Q1. Let \( x_1, x_2 \) and \( x_3 \) be the amounts of units produced in Jan, Feb and Mar respectively, then the problem can be formulated as

\[
\begin{align*}
\text{min : } & \quad c = 10(x_1 + x_2 + x_3) + 3[(x_1 - 10000) + (x_1 - 10000 + x_2 - 40000)] \\
\text{s.t.} & \quad x_1 + x_2 + x_3 = 70000, \quad \text{(no ending inventory)} \\
& \quad x_1 \geq 10000, \\
& \quad x_1 + x_2 \geq 50000, \\
& \quad 0 \leq x_1, x_2, x_3 \leq 30000.
\end{align*}
\]

or equivalently

\[
\begin{align*}
\text{min : } & \quad c = 6x_1 + 3x_2 + 520000 \\
\text{s.t.} & \quad 20000 \leq x_1 \leq 30000, \\
& \quad 0 \leq x_2 \leq 30000, \\
& \quad x_1 + x_2 \geq 50000
\end{align*}
\]

Using the graphical method (here the step are skipped, which have been discussed in tutorial), we find that the cost attain minimum at points \( (x_1, x_2) = (20000, 30000) \), i.e.

\[
c_{\text{min}} = 6 \times 20000 + 3 \times 30000 + 520000 = 730000.
\]

Moreover, \( x_3 = 70000 - (x_1 + x_2) = 20000. \)

If the cost is \( 10x + 10 \), then instead of the above \( c \),

\[
c = 6x_1 + 3x_2 + 520010,
\]

and the constraints remain the same. Using the same method as the first case, you can get the answer, which here we leave as an exercise.

Q2. Let \( v \) be the initial velocity and \( \theta \) be the angle of release, neglecting the effect of air resistance and the height of throw point, then moving time of the shot is

\[
t = \frac{2v \sin(\theta)}{g},
\]

where \( g \) is the gravity constant. Hence the distance thrown is

\[
d = v \cos(\theta)t = \frac{2v^2 \sin(\theta) \cos(\theta)}{g} = \frac{v^2 \sin(2\theta)}{g}.
\]

Assuming the initial velocity at any angle of release is fixed, then when \( \theta = \frac{\pi}{4} \) the distance thrown reaches its maximum.

If the athlete cannot maximize the initial velocity with \( \pi/4 \) as the angle of release, let \( \Delta v > 0 \) be the difference between the maximum velocity he or she can make at angle \( \pi/4 \), and \( \Delta \theta > 0 \) be the difference between the angle, at which he or
she can make maximum velocity, and angle \(\pi/4\), consider the following differences in distance thrown,

\[
\begin{align*}
(\Delta d)_1 &= v^2 - (v - \Delta v)^2 = 2v\Delta v - (\Delta v)^2, \\
(\Delta d)_2 &= v^2(1 - \cos(2\Delta \theta)).
\end{align*}
\]

Then, when

\[
\frac{\Delta v}{v} \leq 1 - \sqrt{\cos(2\Delta \theta)},
\]

it is clear \((\Delta d)_1 \leq (\Delta d)_2\).

Q3. Let \(x_1\), \(x_2\) and \(x_3\) be the weights of 3 cargo for shipping each day, suppose daily cost is fixed, the problem can be formulated as

\[
\begin{align*}
\text{min :} & \quad r = 250(x_1 + x_2 + x_3) \\
\text{s.t.} & \quad 550x_1 + 800x_2 + 400x_3 \leq 50000 \\
& \quad 0 \leq x_1 \leq 30, \\
& \quad 0 \leq x_2 \leq 40, \\
& \quad 0 \leq x_3 \leq 50, \\
& \quad x_1 + x_2 + x_3 \leq 100,
\end{align*}
\]

which is a linear programming problem. It is easy to find that when \(x_1 = 30\), \(x_3 = 50\) and \(x_2 = 16.875\), \(r\) reaches its maximum.

Note that freight 1 and 3 have already reached their available amounts, and \(x_1 + x_2 + x_3 = 96.875\), then if the firm reconfigure one older plane it can ship 2.5tons of freight 2 more, and if it reconfigure two planes it can ship 3.125 tons of freight 2 more.

- If reconfigure one plane, cost is $200,000, and the increased revenue during the following 5 years is

\[
250 \times 2.5 \times 250 \times 5 = 781,250 > 200,000;
\]

- If one more plane is reconfigured, in addition to the above increased revenue,

\[
250 \times (3.125 - 2.5) \times 250 \times 5 = 195,312.5 < 200,000.
\]

Therefore, it would be worthwhile to make the alternations, and reconfiguring one plane brings more profile than reconfiguring two ones.