GEK1506
Project: Eclipse

NUS
National University
of Singapore

Year 2003/2004

Group 26

Group Members, Matric Number:
Yeo Kian Peen, U028075E
Thum Sok Yee, U028074U
Wong Kim Jyh, U028065J
Boon Kia Khoon, U028096J
Wong Yin Peng, Alina, U028086M
Lim Ban Thian, U028053N

Contents Page

Appendices:

The Moon’s Motion, Ecliptic and Moon Phase

The Moon’s Motion and the Ecliptic

Before we can start discussing about how the eclipse happens, we start with discussing about the Moon’s motion and the ecliptic.

The Moon orbits around the Earth in an elliptical path. An ellipse is an elongated circle and the degree to which it is elongated is called its eccentricity. The eccentricity of the Moon’s orbit is very slight.

Over time, the elliptical shape of the Moon’s orbit shifts slightly, causing the shortest and furthest distance (from the Earth to the Moon) in any given year to change. However for most purposes, the distances from Earth in the orbit are averaged, with a mean perigee (shortest distance from the Earth to the Moon) and apogee (longest distance from the Earth to the Moon) used to express the extremes.

The Moon and the Earth are actually orbiting around a common center of gravity called a barycenter that is created by combination of the two masses. The barycenter of the Earth-Moon system is in constant motion because both bodies are in motion, one orbiting around each other, and both rotating around their axis.

As the Earth is much larger than the Moon, “pulls” the barycenter much closer to itself than to the Moon.

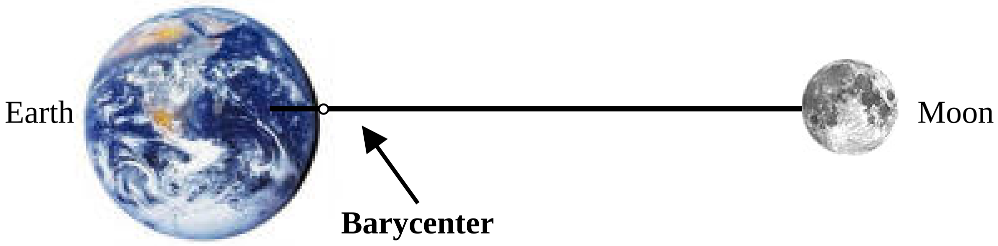


Figure 1: The combined masses of two bodies in a binary system have a common center of mass, which is known as Barycenter. The Barycenter of the Earth and the Moon is located about 3000 miles out from the center of the Earth, which is 900 miles under the surface. The actual location will change constantly because the distance between the Earth and the Moon changes during the Moon’s monthly orbit.

When the Moon is furthest away in its orbit, the barycenter is also at its extreme distance. And when the Moon is closest in its orbit, the barycenter also moves closer to the center of the Earth. Due to the fact that the Moon’s orbit is an ellipse, the motion of the barycenter also forms an ellipse.

The Moon makes one rotation around its axis every 29.5 days. This is also the same time taken for the Moon to make one revolution around the Earth. The Earth rotates at about 1000 miles an hour (measured from a point on the surface at an equator). Where else the Moon rotates at only 10 miles an hour. However the Moon’s orbital speed is much faster than its rotation. The Moon (monthly trip) is moving on an average speed of 2287 miles an hour around the Earth. The effect of an elliptical path on the speed of an orbiting object is to change the speed at different parts of the orbit.

When the Moon is closer to the Earth, it is traveling at its maximum speed and is traveling at its slowest when it is at the furthest point from the Earth.

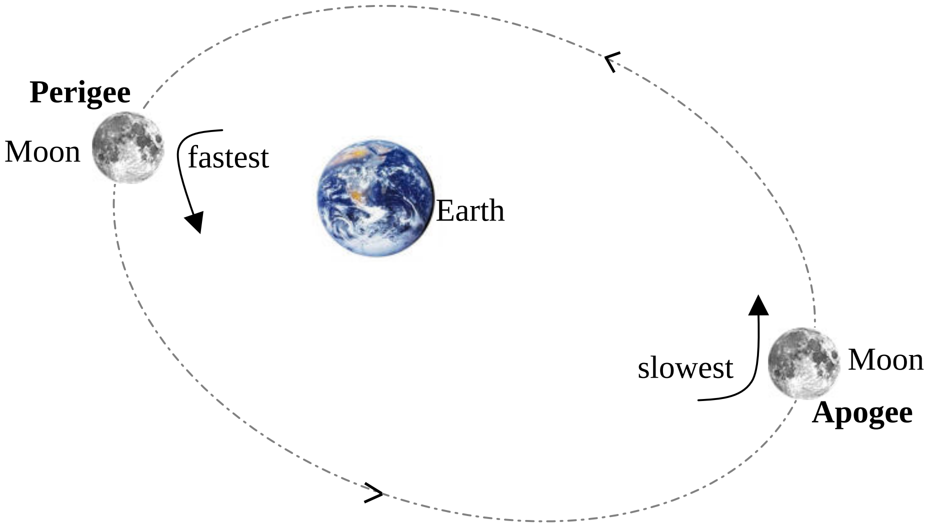


Figure 2: Moon's speed during Perigee and Apogee

The apparent motion of the Moon across the sky is mostly the result of the rotation of the Earth. The speed of the Earth's rotation accounts for about 96 percent of the Moon's visible motion. Only 4 percent is from the Moon's actual movement in orbit. From one night to the next, the Moon "lags" behind about 13 degrees to the east.

Next we start discussing about the ecliptic. Ecliptic is the Sun's path across the sky. This is caused by the revolution of the Earth around the Sun.

The Moon's path is titled to the ecliptic by about 5 degrees. Since the Moon is moving almost 12 times faster than the Sun around this stellar track, it makes a complete circuit in less than a month, a journey that takes the Sun one year to complete.

The plane of the Moon's orbit and the plane of the Earth's orbit intersect at two points and these points are called nodes. If the nodes happen to fall on the line between the Earth and the Sun, a full Moon or a new Moon will result in an eclipse (will further discuss in the later part of the report). When the new Moon is at the node between the Sun and the Earth, the result is a solar eclipse. A lunar eclipse happens when a full Moon is at the opposite node.

Moon Phases

Before going into the eclipse, we feel that we should touch a bit on the Moon phases. As we know that the Moon appears to change the shape as it move but this is not true. The Moon itself does not change shape however the part that is illuminated by the Sun does change. There are 4 official phases that are included in the calendars and almanacs are new Moon, first quarter Moon, full Moon and last Moon. For this topic, all the views we are going to discuss are from the North Pole view.

The new Moon cannot be spotted or is actually invisible. This is because it is defined as an instant of time when the Moon is between the Sun and the Earth and therefore lost in the bright light of the Sun. At this stage, not even the thin crescent can be seen.

The first and last quarter Moons mark the halfway points between the new Moon and full Moon. The first quarter Moon can be distinguished by the illuminated half of the Moon surface begin on the right hand side. The last quarter can be differentiated by the reverse order, which in this case the illuminated half should be on the left hand side.

The lighted part of the Moon always points to the Sun. If the lighted half is on the right then the Sun must be on the right which means that the Sun is ahead of the Moon. If the lighted half is on the left side then the Sun is on the left which means that the Moon is ahead of the Sun (refer to figure 3).

The sequence of the Moon phases always start from the lighted part of the Moon growing from right to left until the full Moon, then lighted part will reduce from right to left until the new Moon. We can always tell where the Moon is in its cycle if we remember the changing part of light and dark always goes from right to left (refer to figure 4 for a clearer picture of how a new Moon, first quarter Moon, full Moon and last quarter Moon looks like).

During the new Moon, the Moon follows the Sun very closely and after the new Moon, it starts to fall behind the Sun. This is the time we will be able to see a right crescent Moon and this is usually two or three days after the new Moon. The Moon will rise a few minutes later after the Sunrise.

As the Moon “grows”, the lighted portion on the right gradually increases until it forms an almost perfect half circle and this is called the first quarter Moon. At this point, the Moon is 90 degree away from the Sun. In another words, the Moon will rise six hours later after the Sunrise.

The Moon will continue to “grow” until a full Moon. At this point the Moon is 180 degree away from the Sun. The Moon will rise twelve hours later after the Sunrise.

After the full Moon, we can see that the lighted part of the Moon starts to reduce from right to left until it forms an almost perfect half circle and the lighted part is at the opposite of the first quarter Moon. This is called the last quarter Moon. During this time, the Moon is 270 degree away from the Sun. The Moon will rise eighteen hours later after the Sunrise.

Then the Moon will continue to reduce until a left crescent is seen and it will continue to reduce until no Moon is seen (back to the new Moon). The cycle will then continue.

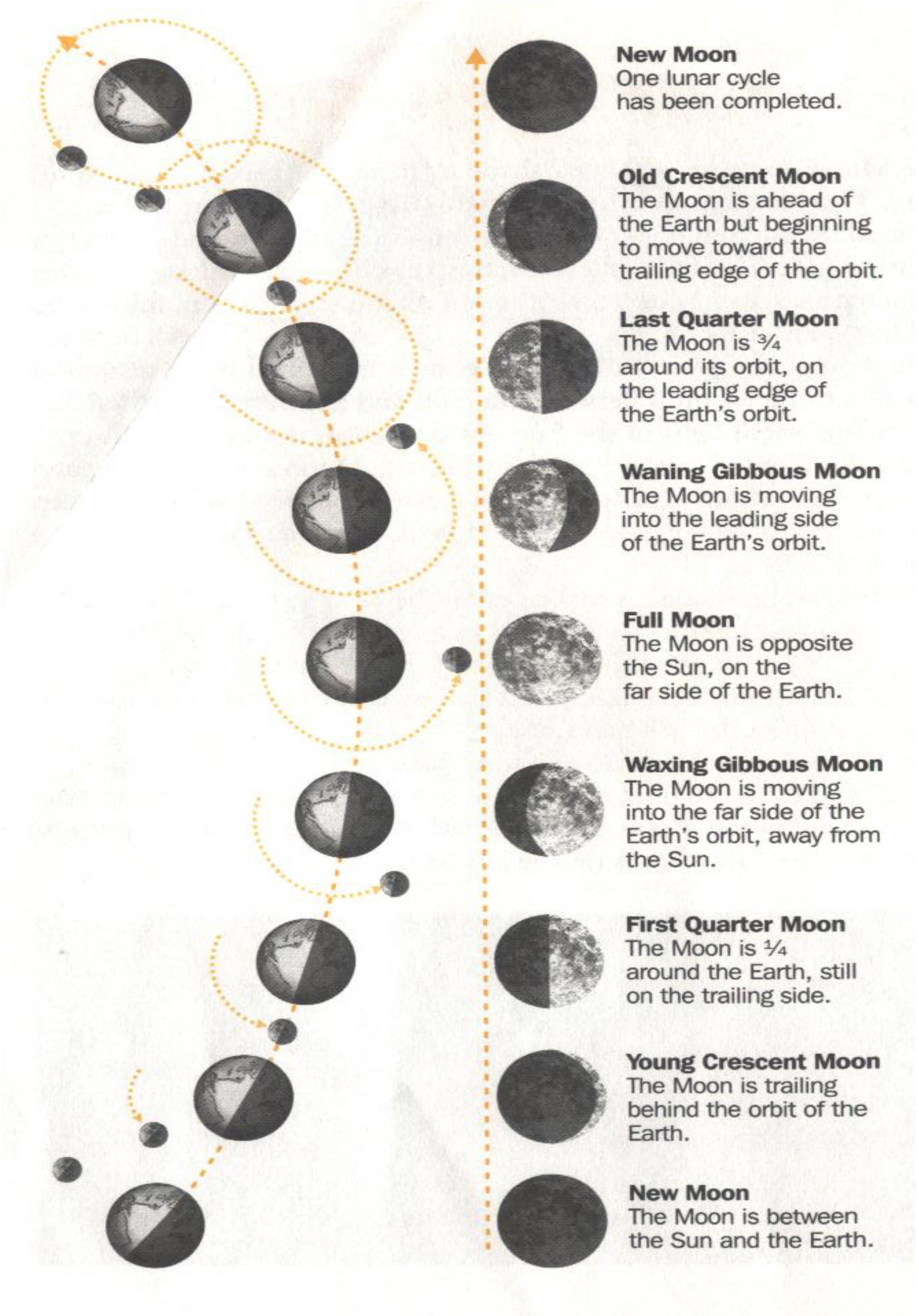
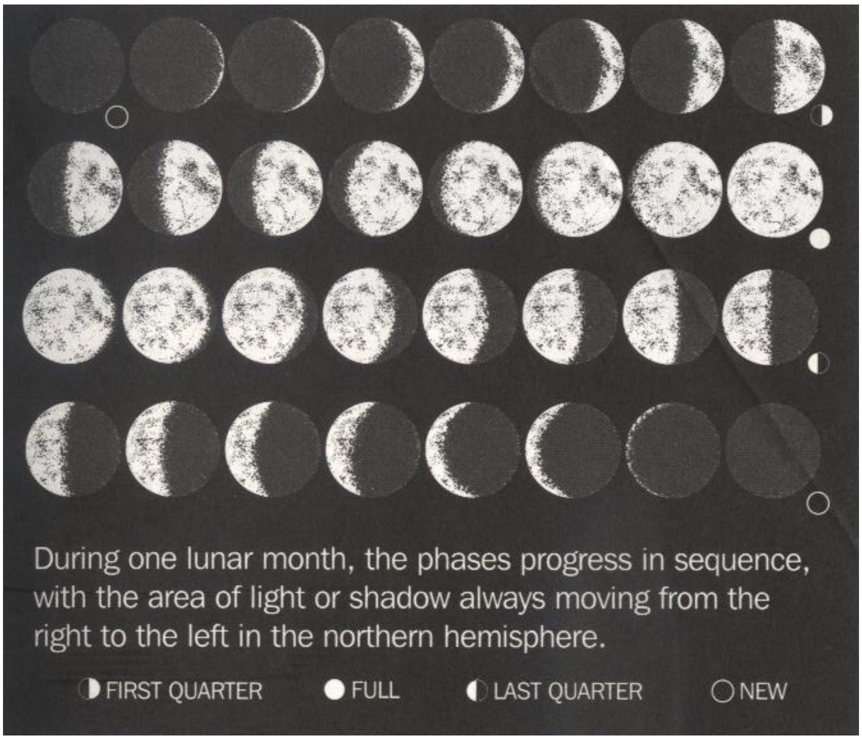


