Syllabus

Lecturer: Dr. Dai Min
Office hours: By appointment via email
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Recommended background reading:

Recommended texts: (Except for the last two, these books have been set as RBR books, available at Science Library)

- Shreve, S.E. (2004), Stochastic Calculus for Finance: The Binomial Asset Pricing Model (Vol I); Continuous-Time Models (Vol II), Springer Verlag, New York


We are now able to talk about “mathematical finance”, “financial engineering” or “modern finance” only because of two revolutions that have taken place on Wall Street in the latter half of the twentieth century. The first revolution in finance began with the 1952 publication of “Portfolio Selection”, an early version of the doctoral dissertation of Harry Markowitz, where he employed the so-called “mean-variance analysis” to understand and quantify the trade-off between risk and return inherent in a portfolio
The implementation of Markowitz’s idea was aided tremendously by William Sharp who developed the Capital Asset Pricing Model. For their pioneering work, Markowitz and Sharp shared with Merton Millier the 1990 Nobel Prize in economics, the first ever awarded in finance. Markowitz and Sharp’s portfolio selection work is for one-period models. Thanks to Robert Merton and Paul Samuelson, one-period models were replaced by continuous-time (Brownian-motion-driven models), and the quadratic utility function implicit in mean-variance optimization was replaced by more general increasing, concave utility functions. Model-based mutual funds have taken a permanent seat at the table of investment opportunities offered to the public.

The second revolution in finance is regarding what we are going to address in this course, the option pricing theory, founded by Fisher Black, Myron Scholes, and Robert Merton in the early 1970s. This leads to an explosion in the market for derivatives securities. Scholes and Merton won the 1997 Nobel Prize in economics. Black had unfortunately died in 1995.

**Preliminary knowledge:** MA3245 (Financial Mathematics I) or at the least

- Basic concepts: derivatives, options, futures, forward contracts, hedging, Greeks and so on.
- Elementary stochastic calculus: Brownian motion, Ito integral and Ito lemma.
- Derivation of continuous time model (Black-Scholes) and discrete model (Cox-Ross-Rubinstein): no arbitrage; delta hedging; option replication.

Even if you know little about the preliminary knowledge, don’t worry too much because I am going to give a review in the first two classes.

**Course Grade**
final exam (70%), mid-term exam (20%), one assignment with tutorials (10%).

**Contents:**

- Preliminary
- Option pricing models for European options
Our philosophy

- We value these derivative products using partial differential equations (PDEs), or equivalently, using binomial tree methods. Probabilistic approach will be discussed in MA5248 (Stochastic Analysis in Mathematical Finance, this semester).

- Explicit solutions to the PDE models, if available, will be given. But we skip how to get these solutions. I refer interested students to the module MA4221 (Partial Differential Equations, next semester).

- Since explicit solutions are so rare that fast accurate numerical methods are essential. We shall mainly focus on the binomial tree method which is easy to implement and can be regarded as an explicit finite difference scheme. Monte-Carlo simulation will be briefly discussed. More general finite difference schemes are discussed in MA4255 (Numerical Partial Differential Equations, next semester).

- Matlab coding is preferable, but not required.