Do the following questions from the textbook:

Sect. 2.3: # 2, 3; Sect. 2.5: 3, 8, 12

In addition, do the following problems:

(1) Find all integer solutions to the linear Diophantine equation:

\[ 3x + 4y + 5z = 15. \]

(2) Let

\[ C^n_r = \frac{n!}{r! \cdot (n-r)!}. \]

Show that \( C^n_r \) is an integer (Hint: show that for any prime \( p \), more powers of \( p \) divide the numerator than the denominator).

(3a) Suppose that \( f(x) = x^k + a_{k-1}x^{k-1} + \ldots + a_1x + a_0 \) is a polynomial with integer coefficients whose leading coefficient is 1. If \( \alpha \) is a root of \( f(x) \) and \( \alpha \) is not an integer, show that \( \alpha \) must be irrational. (Hint: suppose by contradiction that \( \alpha = \frac{m}{n} \) is rational, with \( m \) and \( n \) relatively prime. Clear denominators and try to obtain a contradiction.)

(b) Using (a), show that \( \sqrt{2} + \sqrt{5} \) is irrational.

(4a) Show by mathematical induction that the product of \( n \) integers of the form \( 4k + 1 \) is itself of this form.

(b) Using (a), show that there are infinitely many primes of the form \( 4k - 1 \). (Hint: Imitate Euclid’s proof that there are infinitely many primes. If there are only finitely many primes \( p_1, \ldots, p_r \) of the form \( 4k - 1 \), consider the number \( N = 4p_1 \ldots p_r - 1 \).)