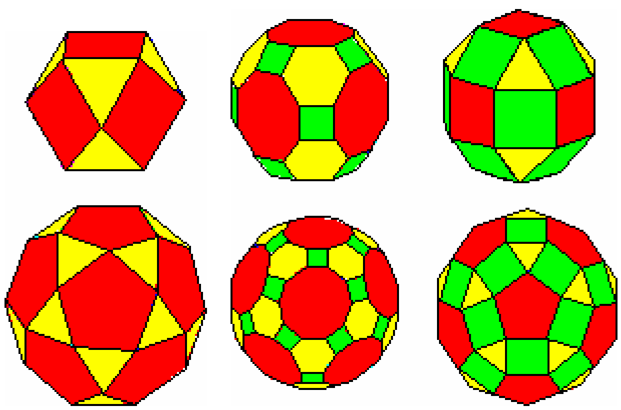


For the truncated Platonics, *d=3*. There are two types of faces: *F 2s*-gons and *V d*-gons.

	Vertices	Edges	Faces	Shape	Degree
Truncated tetrahedron			4	6	3

			4	3	
Truncated cube			6	8	3
			8	3	
Truncated octahedron			8	6	3
			6	4	
Truncated dodecahedron			12	10	3
			20	3	
Truncated icosahedron			20	6	3
			12	5	

You can now use formula (B) to fill in *V* and *E*.



The cuboctahedron and icosidodecahedron are midpoint-truncated Platonic solids. It is easy to see that $d=4$ and that they have *F s*-gons and *V d*-gons.

	Vertices	Edges	Faces	Shape	Degree
Cuboctahedron			6	4	4
			8	3	
Icosidodecahedron			12	5	4
			20	3	

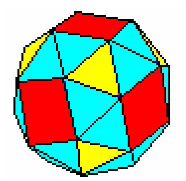
Again, you can now use formula (B) to fill in *V* and *E*.

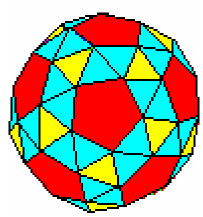
The great rhombicuboctahedron is obtained by truncating the cuboctahedron and then stretching the rectangular faces into squares. It is sometimes called the rhombitruncated cuboctahedron. The rhombicuboctahedron can then be thought of as a midpoint-truncated cuboctahedron. It is sometimes called just the rhombicuboctahedron. If follows that the great rhombis have degree 3 while the rhombis have degree 4, since the former are truncations and the latter are midpoint-truncations. The shape of the faces is still *F 2s*-gons and *V d*-gons, but now there are two types of *2s*-gons.

	Vertices	Edges	Faces	Shape	Degree
Cuboctahedron			6	4	4
			8	3	

Great rhombicuboctahedron			6	8	3
			8	6	
			12	4	
Rhombicuboctahedron			8	3	4
			18	4	
Icosidodecahedron			12	5	4
			20	3	
Great rhombicosidodecahedron			12	10	3
			20	6	
			30	4	
Rhombicosidodecahedron			12	5	4
			20	3	
			30	4	

If you have made it this far, you should know that you now again use formula (B) to fill in *V* and *E*.





For the snubs, remember that you start by breaking the original faces apart and replacing each edge by a pair of triangles. Then you replace each vertex by a *d*-gon. It follows that the number of triangles is $V+2E$. You can easily check that the degree is 5 and then you use formula (B) to fill in *V* and *E*.

	Vertices	Edges	Faces	Shape	Degree
Snub cube	24	60	6	4	5
			32	3	
Snub dodecahedron	60	150	12	5	5
			80	3	

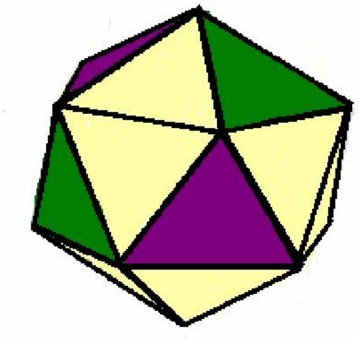
Finally, you should use formula (A) to check all your computations.

You probably will get lost, but whenever you get lost, just look at your models.

Good luck!

2. We have talked about the snub cube and the snub dodecahedron. What would a snub octahedron, snub icosahedron or snub tetrahedron look like?

The snub octahedron is the same as the snub cube, the snub icosahedron is the same as the snub dodecahedron, while the snub tetrahedron is the icosahedron.



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