

## GEM1518 Mathematics in Art and Architecture, 2003/04 II, Tutorial 4 Solutions

1. Why there are only five Platonic solids? Compare it with the proof that there are only three Platonic tilings of the plane.

**Solution:** Then interior angle of a regular  $n$ -gon is  $\frac{\pi(n-2)}{n}$ . In a Platonic solid where  $d$   $n$ -gons meet at each vertex, we have

$$d \frac{\pi(n-2)}{n} < 2\pi,$$

$$(n-2)(d-2) < 4.$$

The only possible solutions are:

$n$	$d$	Solid
3	3	Tetrahedron
3	4	Octahedron
3	5	Icosahedron
4	4	Cube
5	3	Dodecahedron

2. The cube and the octahedron are dual, and midpoint-truncation of either will give the cuboctahedron. In the same way, the dodecahedron and the icosahedron are dual, and midpoint-truncation of either will give the icosidodecahedron. What do you get if you do a midpoint-truncation of the tetrahedron?

**Solution:** Shrinking the hexagons in a truncated tetrahedron leaves us with eight triangles forming an octahedron.



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