1. Consider the advection problem:

\[
\begin{aligned}
&u_t = a \, u_x, \quad 0 < x < 1, \quad (1) \\
&u(0, t) = 0, \quad t \geq 0, \quad (2) \\
&u(x, 0) = u_0(x), \quad 0 < x < 1; \quad (3)
\end{aligned}
\]

where \( a > 0 \).

a). Write down a discretization with second-order centered difference for spatial derivatives and forward Euler scheme for time derivative.

b). Analyze the accuracy.

c). Find the stability condition.

2. Consider the advection-diffusion problem:

\[
\begin{aligned}
&u_t = u_x + \nu \, u_{xx}, \quad 0 < x < 2\pi, \quad (4) \\
&u(0, t) = u(2\pi, t) = 0, \quad t \geq 0, \quad (5) \\
&u(x, 0) = u_0(x), \quad 0 < x < 2\pi; \quad (6)
\end{aligned}
\]

where \( \nu > 0 \).

a). Write down a discretization with second-order centered difference for spatial derivatives and Crank-Nicolson scheme for time derivative.

b). Analyze the accuracy.

c). Express the difference equations in linear system form.

d). Find the stability condition.