Figure 3: picture from THE MOON BOOK.



During one lunar month, the phases progress in sequence, with the area of light or shadow always moving from the right to the left in the northern hemisphere.

Figure 4: picture from THE MOON BOOK.

Historical Eclipses, Discoveries, Myths and Legends

Historical Eclipses and Discoveries

The first dated observations of these celestial phenomena come from Indian, Chaldean, Babylonian and Chinese sources. The demands of astrology and the calendar led to the first human efforts to predict eclipses.

This activity required painstaking study of former observations. Thus there were a large numbers of astronomical tablets giving past details of past eclipses and attempting to predict others.

Eclipses of the Sun and Moon have marked out the course of human history and have sometimes even changed it. We shall look at some of the best known legends and history concerning eclipses.

Myths and Legends

There are many ancient myths and legends from different cultures which attempts to explain eclipses. Most of them tell of creatures devouring the Sun during some celestial repast. The Indonesians and the Chinese believed that an enormous dragon ate the Sun (a concept also found in Greek and Roman legend, and later among the Balts).

This theme of supernatural beasts devouring the Sun and Moon during their periodic disappearances recurs in many cultures however the beast differs for the different cultures. In Siberia, the culprit was a vampire; in Serbia, a werewolf; in Vietnam; a giant frog; in Paraguay and Argentina, a jaguar; and in Bolivia, a huge dog.

One of the most interesting legends is told on the island of Bali, in Indonesia. It was inspired by the Hindu epic, the Mahabharata. The malevolent Kala Rau (Rahu in the Indian version) is jealous of the immortal and omniscient gods inhabiting Nirvana. Kala Rau lays his plans to achieve immortality. Disguises as a woman, he can be let in to the gods' banquet, serving their magical elixir. Taking advantage of a disturbance, he takes a mouthful of the drink. Vishnu who is also present at the banquet is aware of the crime and therefore cuts off his head immediately. Kala Rau's decapitated body dies, but his head has been made immortal by the potion. Ever since, the head has chased the Sun and the Moon through the sky in an attempt to catch and eat them. However when it succeeds they reappear, after a brief absence, through his open throat.

Looking back into the past, we find that the Chinese were certainly capable of predicting eclipses of the Sun and Moon as early as 2800 BC, due to their knowledge of the saros (further discussion of saros in the later part of the report). The oldest Chinese text which dated as far as 1300 BC shows an eclipse figures is inscribed on the bone and was discovered at Anyang. Also from China comes the story of two luckless royal astrologers, His and Ho whose task was to scrutinize the sky for signs and portents. They failed to predict an eclipse of the Sun and were therefore beheaded for their lack of alertness. Most of the astronomers have heard of this legend and have taken to heart its lesson: instead of relying on mere speculation about future events, they have solved the problems of eclipse calculation in unbelievably vast detail. The astronomers of the Chinese court were given task of organizing demonstrations during eclipses, with archers firing arrows in the direction of the Sun, and drummers trying to frighten the dragon away. A total eclipse of the Sun is recorded in the treatise on astronomy of Sung-shu, part of the official history of the Liu-Sung dynasty. The date given corresponds to 10 August 457 AD and the location of this observation was in Jian-kang (now Nan-ching).

For the Babylonian astronomers certainly observed the lunar eclipse of 2283 BC, and knew the periodicity of the saros, a development which accelerated progress in the science of celestial mechanics. One of the oldest texts, on a clay tablet bearing a description of an eclipse of the Sun, was discovered during excavations of the ancient Mesopotamian town of Ugarit, near the modern day Syrian border. The text describes the eclipse of 3 May 1375 BC, and reads: ‘On a day of the new Moon, in the month of Hiyar, the Sun was ashamed, and hid itself in the daytime, with Mars as witness.’ Around 750 BC, Babylonian astronomers were systematically observing and cataloguing eclipses of the Sun and the Moon, together with many other celestial events. There are only about 1000 fragments of clay tablets, covered with cuneiform characters have survived and are mostly found in the British Museum.

Solar eclipses have altered the course of human history. According to the Greek historian Herodotus that in 585 B.C. while the Lydians and the Medes were doing battle in what is present day Turkey and suddenly darkness fell upon the land (solar eclipse). Both armies took this as a sign telling them to stop fighting instantly and ended up signing a peace treaty instead.

Lunar eclipses do have interesting tales as well. Many ancient cultures had legends that associated lunar eclipses with plagues, earthquakes and other disasters. One of the most famous lunar eclipse anecdotes comes from the exploits of Christopher Columbus. During his fourth voyage to the New World, there was an epidemic of shipworms which turned his fleet into a collection of sieves. The situation was so bad until he was forced to beach his sinking armada on Jamaica. The natives provided the castaways with food and shelter. However tension mounted among the shipwrecked crew as the passing weeks turned into months. Six months passed by and half of the crew mutinied, attacking the remaining crew, murdering the natives and stealing their food. Columbus knew from an astronomical almanac he brought along on the voyage that a total lunar eclipse would be seen from Jamaica in just a few days and he knew that the Jamaicans were terrified by such events. Taking advantage of this situation, Columbus told the native chiefs that unless they immediately gave his crew food, the angry Christian God would turn the Moon blood red. When that night came, sure enough the eclipse happened as predicted. The terrified natives quickly made amends with ample of food offerings and continued keeping the crew well fed until help from home arrived.

Eclipses

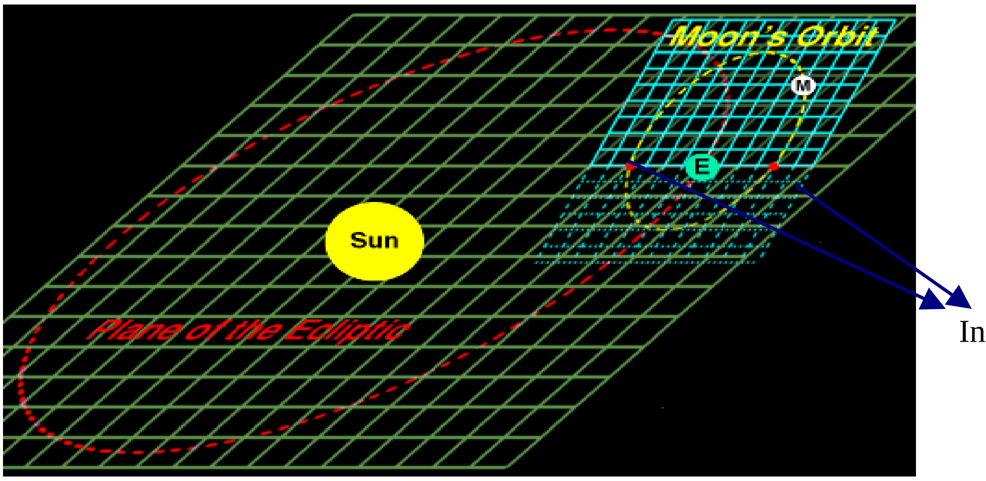
Brief Description of Eclipse

Over the centuries, mankind has gained much knowledge of the eclipses through the help of our advance scientific technology. The solar eclipse, resulting from the Moon coming inline between the Sun and the Earth. The lunar eclipse, resulting from the Earth coming inline between the Moon and the Sun. However, eclipses only happen at certain conditions are a bit more complicated.

When the Earth circles around the Sun, it stays within a flat plane, known as the *ecliptic*. As the Earth moves in its orbit around the Sun, it stays on the ecliptic. Now, you can represent the Moon's orbit by putting a pea near the Earth, circling around it. The Moon orbits the Earth in the plane of the ecliptic, just like the Earth orbits the Sun. But in reality, that’s is not the case. If we consider the Sun and the Earth’s orbit around it to be the same plane as on the same piece of paper, the Moon’s orbit is tilted by about five degrees to that plane and this angle is called the *inclination.*

How do Eclipse comes about.

When the Moon is aligned with the Sun, we say it is at *conjunction*, and when it is at 180 degrees from that point is at it *opposition*. This means that to get an eclipse requires a quite stringent alignment. An eclipse occurs only if the Moon crosses the ecliptic when very close to either *conjunction* or *opposition*, producing solar and lunar eclipses. During at each intercept of the Moon crosses the Earth’s ecliptic, it travels in a upwards and downwards direction. These are called the *nodes* of the orbit named, *ascending* and *descending* nodes respectively. An eclipse can occur only at a node. This diagram illustrates how the Earth moves in the Ecliptic plane and also the movement of the Moon on its orbit.



Interception nodes

Figure 5: The interception of the plane of the Ecliptic with the Moon’s orbit.

Types of Eclipse :

(1)Solar Eclipses

A Solar eclipse is only seen at New Moon, when the Moon moves between Earth and Sun. When all the three bodies are aligned, the Moon casts its shadow across a portion of our world's surface, blocking some or all of the Sunlight from reaching the affected region. Just how much of the Sun will be hidden depends on where the observer stands, relative to the Moon's shadow.

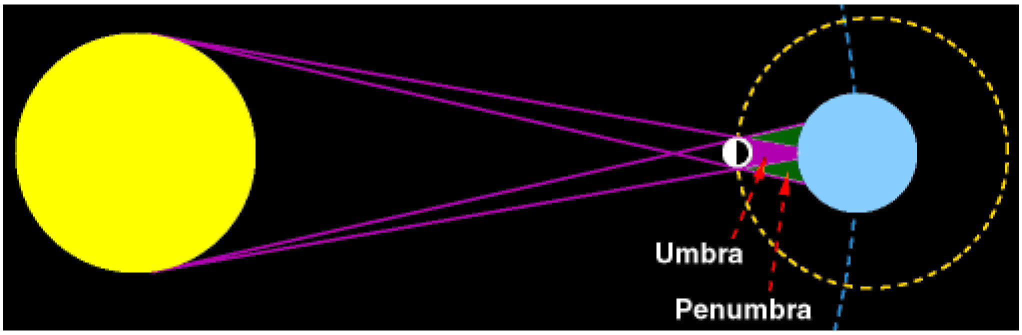


Figure 6: The illustration of how Total and Partial Eclipses are formed.

The dark, central, cone-shaped region shown in figure 6 is called the *umbra*. People standing on those parts of the Earth, within the Umbra, see the Sun's face completely hidden by the Moon shown in figure 7 -- a **Total Eclipse of the Sun**. While on the other two green, fan-shaped area is the *penumbra*. It covers a much larger area of the Earth than the umbra. The people who are within the penumbra see a **Partial Eclipse of the Sun** shown in figure 8. If an observer is located outside of the penumbra, then no eclipse will be seen.

