

MA5251 Homework and Project 1
(Due date: 10:00pm Monday 16 February 2009)

1. Consider the following Elliptic problem

$$\begin{aligned} -u''(x) + a u(x) &= f(x), & 0 < x < \pi, \\ u(0) &= u(\pi) = 0, \end{aligned}$$

where a is a given constant and f is a given function.

i). Design a second-order finite difference discretization for the problem and express the difference equations into a linear system form.

ii). Design a spectral discretization by using the sine series.

iii). Implement the above two discretizations and find the order of convergence numerically for the following data:

1. $a = 1$ and $f(x) = \sin(5x)$;
2. $a = 1$ and $f(x) = e^{2x}$;
3. $a = 1$ and $f(x) = 1$ when $0 \leq x \leq \pi/2$ and $f(x) = 0$ when $\pi/2 < x \leq \pi$.

what conclusions can you obtain from your numerical results? Why?

2. Given a function $f(x)$ on $[0, 2\pi]$, write a code to find its Fourier interpolation function $I_N f$ by using $N + 1$ uniform grid points and find the interpolation error $\|f - I_N f\| := \int_0^{2\pi} |f(x) - I_N f(x)|^2 dx$.

Find the interpolation error and plot the function f and its interpolations $I_N f$ with $N = 16, 32, 64, 128$ and 256 for the following functions:

1. $f(x) = \frac{3}{6+4\cos(x)}$;
2. $f(x) = \sin(64x)$;
3. $f(x) = \frac{1}{1+x^2}$;
4. $f(x) = \begin{cases} 0 & 0 \leq x < \pi/2, \\ 1 & \pi/2 \leq x < 3\pi/2, \\ 0 & 3\pi/2 \leq x \leq 2\pi. \end{cases}$

What conclusions can you obtain from your numerical results? Why?

3. Write a code to find the Legendre-Gauss points $\{x_j\}_{j=0}^N$ and the corresponding weights $\{\omega_j\}_{j=0}^N$ for a given integer $N > 0$.