Do the following questions from the textbook:

Misc. Exercises (Pg. 108): # 1

(1) Find all different solutions modulo 60 of the quadratic congruence:
\[ x^2 + 26x + 33 \equiv 0 \pmod{60}. \]
(Hint: 60 = 3 · 4 · 5).

(2a) We showed in class that Euler’s function \( \phi \) is multiplicative. Determine the function
\[ F_\phi(n) = \sum_{d|n} \phi(d). \]
(b) Deduce from (a) that
\[ \sum_{d|n} \phi(d) = n. \]

(3) Let \( \left( \frac{a}{p} \right) \) denote the Legendre symbol. Compute the value of
\[ \left( \frac{5}{11} \right), \left( \frac{7}{11} \right), \left( \frac{8}{11} \right). \]

(4) Let \( p \geq 7 \) be a prime number. Show that one of 2, 5 and 10 is a quadratic residue mod \( p \). Deduce from this that there are two consecutive integers which are quadratic residues mod \( p \).

(5) Find all odd primes \( p \) such that \( -2 \) is a quadratic residue mod \( p \).

(6) Show that there are infinitely many primes of the form \( 8k + 3 \). (Hint: if there are only finitely many such primes \( p_1, \ldots, p_r \), consider \( Q = (p_1 \ldots p_r)^2 + 2 \) and use Question 5).

(7) Find all odd primes \( p \) such that the Legendre symbol \( \left( \frac{3}{p} \right) = -1. \)

(8) Show that there are infinitely many primes of the form \( 5k + 4 \). (Hint: if there are only finitely many such primes \( p_1, \ldots, p_r \), consider \( Q = 5(2p_1 \ldots p_r)^2 - 1 \)).

Questions 6 and 8 are special cases of Dirichlet’s theorem on primes in arithmetic progression, which says that if \( \gcd(a, n) = 1 \), then there
are infinitely many primes which are $\equiv a \mod n$. The proof of this is beyond the scope of this course, but it may be covered in Math104C.