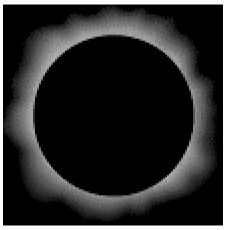


Figure 7: Total Eclipse of the Sun

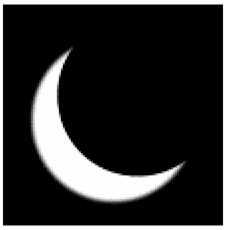


Figure 8: Partial Eclipse of the Sun

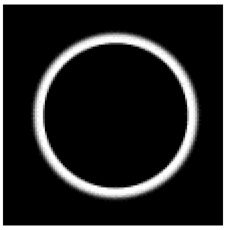


Figure 9: Annular Eclipse of the Sun

Sometimes the people who are within the umbra doesn't experience a Total Eclipse as in the Sun is not totally covered by the Moon. This is because the Moon's orbit is not circular, so it is sometimes closer to the Earth (at perigee) and sometimes further away (at apogee). When near perigee its disk appears comparatively large and so can cover the Sun completely – *a Total Eclipse*. When it is at apogee, its disk appears smaller and so it's unable to obscure the Sun completely. A bright "Annulus" then appears around the circumference of the Moon, thus the *Annular eclipse* is formed as shown in figure 10. Figure 9 shows a view of the Annular Eclipse.

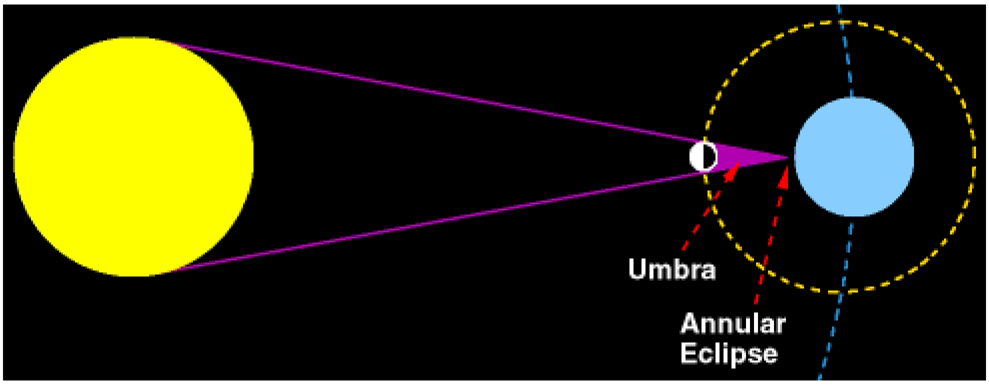


Figure 10: The illustration of Annular Eclipse is formed.

(2)Lunar Eclipses

A Lunar Eclipse is only seen at Full Moon when the Moon passes through the Earth’s shadow. While a solar eclipse is seen over only a small portion of the day side of Earth, and changes in appearance depending on where an observer is standing relative to the point of maximum eclipse, a Lunar Eclipse can be viewed from across the entire night side of the Earth, and will appear exactly the same for all observers on the Earth’s night hemisphere as shown in figure 11. This is why Solar-eclipse chasers may have to travel great distances to be in the center of the umbra during a solar eclipse, lunar-eclipse observers may generally enjoy the event from the comfort of their yards without missing a thing.

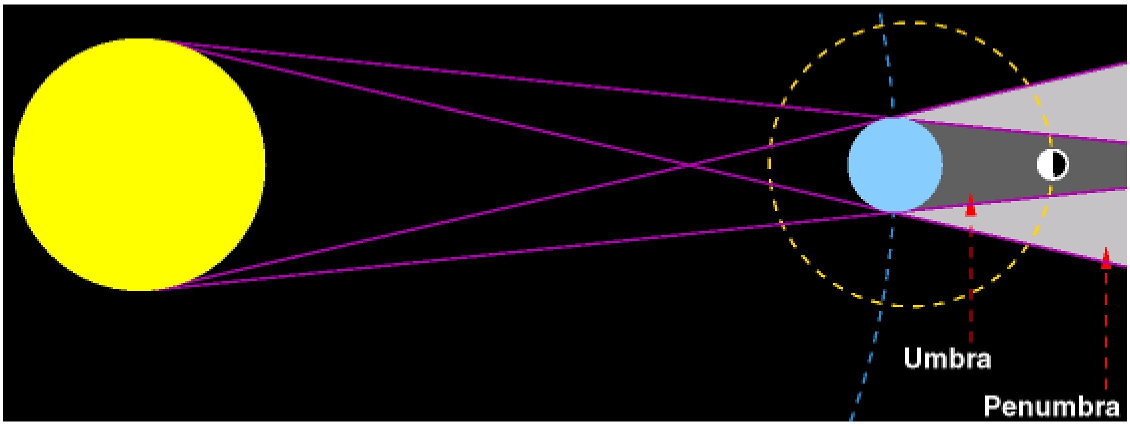


Figure 11: The illustration of how Lunar Eclipse is formed.

Lunar Eclipses come in three varieties: Total, Partial, and Penumbral as shown in figure 12.

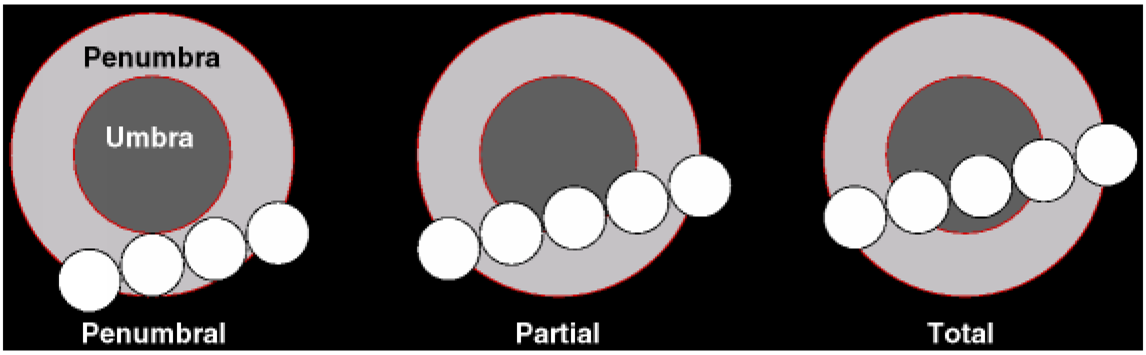


Figure 12: The illustration of how Penumbral, Partial and Total Lunar Eclipses are formed.

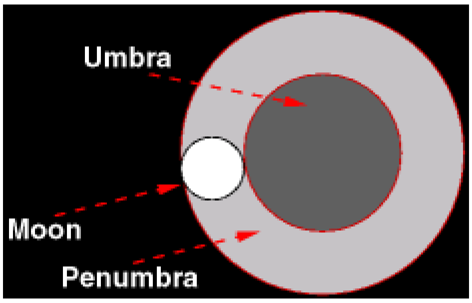


Figure 13: An illustration of a Penumbral Lunar Eclipse.

A Penumbral Lunar Eclipse shown in figure 13 occurs when the Moon only passes through the faint penumbral portion of Earth’s shadow, as it grows dimmer. None of the lunar surface is completely shaded by Earth’s umbra.
In principle, a Penumbral Eclipse can be partial (with only part of the Moon in the penumbra) or total; however, even a Total Penumbral Eclipse is very hard to detect, since the Moon is still quite brightly lit.

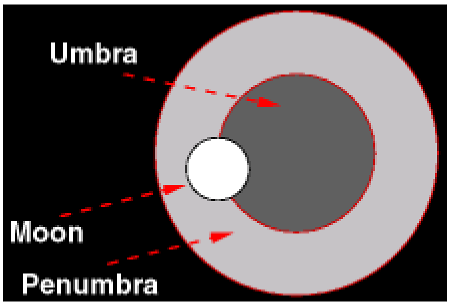


Figure 14: An illustration of a Partial Lunar Eclipse.

A Partial Lunar Eclipse shown in figure 14 occurs when the Moon is oriented such that only a portion of it dips into the Earth’s umbra. Depending on the magnitude of the eclipse, the shadowed portion of the lunar surface may appear a dark red or rust colour, or simply a charcoal gray, because of the sharp contrast between it and the brilliant part of the Moon that remains outside the umbra.

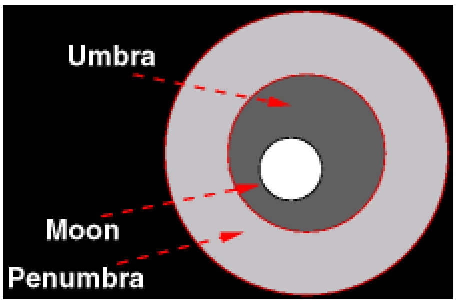


Figure 15: An illustration of a Total Lunar Eclipse.

A Total Lunar Eclipse shown in figure 15 takes place when the Moon’s entire disk is bathed in the Earth’s umbra and no direct light can reach it from the Sun.
However, the Earth's atmosphere refracts -- or bends -- light, at the same time filtering it, so that it illuminates the Moon with a dark red colour. The total phase of a solar eclipse may last only a few minutes, but totality during a Lunar Eclipse can continue for 90 minutes or more.

Saros Cycle

Firstly, we have to introduce the definitions of some important terms used in the calculation of the Saro's cycle:

- Moon's synodic period – from New Moon to new Moon, the time the Moon takes to complete its eastward circuit of the star field and catch up with the Sun again. The duration is 29.53 days.

- Moon's anomalistic month - the time it takes the Moon in its elliptical orbit around the Earth to go from perigee to apogee and back to perigee. The duration is 27.5545 days.

- Eclipse year of the Sun – the period for the Sun(as seen from the Earth) to complete one revolution with respect to a node of the Moon's orbit. The duration is 346.62 days.

- Solar year - the period of time required for the Earth to make one complete revolution around the Sun, measured from one vernal equinox to the next and equal to 365.24 days.

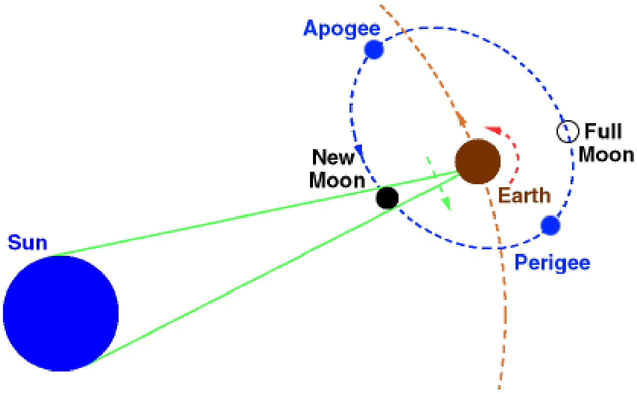


Figure 16 : anomalistic month

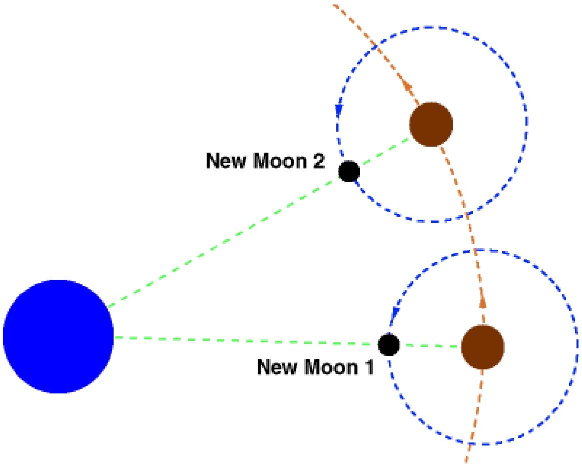


Figure 17 : Synodic month

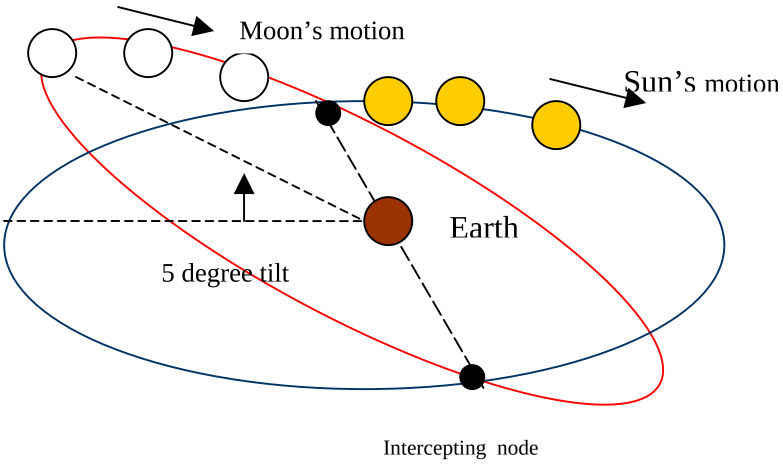


Figure 18 :Eclipse year.

