Q1.
According to the assumptions, the model can be formulated as
\[ \frac{dN(t)}{dt} = \alpha N^2, \quad t > 0. \]
Here we use dozen as the unit of population, so \( N(0) = 1 \) and \( N(10) = 2 \). Solving the initial-value problem, we have
\[ N(t) = \frac{1}{1 - \alpha t}. \]
Plug \( N(10) = 2 \) into the above solution, we get \( \alpha = 1/20 \), i.e.,
\[ N(t) = \frac{1}{1 - t/20}. \]
Set \( N(t) = 4 \), \( t = 15 \), i.e., in 2005 there would be four dozen.

Q2.
\[ \frac{dx(t)}{dt} = 0.8x - 0.004x^2 := f(x), \quad t > 0. \]
(i). Solving the above ODE with some initial data \( x(0) = x_0 \), we find
\[ \begin{align*}
    x(t) &= e^{0.8t} \left[ \frac{1}{x_0} + \frac{1}{200} (e^{-0.8t} - 1) \right]^{-1}, \quad \text{if } x_0 \neq 0, \\
    x(t) &= 0, \quad \text{if } x_0 = 0.
\end{align*} \]
The maximum value depends on the initial data \( x_0 \), and it can be found that the maximum value \( x_{max} \) is
\[ x_{max} = \begin{cases} 
    0, & x_0 = 0, \\
    200, & 0 < x_0 \leq 200, \\
    x_0, & x_0 > 200.
\end{cases} \quad (1) \]
(ii). From \( x_0 = 50 \),
\[ x(t) = \frac{200}{3e^{-0.8t} + 1}. \]
Set \( x(t) = 100 \), \( t \approx 1.3733 \).
(iii). Equilibrium can be founded by setting \( f(x^*) = 0 \),
\[ x^* = 0 \text{ or } 200. \]
For stability,
\[ f'(0) = 0.8 > 0 \implies x^* = 0 \text{ is unstable,} \quad (2) \]
\[ f'(200) = -0.8 < 0 \implies x^* = 200 \text{ is stable.} \quad (3) \]

Q3. According to the assumptions, we have
\[ \frac{dN(t)}{dt} = \alpha \sqrt{N}, \quad t > 0. \]
Solving the above with some initial condition \( N(0) = N_0 \), we have

\[
N(t) = \left( \frac{\alpha t}{2} + \sqrt{N_0} \right)^2.
\]

From \( N(0) = N_0 = 100 \) and \( N'(0) = 20 \), we get

\[
N(t) = (t + 10)^2,
\]

so, \( N(12) = 22^2 \).

Q4.

Solving the IVP of

\[
\frac{dN(t)}{dt} = b_0 e^{-\alpha t} N, \quad N(0) = N_0,
\]

we have

\[
N(t) = N_0 e^{(1-e^{-\alpha t})b_0/\alpha}.
\]

Hence, as \( t \to \infty \), we have

\[
N(t) \to N_0 e^{b_0/\alpha}.
\]