MA3264 Tutorial 3

1. Assume we are considering the survival of whales and that if the number of whales below a minimum survival level $m$ the species will become extinct. Assume also that the population is limited by the carrying capacity $M$ of the environment. That is, if the whale population is above $M$, it will experience a decline because the environment cannot sustain that large a population level. Let $a_n$ represents the whale population after $n$ years. Assume that the change of the population at $n+1$-th year is proportional to the product of $M - a_n$ and $a_n - m$.

   (i) Build a discrete model for the population of whales.
   (ii) Find the equilibrium and their stability.
   (iii) Discuss the long-term behavior.

2. Consider two species whose survival depends on their mutual cooperation. An example would be a species of bee that feeds primarily on the nectar of one plant species and simultaneously pollinates that plant. Let $a_n$ and $b_n$ represent the bee and plant population level after $n$ days, we have the model

   \begin{align*}
   a_{n+1} &= a_n - k_1 a_n + k_2 a_n b_n, \\
   b_{n+1} &= b_n - k_3 b_n + k_4 a_n b_n, \quad n = 0, 1, 2, \ldots,
   \end{align*}

   where the $k_i$ are positive constants.

   (i) Discuss the meaning of each $k_i$ in terms of mutual cooperation.
   (ii) What assumptions are implicitly being made about the growth of each species in the absence of cooperation.
   (iii) Find the equilibrium and their stability.

3. Suppose that a community contains 15000 people who are susceptible to a spreading contagious disease. At time $t=0$, the number of people who have the disease is $N_0 = 5000$ and is increasing by 500 per day. How long will it take for another 500 people to contract the disease? Assume that the change of the number of people who have the disease each day is proportional to the product of the number of those who have the disease and those who do not.

4. A tank initially contains 60 gal of pure water. Brine containing 1 lb of salt per gallon enters the tank at 2 gal/min, and the (perfectly mixed) solution leaves the tank at 3 gal/min; thus the tank is empty after 1 hour.

   (i) Find the amount of salt in the tank after $t$ minutes.
   (ii) What is the maximum amount of salt ever in the tank?