The Saro's cycle way to predict Eclipses.

As mentioned earlier in the myths of ancient eclipse records, the Babylonians displayed great interest in the prediction of eclipses. As time passed and they accumulated more records, the Chaldeans and other ancient people recognized that a specific eclipse occurred a precise number of days after a previous eclipse and before a subsequent one. They exhibited a long term rhythm of their own. The more famous and perhaps, most useful of these eclipse rhythms was the saro, discovered by the Chaldeans and inscribed on clay tablets in their cuneiform writing. The Chaldeans noticed that 6585 days after virtually every lunar eclipse, there was another very similar one. If the first one was a total eclipse, the next eclipse observed was almost always total. Therefore, the Saro cycle is based on this magical number 6585 days to predict the occurrence of eclipses.

Explanation : Let's say a total solar eclipse occurs one day at the Moon's descending node in a particular year, after 6585 days from this particular day, the Moon will have complete 223 synodic months and had returned to new Moon at the same node. In the same period of 6585 days, the Sun will have complete 19 eclipse years as well and returned to the descending node again, therefore, making the next solar eclipse to happen possible. Since these 6585 days is very close to an even 18 calendar years, this solar eclipses occurs at the season of the year as its predecessor, and with the Sun very close to the same position in the zodiac that it occupied at the eclipse 18 years earlier.

Besides the eclipse year and the synodic months used in explaining the occurrence of eclipses, the other important factor is the Moon's anomalistic month. As the anomalistic month is 27.55 days long and 239 times of this period will add up to 6585 days. This determines that if the previous eclipse occurred with the Moon at the perigee, the new eclipse will occur with the Moon near perigee after the 6585 days difference.

The Real Truth:

The 3 important factors that helped predict the occurrence of the eclipses is not totally accurate. As the Moon’s synodic month, the eclipse year and the Moon’s anormalistic month is not exactly the same. The calculations are as follows :

223 synodic months of the Moon at 29.53 days each = 6585.32 days

19 eclipse years of the Sun at 346.6201 days each = 6585.78 days

239 anomalistic months of the Moon at 27.55455 days each = 6585.54 days

As observed, the three calculations different with one another by fractions of a day, and because of this, it indeed results in some consequences.

The 223 synodic periods of the Moon amount to 6585.32 days which is equivalent to 18 years and 11.33 days(18 years 10.33days if 5 leap years intervene). As the saros period is 18 years and 11.33 days, each subsequent eclipse occurs about 0.33 of the way around the world westward from the previous one. The effect of this westward shift on the Lunar eclipse, which is visible to half the Earth, does not push the eclipse out of the view. However, for the solar eclipse, the narrow umbra on the Earth’s surface will be change from the initial location to another location. The viewers at the initial location will not be able to see the solar eclipse now.

As a saro cycle shifts the eclipses to the West from the previous one, after 3 saros cycles, which is 54 years 34 days, the eclipse would have been back to its original longitude but its latitude have been changed about 600 miles northward or southward, making the totality and even major partiality out of the view for the original observer.

The life of the eclipse.

The 223 synodic months amount to 6585.32 days as mentioned above, while the 19 eclipse years of the Sun requires 6585.78 days, so after 18 years and 11.33 days, the Sun will have returned to the node again but not at its exact location as before. It will have shifted to 0.477 degree farther west now. Therefore, if the solar eclipse that happen on 18 years ago was a partial one with the Moon just barely touches it surface lightly when it passes it, with the subsequent saros that occur, the eclipse will become larger and larger partials until the Moon is passing across the center of the Sun’s disk, producing a total or annular eclipses. Thus, after the passing saros had happened, the Sun is proceeding farther west within the eclipse limit, the eclipses will return to partials again. Finally after about 1300 years, the Sun is no longer within the eclipse limits when the Moon, after 223 synodic months arrives. Therefore, we can no longer see any eclipses of this saro cycle, after 1300 years from the first saro experienced, and this saros cycle has died. The photo shown in Figure 19 shows the various cycles of a eclipse.

Figure 19 : Various cycles of a solar eclipse.

Interesting Facts about Eclipses (1) - Solar Eclipses Outnumbered Lunar Eclipses.

There are more solar than lunar eclipses. This fact usually comes as a surprise for many people because most people have seen a lunar eclipse but relatively few have seen the solar eclipse. The reason for this disparity is simple. When the Moon passes into the shadow of the Earth to create a lunar eclipse, the event is visible whenever the Moon is in view, which means half the Earth will see it. The actual fact is that more than half the Earth will see the lunar eclipse as during the course of a lunar eclipse (up to 4 hours),the Earth rotates so that the Moon comes into the view for additional areas. Another reason why the solar eclipses is seldom being seen is that in the brighter days where the partial solar eclipses happen, it is hardly visible to human naked eyes to notice that a solar eclipse is happening and is lasts only a few minutes compared to the lunar eclipses.

However, whenever the Moon passes in front of the Sun, the shadow's creates a solar eclipse, the shadow is small and touches a tiny portion of the Earth surface. Therefore, only the limited locations that the dark shadow (umbra) falls on will experience a total solar eclipse which is rare. But from the either side of the path of the total eclipse, stretching northward and southward 2000 miles and sometimes more, an observer see the Sun partially eclipsed. Even these band of partial eclipse covers a smaller fraction of the Earth's surface compared to the lunar eclipse.

The reason why solar eclipses slightly outnumber lunar eclipse is most easily visualized if you are looking down on the Sun-Earth-Moon system and start by considering only total solar and total lunar eclipses. The Moon will be totally eclipsed whenever it passes into the shadow of the Earth- between c and d. At the Moon's average distance from the Earth, that shadow is about the 2.7 times the Moon's diameter. The eclipse of the Sun will happen whenever the Moon passes between the Earth and Sun-between a and b. The distance between a and b is longer than between the c and d, so total solar eclipse must occur slightly more often as illustrated in Figure 20.

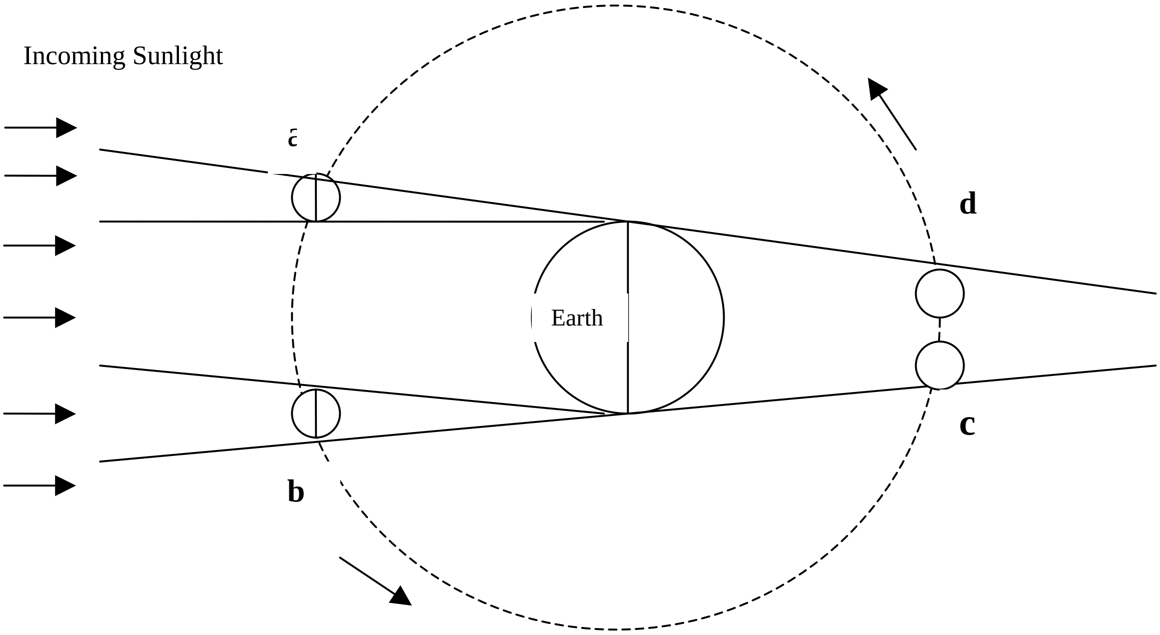


Figure 20: Eclipse limits for Solar Eclipse and Lunar Eclipse

Interesting Facts about Eclipses (2) - Conditions to create the Ultimate Eclipse.

Firstly, the Moon should be having the maximum angular size, that means to say it should be near the perigee, the point in the Moon's orbit that is closest to the Earth. As stated earlier, this happens once every 27.55 days, which is the anomalistic month.

Secondly, the Sun should be near minimum angular size, which is obvious, the Earth have to be near aphelion, the point in the Earth's orbit where it is the farthest from the Sun. This happens once every 365.26 days, which is the anomalistic year.

Third, to prolong the time taken to view the eclipse, the Moon's eclipse shadow must be forced to travel as slowly as possible. At the time of a solar eclipse, the Moon's shadow is moving about 2,100 miles an hour with respect to the center of the Earth. But the Earth is rotating form West to East, the same direction that the eclipse shadow travels. At a latitude of 40 degrees North or South of the equator, the surface of the Earth turns at about 790 miles an hour, slowing the shadow's eastward rush by that amount. At the equator, the Earth's surface rotates at 1040 miles an hour, slowing the shadow's speed to only 1,060miles an hour, thereby prolonging the duration of totality, Baily's Beads, and all phrases of the eclipse.

Lastly, to prolong the eclipse just a few seconds more, the Moon should be directly overhead, so that the full radius of the Earth is used to place ourselves about 4,000 miles closer to the Moon than the limb of the Earth, thus maximizing the Moon’s angular size and therefore its eclipsing power. The latitude where the Moon stands overhead varies with the seasons as the Sun appears to oscillate 23.5 degrees North and South of the equator. Thus the peak duration of totality rarely occurs at the equator but always occurs in the tropics.

Scientific Uses for Eclipses

Early scientific uses for Eclipses

1. Clock the motions of the Moon around the Earth and the Earth around the Sun.
   By trying to predict the exact time and location of the path of a solar eclipse, astronomers could take note of their errors and refine their knowledge of the orbits of the Earth and Moon. This work was pioneered by Edmond Halley in 1715 for a total eclipse crossing southern England.

2. To perceive how hot the interior of the Sun must be.
   More than a century later, in the early days of astrophysics, a second use for total eclipses emerged. The eclipse of 1842, carving a path across southern
   France and northern Italy, gave European scientists a front-row seat to see for themselves the occasionally reported effects surrounding totality: the corona, prominences, and chromosphere. They were awed and realized that the Sun, by covering its face, was revealing physics aspects of itself that were not visible at any other time.

2. To confirm or deny the peculiar new theory of gravity and the structure of the cosmos offered by Albert Einstein
   The first affirmative answer came form an eclipse in 1919, with data from subsequent eclipses over the remainder of the century adding to the certainty.

   Meanwhile, the original scientific uses for total eclipses waned. The U.S. Naval Observatory no longer refines the orbits of the Moon or Earth from eclipse timings. Equipment and techniques developed since 1868 allow astronomers to study prominences of the Sun independent of eclipses. In 1930, Bernard Lyot of France invented the coronagraph, a telescope with a special optical system to create an artificial eclipse so that the brighter regions of the corona can be studied without waiting for a precious few moments of the eclipse totality in some remote corner of the world. And the relativistic bending of starlight can now be checked more precisely by using radio waves, which can be received during broad daylight, without waiting for an eclipse

Modern Scientific Uses for Eclipses

1. Derive the size of the Sun.
   David Dunham is not about to consign eclipses to the historical archives of scientific relics. He and his colleagues Joan Dunham, Alan Fiala, Paul Maley, amateur astronomers David Herald of Australia and Hans Bode of Germany, and other members of the International Occultation Timing Association (IOTA), journey to eclipses around the world for the express purpose of almost missing them ---that is, missing what almost everyone is most anxious to see: the longest possible duration of totality. You will never find Dunham and his colleagues along the center line. Instead, they and any friends and local people who wish to join them position themselves across the northern and southern limits of the eclipse path so that they witness just a few moments of totality, or none at all.
   From their positions at the edges of totality, the top or bottom of the Moon just briefly covers the full face of the Sun. The corona appears and then fades away as the Sun reemerges. Here Dunham finds precisely the information he seeks. He and his coworkers are trying to measure minute changes in the diameter of the Sun, and enlisted the Moon for their service. The size and distance of the Moon are well known. The distance of the Sun is

known to great accuracy. Therefore, by measuring the size of the Moon's shadow, they can derive the size of the body responsible for that shadow: the Sun.

