GEK1518 Mathematics in Art and Architecture, 2003/04 II, Tutorial 8 Solutions

1. The shape of "An" paper is determined by the following rules.
   - "An" is a rectangle.
   - If you fold "An" along its longest side you get "A(n+1)".
   - "An" is similar to "A(n+1)".
   - The area of A0 is 1m^2

What is the size of A4 paper?

Solution: Suppose A0 paper has width w and height h. Let us write that as

$$\text{A0: } (w, h).$$

Since A1 is obtained by folding A0 height-wise, it follows that

$$\text{A1: } (h/2, w).$$

But since the two are similar, we must have

$$\begin{aligned} w/h &= h/2\big/w, \\ 2w^2 &= h^2, \\ 2 &= (h/w)^2, \\ h/w &= \sqrt{2}. \end{aligned}$$

It follows that all the "An" shapes have a height/width ratio of $\sqrt{2}$.

Since A0 has area 1m^2, we have

$$\begin{aligned} hw &= 1, \\ \sqrt{2}w^2 &= 1, \\ w^2 &= 2^{-1/2}, \\ w &= 2^{-1/4}. \end{aligned}$$

It then follows that $h = 2^{1/4}$, so

$$\text{A0: } (2^{-1/4},\ 2^{1/4}).$$

By folding, we see that

$$\text{A1: } (2^{-3/4},\ 2^{-1/4}),$$
$$\text{A2: } (2^{-5/4},\ 2^{-3/4}),$$
$$\text{A3: } (2^{-7/4},\ 2^{-5/4}),$$
$$\text{A4: } (2^{-9/4},\ 2^{-7/4}) = (0.210,\ 0.297).$$

As a check, you see that the area of A4 is $2^{-4} = 1/16$, which is correct since we have divided A0 four times.

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