By stretching a team of observation perpendicular to the expected edge of the eclipse path, typically form 0.5 mile (0.8 kilometer) outside to 1.5 miles (2.4 kilometers) inside, they can determine where the actual edge of totality passes to within a hundred meters. This translates into the ability to measure the angular radius of the Sun to an accuracy of 0.04 arc second, or about 20 miles (30 kilometers). The Sun in 1983 was, according to their measurements, about 0.4 to 0.5 arc second larger than it was in 1979, which means that the Sun had increased in radius by about 180 miles (290 kilometers). However, the Sun seemed to be 0.5 arc second similar in 1979 than it was in 1715or even 1925, a decrease in the solar radius by about 230 miles (375 kilometers).

It is a little surprising to think that our Sun might be expanding, shrinking, or pulsating. But then, it was a shock to people almost four centuries ago when Galileo used his telescope on the Sun, a celestial body thought to be "pure" and immune to change, and found spots on the Sun that appeared, changed, disappeared and showed that the Sun was rotating. Over the centuries, scientists recognized more and more changes on the Sun: the shape of the corona, prominences, flares, the Sunspot cycle. What is most surprising is not the Sun changes but that changes on the Sun are not reflected more dramatically (and catastrophically) in short-term climate changes on Earth.

Do the Dunham, Fiala and their colleagues regret missing the "main attraction" of a total eclipse? No, says Dunham, he actually prefers the view from the edge and chides "center line prejudice". He sacrifices a long look at the corona and the eerie twilight at the center of the eclipse path, but he gains a maximum-duration view of Baily's Beads, the chromosphere and the shadow bands, which he sees without special effort at virtually every eclipse. Dunham has observed thirteen eclipses in this fashion and, funds permitting, he plans to attend many more.

If this variation in the diameter of the Sun is real, and he and his colleagues think it is, they suspect that it may be a cyclic pulsation tied to the Sunspot cycle of approximately 11 years. Why, Dunham does not know. He refers theoretical questions to others, such as Sabatino Sofia. Dunham concentrates on refining his equipment and techniques to hold experimental error to low enough levels so that his data will have clear meaning.

The task is simple in theory but very complicated in practice. This raw-data—what was seen from each precisely marked viewing position at precisely what time---are just the start of the project. To compute the angular size of the Sun to see if it has changed, one must take into consideration a host of factors, the most complex of which is the landscape of the Moon. Because of the mountains and valleys on the Moon. Its limb is japed, producing Baily's Beads as totality nears and the only light from the Sun's photosphere that reached the Earth is passing through the deepest valleys. But because the Moon has an elliptical and inclined orbit, the features visible along the Moon's edge change from one eclipse to the next.

For the radius of the Sun to be measured accurately using the Moon's shadow during the eclipse, the exact height of the north and south limb features of the Moon as they appear during each specific eclipse must be calculated to high accuracy; otherwise the fraction of-a-mile uncertainty in the bead-creating diameter of the Moon washes out the accuracy in the measurement of the Sun's radius. Critics of a solar radius variation claims say that practitioners are overestimating the accuracy of their measurement.

2. Solar eclipses allows occultation curve observations to be made, that can provided scientists with remarkably detailed profiles of the structure of the solar chromosphere in radiation

Dunham and IOTA members are by no means the only astronomers who still see the important scientific value in solar eclipses. In fact, solar eclipses provide so many unique opportunities to study the Sun’s atmosphere that this chapter can present only a small sampling of the research in progress.

Spectroheliographs and coronagraphs allow photographic inspection of many aspects of the Sun’s chromosphere, prominences, and lower corona outside of an eclipse. But observing in special wavelengths or by placing an occulting disk at the focal plane to block the face of the Sun cannot compensate for the turbulence in the Earth’s atmosphere that causes he Sunlight to be refracted so that it comes in from very slightly different angles. This deflection of the light blurs the image enough to suppress viewing of fine detail in the corona and prominences altogether, leaving only the most conspicuous solar atmospheric features visible.

For more than a century, astronomers have used occultation by sharp lunar limb to detect detail in sources that is much too fine to resolve with the most powerful telescopes. What appears to the telescope as a single star can be identified as a close binary star system as the Moon passes in front of it.

The basis of this technique is to measure the “occultation curve” of the source---how rapidly the intensity of the source falls off as the solid surface of the Moon occults it. If the source is relatively broad and diffuse, the intensity decreases relatively gradually, over many seconds, as the Moon slides of it and covers it up. If the source is extremely sharp and fine, and the intensity drops to zero in a few seconds or less. Eclipses allow scientists to apply occultation technique directly to the solar limb, resolving the density and temperature profile of the solar chromosphere in fine detail.

Alan Clark and John Beckman pioneered this technique in the infrared wavelengths, observing solar eclipses with telescopes abroad high-flying aircraft. Charles Lindsey and Eric Becklin refined their techniques, using NASA’s Kuiper Airborne Observatory, a military cargo jet converted to carry a 36-inch (0.9-meter) infrared telescope.

Lindsey, Becklin, and their colleagues from the United States, the United Kingdom, and Canada applied the same technique to the spectacular eclipse of July 11, 1991, which passed directly over Hawaii’s Mauna Kea Observatory, perched at an elevation of 13,860 feet (4,225 meters). Because of exceptionally dry conditions atop high mountains, infrared wavelengths meter-diameter) James Clerk Maxwell Telescope can be recorded there without the use of aircraft. These observations were made with the 50-foot-diameter (15-, a radio observatory operated on Mauna Kea by the United Kingdom, the Netherlands, and Canada.

3. Eclipses provide the resolution necessary to help clarify the contribution of magnetic fields to the structure of the Sun’s lower atmosphere

Spectroscopy of the lower atmosphere of the Sun reveals the presence of carbon monoxide (CO) ---a molecule. That is weird. Temperatures near the surface of the Sun should instantly split carbon monoxide into carbon and oxygen unless this molecular gas has a temperature as low as Sunspots, some 2,700ºF (1,500ºC) cooler than the 10,000ºF (5,500ºC) photosphere.

Exactly where it this “cold” material in the solar atmosphere and how high does it extend? Attempts to pin down its precise vertical extent had been thwarted by fluctuations in the solar image caused by the Earth’s turbulent atmosphere. This turbulence causes stars to appear to twinkle and the Sun’s disk to appear to quiver.

Alan Clark thought it might be possible to avoid this “seeing” problem and the pinpoint the position of the carbon monoxide by using the geometry of a solar eclipse. He and Rita Boreiko made initial observations at eclipses in the 1980s by flying in a small Learjet above most of the infrared-radiation-absorbing water vapor in the Earth’s atmosphere. They got mixed results.

But when 1991 brought a total eclipse across the summit of Mauna Kea in Hawaii, Clark and the David Naylor exploited the opportunity by using infrared photometry to show that the carbon monoxide lay in a narrow band above the photosphere.

The annular eclipse of May 10, 1994, however, provided Clark with an almost ideal opportunity to pinpoint the height of this surprisingly cold atmospheric component. The Moon's shadow swept close to the world's largest solar telescope, the McMath-Pierce instrument on Kitt Peak in southern Arizona. Furthermore, this telescope was equipped with a powerful infrared spectrograph and imaging camera. As the Moon progressively eclipsed the solar atmosphere, Clark, Charles Lindsey, Douglas Rabin, and William Livingston were able to watch specific spectral lines of carbon monoxide molecules as they changed from absorption lines when seen against the hotter background of the solar disk to emission lines when seen against the background of space at the solar limb. They could determine the position of the process to within about 50 mile (80 kilometers). Such observations are relatively unhindered by "seeing" fluctuations because the obscuring Moon is above the Earth's atmosphere. Most of the carbon monoxide was concerned within 280 miles (450 kilometers) of the Sun's surface, although very small amounts were detected up to about 625 miles (1,000 kilometers).

The remaining problem is to explain how this cool layer can exist, in which carbon monoxide survives dissociation into atoms while surrounded by fierce temperatures. Clark calls this the "baked Alaska problem." How can the ice cream (the carbon monoxide) remain frozen within the cake while in an oven (the chromosphere) hot enough to bake the meringue?

The present idea is that a network structure of hot, concentrated plasma containing magnetic fields is spread like a fishnet across the solar surface. The carbon monoxide appears to survive in cold pools within the cells of this network, cooling itself by emitting intense infrared radiation by which is detected along the solar limb. It survives only up to the attitude at which the network magnetic fields spread out to cover the solar surface uniformly. These measurements using eclipses thereby establish the "canopy height" for the chromospheric network magnetic fields.

4. Use eclipses to make observations of the solar atmosphere in infrared wavelengths

During the 1998 eclipses, Jeffrey Kuhn and his colleagues instrumented a C-130 cargo plane with a hole in its roof through which could track the Sun. This National Center for Atmospheric Research aircraft allowed them to climb above much of the water in the atmosphere that absorbs the Sun's infrared radiation. Kuhn's team flew from Panama out over Pacific Ocean to intercept the eclipse. By flying eastward along the eclipse path, they were able to extend the duration of totality from 4 minutes to 5. Through these observations, Kuhn and his coworkers discovered a new line in the spectrum of the Sun created by silicon with 8 of its 14 electrons missing. The spectral line is strong enough to use in future eclipses as a probe to trace and measure the magnetic fields of the corona.

Infrared studies of the Sun's corona during eclipses have also allowed Kuhn and his colleagues to detect fine dust particles deposited in the inner solar system by outgassing comets and colliding asteroids. Sunlight reflecting from this dust provides the faint, hazy zodiacal light can be seen under dark-sky conditions rising before the Sun or setting after the Sun. The dust that reflects the zodiacal light lies primarily in and near the plane of the system and gradually spirals in closer to the Sun. With their infrared measurements during eclipses, Kuhn and his coworkers have detected zodiacal dust within the corona, but, as it spirals into the Sun, this dust does not pile up in rings in the outer corona, as some astronomers had suspected

5. Eclipses permit scientist to get a view of the lowest regions of the corona

Serge Koutchmy and Laurence November wait for the great natural coronagraph in-space, the Moon, to eclipse the Sun to permit them a view of the lowest regions of the corona. They hunt for narrow, bright coronal loops indicative of powerful magnetic fields. Studying these loops carefully in ordinary white light, they have observed bright and dark "threads" within the loops ---and have found that the dark threads must be essentially vacuums, void of material. Full evacuation requires a magnetic field of a specific strength. The evacuated dark threads provide November with a new perspective on how the magnetic field may determine the temperature of the corona.

When the total solar eclipse of July 11, 1991 passed over Hawaii and the huge observatory complex atop the extinct volcano Mauna Kea, Koutchmy, November, and their coworkers seized the opportunity to observe the corona with the largest optical telescope ever to record a total eclipse, the 142-inch (3.6-meter) Canada-France-Hawaii Telescope. With so large a telescope, they were able to see details in the corona too small to be seen before. They think they observed a coronal plasmoid, a high-density bubble of ionized gas in the corona, about 900 miles (1,400 kilometers) in diameter, moving at 60 miles per second (100 kilometers per second) outward from a magnetic loop prominence that probably spawned it. During the four minutes of the eclipse, the plasmoids, Koutchmy, November, and their colleagues suspect that it is these plasmoids, small-scale ejection events, not the large coronal mass ejections, which supply most of the material to the upper corona that the Sun loses to space as the background solar wind.

The atmosphere of Earth is controlled by three primary forces: gravity pressure variations, and the Earth's rotation. The Sun's atmosphere is shaped not only by gravity, pressure, and rotation, but also by a fourth force: magnetism. A total eclipse offers the best chance to see how these magnetic fields sculpt the atmosphere of the Sun. By revealing the corona, totality also provides a chance to study high temperature plasma under conditions that cannot be duplicated on Earth: extremely hot ionized gases at extremely low density.

A total eclipse offers as well an opportunity to see how the corona changes with time and to what degree these fluctuations correlate with magnetic activity. Richard R. Fisher is one of a team of scientists who like to use eclipses to record the temperature and structure of the corona with far more precision than observations of the Sun outside of eclipses provide.

Another astronomer still using eclipses to probe the Sun is Jay Pasachoff. He is eager to understand how the corona is heated to temperatures more than 300 times hotter than the photosphere. The energy of the Sun is created at its core, where the temperature is about 27 million ºF (15 million ºC). By radiation and then convection the photons of light energy make their way to the surface, gradually cooling on route, so that the Sun's photosphere glows with a temperature of only about 10,000 ºF (5,500ºC). The base of the chromosphere, the lower atmosphere of the Sun, is cooler still, about 7,200ºF (4,000ºC). But the temperature in the corona exceeds one million degrees. Temperature is a measure of the random motion of atoms and subatomic particles, and the particles in the corona are indeed traveling at great speed. But the corona has a very low density so the total amount of hat in the corona is quite small.

What causes the temperature in the corona to be so much higher than that of the visible surface of the Sun? These surprising temperatures seem to be caused by magnetic fields. The magnetic fields that cause Sunspots in the photosphere pervade the corona as well. They are responsible for streamers and other coronal features; they also shape the prominences and filaments.

A favored explanation for the high temperature of the corona is that magnetic waves (for example, Alfven waves), excited by motion in the convective region beneath the photosphere, propagate energy into the corona, where it is then dissipated into heat. Such waves might manifest themselves as local oscillation in coronal brightness with periods in the range of about 1/10 to 10 seconds.

Jay Pasachoff and his students search the corona during total eclipses, such as the one of February 26, 1998, on Aruba. Some coronal heating theories predict the existence of waves along coronal loops. If magnetic waves are present, observers of the loops should see rapid oscillations in the density of the plasma (ionized gases) as the waves pass through the gases. Because the coronal gases are so hot, they give off light at distinctive wavelengths (such as a green spectral line that iron emits when half its 26 electrons are missing).

The coronal plasma is quite rarefied, however, so glow is weak. But if waves pass through a point within these gases, the waves briefly increase the gas density at that point, thereby briefly increasing the brightness of the gas, and this fluctuation in brightness would give evidence that, indeed, waves are present.

Pasachoff and his students use the total phase of a solar eclipse to make a sequence of images of coronal loops using the light of the coronal green line. They watch the periodic brightness variations in small regions of the loops that would suggest the passage of waves. They think they may have detected oscillations with periods in the 0.2-5 second range. The analysis of data continues.

Pasachoff uses every accessible total eclipse to search for these oscillations in the corona of his favorite star. He hopes this research will help to identify what kind of wave is responsible for the surprisingly high temperatures of the corona and how that heating occurs.

6. <u>Observation rockets are launched during an eclipse to recorded extremely short ultraviolet wavelengths from the Sun's corona before they are absorbed in the Earth's atmosphere</u>

NASA Black Brant suborbital rockets carry their ultraviolet cameras and spectrometers to an altitude of about 200 miles (325 kilometer), where extremely short ultraviolet wavelengths from the Sun's corona can be recorded before they are absorbed in the Earth's atmosphere. Because Rottman, Orrall, and Hassler were collecting the shortest ultraviolet wavelengths, they could have launched rockets to make these observations of the corona without waiting for a total eclipse. The high temperature of the corona gives it a brightness at very short wavelengths that overwhelms the brightness of the comparatively cool photosphere, allowing the corona to be seen even thought the disk of the Sun is in full view. But total eclipses are the best time for rocket flights because the state of the corona they record in their five minutes above the Earth's atmosphere can be compared with the corona seen nearly simultaneously from the ground in visible and radio wavelengths, greatly increasing the scientific value of all the observations. Each wavelength provides its own revelations about the corona and, studied together, they provide powerful tests of the theories that attempt to explain peculiar features of the corona: its chemical abundances, its density irregularities (clumpiness), how it is heated to exceptionally high temperatures, and the mechanism by which the Sun drives the solar wind.

Advantages of using eclipses over space observations for solar research

The traditional advantages of eclipses over space observations for solar research are flexibility and cost. New observation and instruments can be incorporated into an eclipse expedition on short notice. State-of –the-art and bulky equipment can be transported to eclipse sites far more cheaply than to space. Eclipse equipment does not have to meet launch standards of sturdiness. Further, eclipse instruments can be mounted on steady bases, with the Earth as a platform, and can be adjusted at the last minute by qualified scientists.

The SOHO solar telescope cost hundreds of millions of dollars. For less than one-tenth of one percent of these costs, a well-equipped modern ground-based eclipse expedition can be mounted. Even allowing for some of the eclipses to be clouded out, eclipses are a very cost-efficient way of doing astronomy research.

Appendix I: Calculation of Angles Related to Solar Eclipse

Note: In all the calculation, we have assumed the Sun; the Earth and the Moon are spherical with constant radiuses. All diagrams are not draw to scale unless otherwise stated.

In order to calculate of the time of eclipse, we need to understand the relativity motion of the Sun, the Earth and the Moon. Firstly, we look at the some information on our Earth, Moon and Sun in table i1.

Description	**Data**
The size of the Sun (radius)	695,500 km
The size of the Earth (radius)	6,377 km
The size of the Moon (radius)	1,737.4 km
The Sun and the Earth	
Distance at Perihelion (dist. closest to the Sun)	147.09 x 10^6 km
Distance at Aphelion (dist. furthest to the Sun)	152.10 x 10^6 km
Maximum orbital velocity (speed along the orbit around the Sun)	30.29 km/s
Minimum orbital velocity (speed along the orbit around the Sun)	29.29 km/s
The Moon and the Earth	
Distance at Perigee (dist. closest to the Earth)	363.3 x 10^3 km
Distance at Apogee (dist. furthest to the Earth)	405.5 x 10^3 km
Maximum orbital velocity (speed along the orbit around the Earth)	1.076 km/s
Minimum orbital velocity (speed along the orbit around the Earth)	0.964 km/s

Table i1 Information on our Earth, Moon and Sun

How solar eclipse occurs?

Before answering the question to how solar eclipse occurs, we can ask ourselves how come solar eclipse doesn't occur. Figure i1 shows the position of the Moon and the Sun during new Moon and there is no eclipse. For eclipse to happen the following must be met.

1. The Moon must be within a certain degree above/below ecliptic to create the eclipse.
2. The Moon must be near new Moon. (Moon is between the Sun and the Earth)

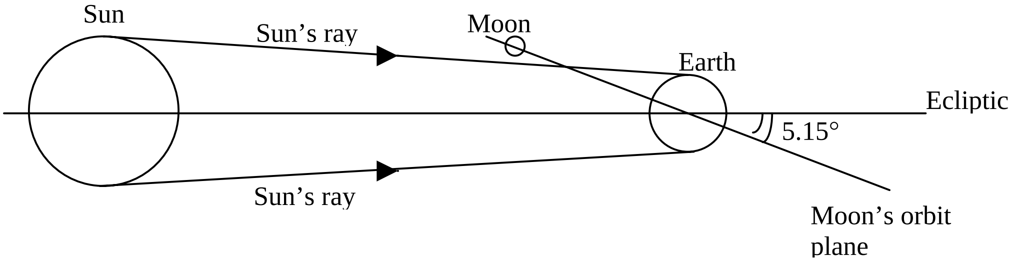


Figure i1: Position of the Moon, Earth and Sun without a solar eclipse.

What is the minimum degree above/below the ecliptic must the Moon be to create a solar eclipse?

An eclipse will happen when the Sun’s ray is block by the Moon.

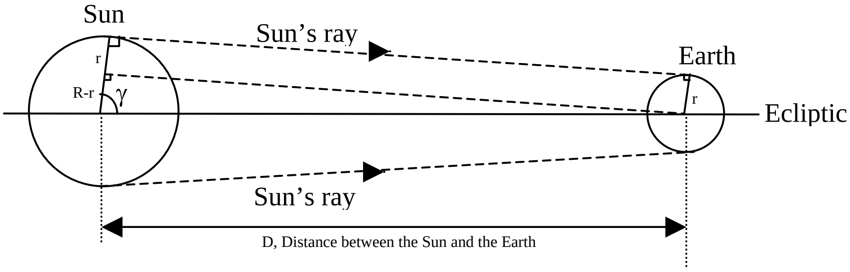


Figure i2: The Sun with R, radius, the Earth with r, radius and the distance between Earth and Sun.

Calculation for γ which depend the distance between the Earth and the Sun.
R = 695,500km (Radius of the Sun)
r =6,377km (Radius of the Earth)
D = 147.09 x 10^6 km at Perihelion, 152.10 x 10^6 km at Aphelion

$$\cos\gamma = \frac{R-r}{D}$$

$$\gamma = \cos^{-1}\left(\frac{R-r}{D}\right)$$

The value of γ depends only on the distance between the Earth and the Sun.

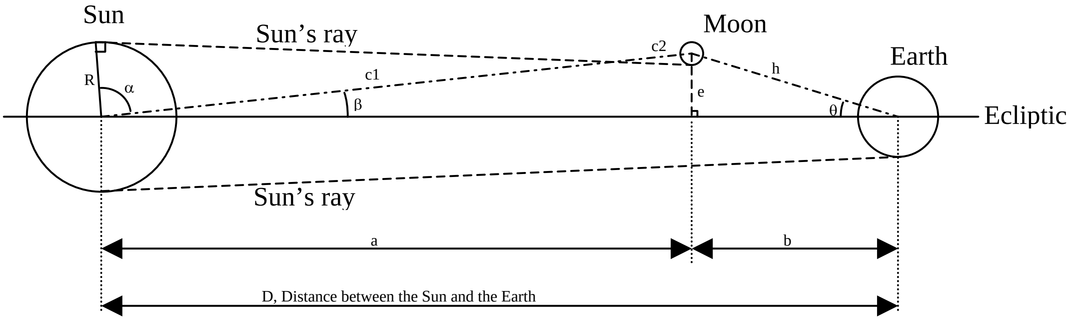


Figure i3: The angle and distance between the Sun, Moon and the Earth.

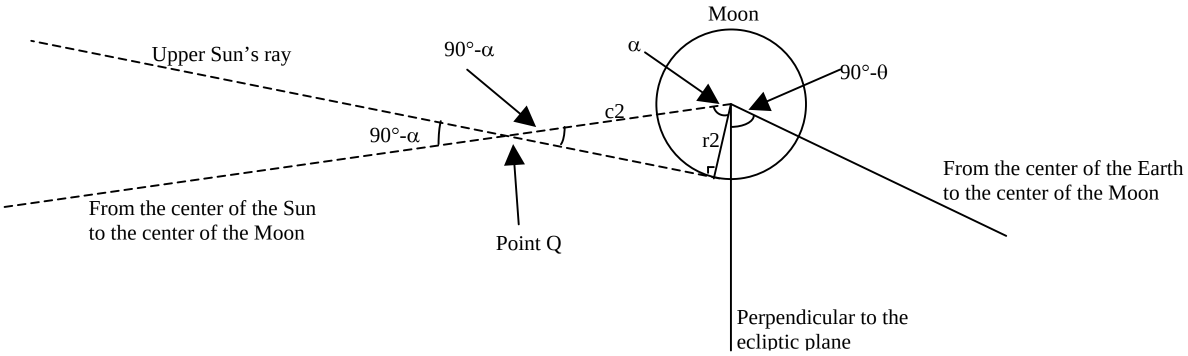


Figure i4: Enlarged view of the angle and distance related to the Moon.

R - 695,500km (Radius of the Sun)

r2 - 1,737.4 km (Radius of the Moon)

a - distance from the center of the Sun to the point on the ecliptic where the center of the Moon is perpendicular to the ecliptic plane

b - distance from the center of the Earth to the point on the ecliptic where the center of the Moon is perpendicular to the ecliptic plane

Q - the point where the tangent line formed by the surface of the Sun and the surface of the Moon intersect the line formed by the center of the Sun and the center of the Moon.

C - distance from the center of the Sun and the center of the Moon

c1 - distance from the center of the Sun to the point Q

c2 - distance from the center of the Moon to the point Q

D - 147.09 x 10^6 km at Perihelion, 152.10 x 10^6 km at Aphelion

e - distance from the center of the Moon perpendicular to the ecliptic plane

h - distance between the center of the Moon and the center of the Earth

θ - angle between the ecliptic plane and the line formed by the center of the Earth and the center of the Moon

α - angle between line formed by the center of the Sun to the center of the Moon and the tangent line formed by the surface of the Sun and the surface of the Moon

β - angle between the ecliptic plane and the line formed by the center of the Sun and the center of the Moon

We want to find out what is the maximum value for θ to have an eclipse. We know:

$$b = h\cos\theta$$

$$e = h\sin\theta$$

$$a = D - b$$

$$\beta = \arctan\left(\frac{e}{a}\right) = \arctan\left(\frac{h\sin\theta}{D-b}\right)$$

Where θ is between 0° and 5.15°.

$$\cos\alpha = \frac{R}{c1} = \frac{r2}{c2}$$

$$\frac{R}{c1} = \frac{r2}{c2}$$

$$\frac{R}{r2} = \frac{c1}{c2} = \frac{C - c2}{c2}$$

$$c2\frac{R}{r2} = C - c2$$

$$c2\left(\frac{R}{r2} + 1\right) = C$$

$$c2 = C\left(\frac{r2}{R + r2}\right)$$

Therefore,

$$\cos\alpha = \frac{r2}{c2} = \frac{r2}{C\left(\frac{r2}{R + r2}\right)} = \frac{R + r2}{C}$$

$$\alpha = \arccos\left(\frac{R + r2}{C}\right)$$

The maximum value of θ to have an eclipse is bounded by the β + α = γ. Eclipse will happen when β + α is less than or equal to γ.

Using a matlab's script named "cal_solar_theta.m" in appendix a, the max angle θ is given in table i2.

	Sun at Perihelion Moon at Perigee	Sun at Perihelion Moon at Apogee	Sun at Aphelion Moon at Perigee	Sun at Aphelion Moon at Apogee
Max angle θ	1.54808°	1.41480°	1.53928°	1.40600°

Table i2: The maximum angle θ versus the distance between the Sun and Earth and the distance between the Earth and the Moon

Assume the line formed by the center of the Sun and the center of the Earth as *x*.

Assume the line formed by the center of the Moon and the center of the Earth as *y*.

We have found that if the angle θ formed by *x* and *y* is less than 1.40600°, we will definitely have an eclipse.

A node is a point where the ecliptic plane and the orbit of the Moon meet. There are two kind of nodes namely, the ascending node and descending node. What about the angle from a node to the point on the ecliptic plane will there be an eclipse?

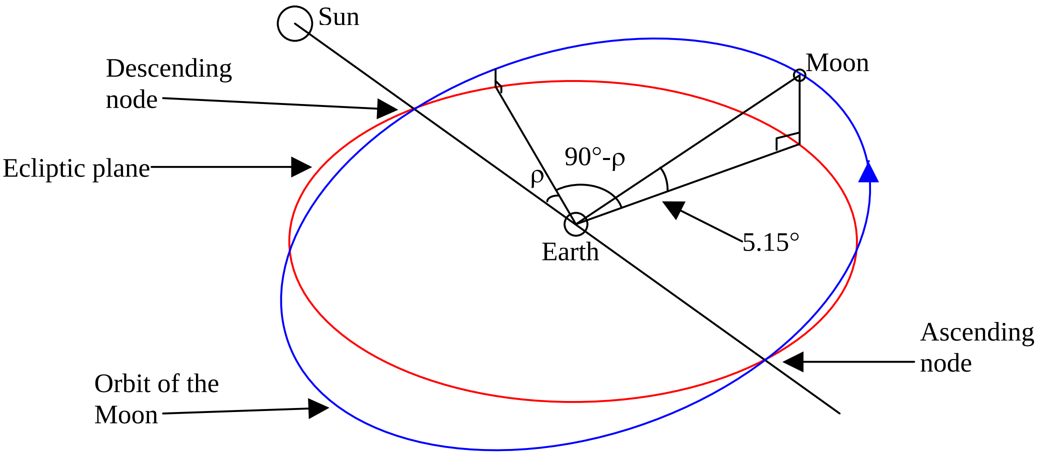


Figure i5: Angle ρ is the maximum angle from the node to the point where eclipses can occur.

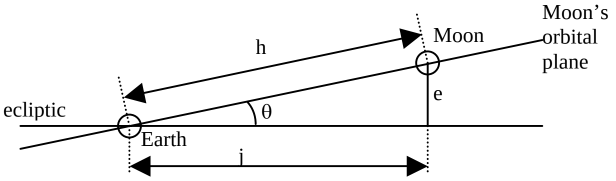


Figure i6: Distance e from the center of the Moon to the point perpendicular to the ecliptic plane and distance j from the center of the Earth to the point perpendicular to the ecliptic plane.

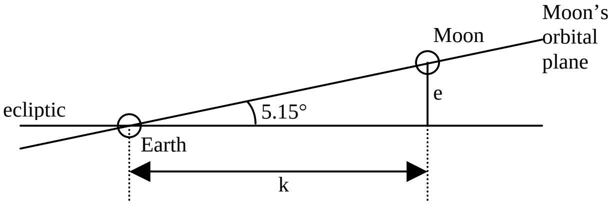


Figure i7: Distance k with angle 5.15° and value e.

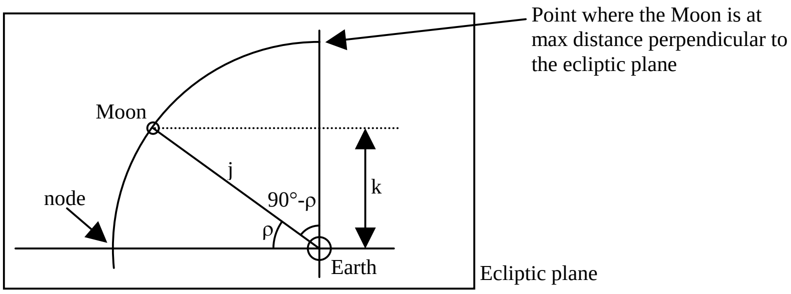


Figure i8: Calculation of angle ρ using j and k.

Therefore,

$$e = h\sin(\theta)$$

$$j = h\cos(\theta)$$

$$k = \frac{e}{\tan(5.15°)} = \frac{h\sin(\theta)}{\tan(5.15°)}$$

$$90° - \rho = \arccos\left(\frac{k}{j}\right)$$

$$\rho = 90° - \arccos\left(\frac{k}{j}\right)$$

Using a matlab's script named "cal_solar_rho.m" in appendix b, the max angle ρ is given in table i3.

	Sun at Perihelion Moon at Perigee	Sun at Perihelion Moon at Apogee	Sun at Aphelion Moon at Perigee	Sun at Aphelion Moon at Apogee
Max angle ρ	17.44926°	15.90445°	17.34686°	15.80289°

Table i3: The maximum angle ρ versus the distance between the Sun and Earth and the distance between the Earth and the Moon

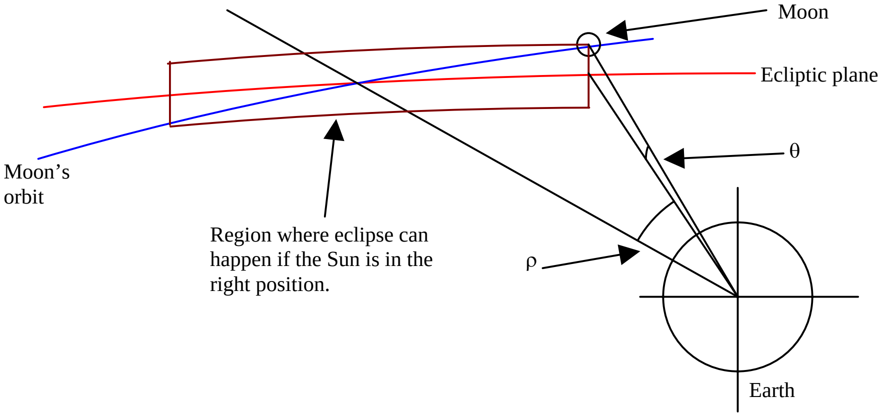


Figure i9: Region where eclipse can happen given the Sun is at the right position.

As you can see from figure i9, a region where eclipse will happen can be define by the angle θ and ρ.

Appendix II: Calculation of Angles Related to Lunar Eclipse

Note: In all the calculation, we have assumed the Sun; the Earth and the Moon are spherical with constant radiuses. All diagrams are not draw to scale unless otherwise stated.
Information on our Earth, Moon and Sun is given in appendix I, table i1.

How Lunar eclipse occurs?

Before answering the question to how lunar eclipse occurs, we can ask ourselves how come lunar eclipse doesn't occur. Figure ii1 shows the position of the Moon and the Sun during full Moon and there is no eclipse. For eclipse to happen the following must be met.

3. The Moon must be within a certain degree above/below ecliptic to create the eclipse.
4. The Moon must be near full Moon. (directly opposite of new Moon)

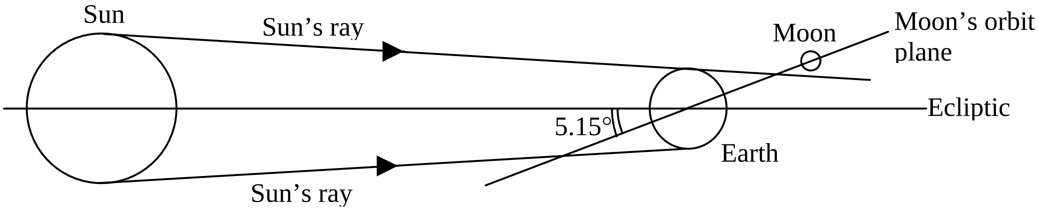


Figure ii1: Position of the Moon, Earth and Sun without a lunar eclipse.

What is the minimum degree above/below the ecliptic must the Moon be to create a solar eclipse?

A lunar eclipse will happen when the Sun's ray is block by the Earth during full Moon.

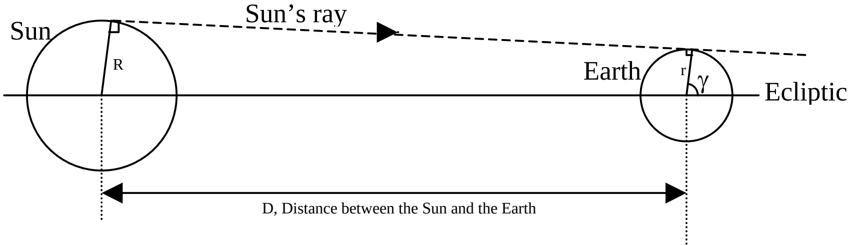


Figure ii2: The Sun with R, radius, the Earth with r, radius and D, the distance between Earth and Sun.

Calculation for γ which depend the distance between the Earth and the Sun.
R = 695,500km (Radius of the Sun)
r = 6,377km (Radius of the Earth)
D = 147.09×10^6 km at Perihelion, 152.10×10^6 km at Aphelion

$$\cos\gamma = \frac{R-r}{D}$$

$$\gamma = \cos^{-1}\left(\frac{R-r}{D}\right)$$

The value of γ depends only on the distance between the Earth and the Sun.

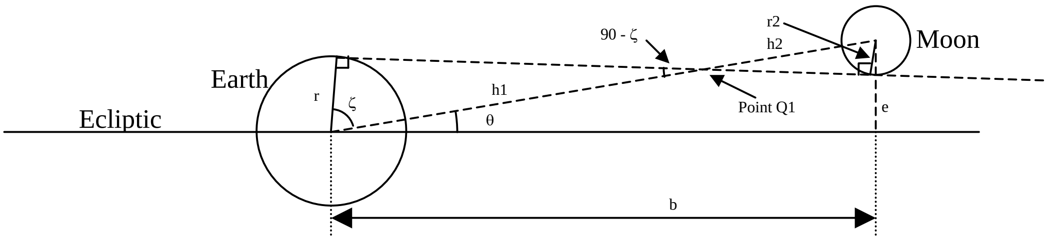


Figure ii3: The angle and distance between the Moon and the Earth.

r - 6,377km (Radius of the Earth)

r2 - 1,737.4 km (Radius of the Moon)

b - distance from the center of the Earth to the point on the ecliptic where the center of the Moon is perpendicular to the ecliptic plane

e - distance from the center of the Moon perpendicular to the ecliptic plane

h - distance between the center of the Moon and the center of the Earth

h1 - distance from the center of the Earth to the point Q1

h2 - distance from the center of the Moon to the point Q1

θ - angle between ecliptic plane and the line formed by the center of the Earth and the center of the Moon

ζ - angle between the line formed by the center of the Earth and the center of the Moon and the tangent line formed by the surface of the Earth and the surface of the Moon.

We want to find out what is the maximum value for θ to have a lunar eclipse. We know:

$$b = h\cos\theta$$

$$e = h\sin\theta$$

$$h = h1 + h2$$

Where θ is between 0° and 5.15°.

$$\cos\zeta = \frac{r}{h1} = \frac{r2}{h2}$$

$$\frac{r}{h1} = \frac{r2}{h2}$$

$$\frac{r}{r2} = \frac{h1}{h2} = \frac{h-h2}{h2}$$

$$h2\frac{r}{r2} = h - h2$$

$$h2\left(\frac{r}{r2}+1\right) = h$$

$$h2 = h\left(\frac{r2}{r+r2}\right)$$

Therefore,

$$\cos\zeta = \frac{r2}{h2} = \frac{r2}{h\left(\frac{r2}{r+r2}\right)} = \frac{r+r2}{h}$$

$$\zeta = \arccos\left(\frac{r+r2}{h}\right)$$

The maximum value of θ to have a lunar eclipse is bounded by the $\theta = \gamma - \zeta$. Eclipse will happen when θ is less than or equal to $\gamma - \zeta$.

Using a matlab's script named "cal_lunar_theta.m" in appendix c, the max angle θ is given in table ii2.

	Sun at Perihelion Moon at Perigee	Sun at Perihelion Moon at Apogee	Sun at Aphelion Moon at Perigee	Sun at Aphelion Moon at Apogee
Max angle θ	1.01139°	0.87818°	1.02023°	0.88702°

Table ii2: The maximum angle θ versus the distance between the Sun and Earth and the distance between the Earth and the Moon

Assume the line formed by the center of the Sun and the center of the Earth as *x.*

Assume the line formed by the center of the Moon and the center of the Earth as *y*.

We have found that if the angle θ formed by *x* and *y* is less than 0.87818°, we will definitely have a lunar eclipse.

A node is a point where the ecliptic plane and the orbit of the Moon meet. There are two kind of nodes namely, the ascending node and descending node. What about the angle from a node to the point on the ecliptic plane will there be a lunar eclipse?

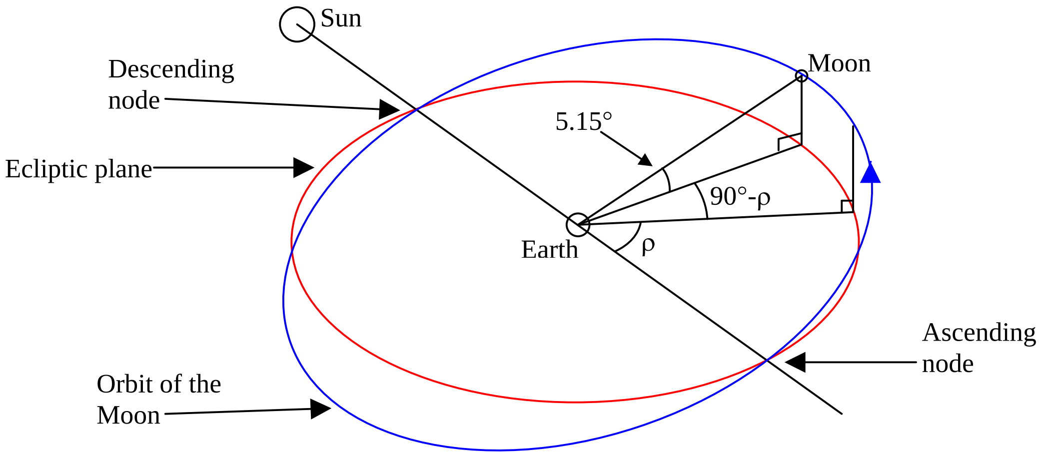


Figure ii5: Angle ρ is the maximum angle from the node to the point where eclipses can occur.

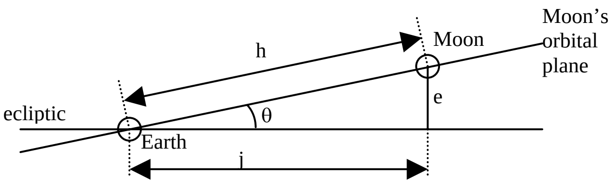


Figure ii6: Distance e from the center of the Moon to the point perpendicular to the ecliptic plane and distance j from the center of the Earth to the point perpendicular to the ecliptic plane.

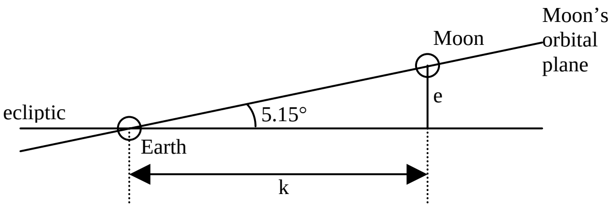


Figure ii7: Distance k with angle 5.15 ° and value e.

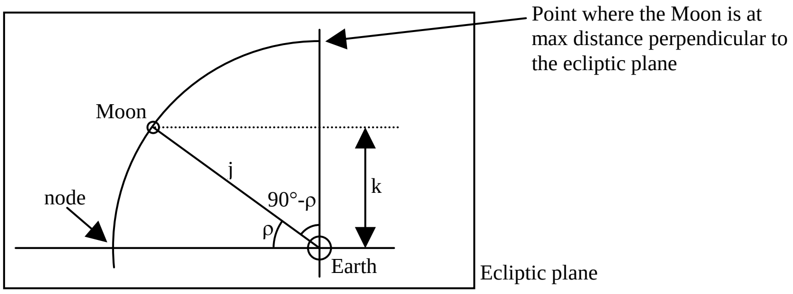


Figure ii8: Calculation of angle ρ using j and k.

Therefore,

$$e = h\sin(\theta)$$

$$j = h\cos(\theta)$$

$$k = \frac{e}{\tan(5.15^\circ)} = \frac{h\sin(\theta)}{\tan(5.15^\circ)}$$

$$90^\circ - \rho = \arccos\left(\frac{k}{j}\right)$$

$$\rho = 90^\circ - \arccos\left(\frac{k}{j}\right)$$

Using a matlab's script named "cal_lunar_rho.m" in appendix d, the max angle ρ is given in table ii3.

	Sun at Perihelion Moon at Perigee	Sun at Perihelion Moon at Apogee	Sun at Aphelion Moon at Perigee	Sun at Aphelion Moon at Apogee
Max angle ρ	11.29599°	9.79214°	11.39606°	9.89171°

Table ii3: The maximum angle ρ versus the distance between the Sun and Earth and the distance between the Moon and the Earth

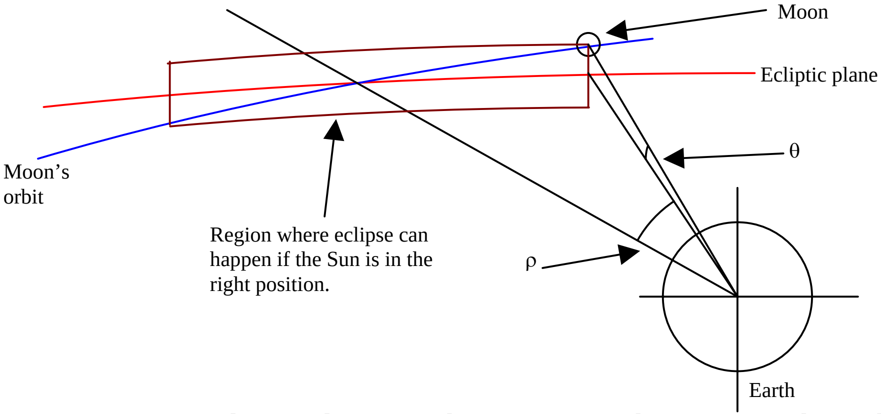


Figure ii9: Region where eclipse can happen given the Sun is at the right position.

As you can see from figure ii9, a region where eclipse will happen can be define by the angle θ and ρ.

Appendix III: Other information related to Eclipse

Note: In all the calculation, we have assumed the Sun; the Earth and the Moon are spherical with constant radiuses. All diagrams are not draw to scale unless otherwise stated.

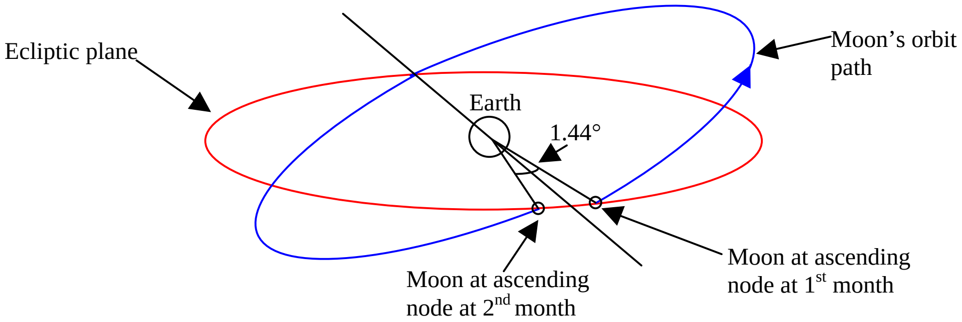


Figure iii1: change in the position of ascending node in two months.

Figure iii1 shows the change in the position of ascending node each time it cut through the ecliptic plane at the ascending node. This is known as regression. The position of the ascending node is moving counter clockwise 1.44° every nodical month. It takes 250 nodical months, which is about 18.61 year for the ascending node to return to its original position.

Appendix A: cal_solar_theta.m

```
%program to calculate the angles for the solar ecilpse
%=====================================================
format long
disp('Calculation for theta in Solar Eclipse');
disp('======================================');
Sun_dist = [147.09*10^6 147.09*10^6 152.10*10^6 152.10*10^6];
Moon_dist = [363.3*10^3 405.5*10^3 363.3*10^3 405.5*10^3];
%Constant
%==========================================
R = 695500;             %radius of the Sun
r = 6377;               %raidus of the Earth
r2 = 1737.4;            %raduis of the Moon

for i=1:4
        D = Sun_dist(i);        %distance from the center of the Earth to center of the Sun
        h = Moon_dist(i);         %distance from the center of the Earth to center of the Moon
        gamma = (acos((R-r)/D))*180/pi;

        start_theta = 0;
        end_theta = 5.15;
        cur_theta = (end_theta - start_theta) / 2;
        ok = 0;
        while ok == 0
                b = h*cos(cur_theta*pi/180);
                e = h*sin(cur_theta*pi/180);
                a = D-b;
                C = sqrt(a^2 + e^2);

                beta = atan(e/a)*180/pi;
                alpha = acos((R+r2)/C)*180/pi;
                diff = beta + alpha - gamma; % + mean the Moon is too high, - mean the Moon is too low

                if diff > 0     % the Moon is too high
                        end_theta = cur_theta;
                        cur_theta = (cur_theta - start_theta)/2 + start_theta;
                else            %the Moon is too low
                        start_theta = cur_theta;
                        cur_theta = (end_theta - cur_theta)/2 + start_theta;
                end

                if (diff < 0.000001) & (diff > -0.000001)
                        if mod(i,2)==1
                                s2 = 'Perigee';
                        else
                                s2 = 'Apogee';
                        end
                        if i < 3
                                s1 = 'Perihelion';
                        else
                                s1 = 'Aphelion';
                        end
                        disp(sprintf('Sun at %s and Moon at %s', s1, s2));
                        disp(sprintf('Theta = %3.5f deg\n', cur_theta));
                        ok = 1;
                end
        end
end
```

Appendix B: cal_solar_rho.m

```
%calculation of rho in solar Eclipse
%===================================
disp('Calculation for rho in Solar Eclipse');
disp('====================================');
h = [363.3*10^3 405.5*10^3];  %max dist from the Earth to the Moon
theta = [1.54808 1.41480 1.53928 1.40600];
for i=1:4
        if mod(i,2)==1
                e=h(1)*sin(theta(i)*pi/180);
                j=h(1)*cos(theta(i)*pi/180);
        else
                e=h(2)*sin(theta(i)*pi/180);
                j=h(2)*cos(theta(i)*pi/180);
        end
        k=e/tan(5.15*pi/180);
        rho=(pi/2-acos(k/j))*180/pi;

        if mod(i,2)==1
                s2 = 'Perigee';
        else
                s2 = 'Apogee';
        end
        if i < 3
                s1 = 'Perihelion';
        else
                s1 = 'Aphelion';
        end
        disp(sprintf('Sun at %s and Moon at %s', s1, s2));
        disp(sprintf('Rho = %3.5f deg\n', rho));
end
```

Appendix C: cal_lunar_theta.m

```
%program to calculate the angles for the lunar ecilpse
%=====================================================
format long
disp('Calculation for theta in lunar Eclipse');
disp('======================================');
Sun_dist = [147.09*10^6 147.09*10^6 152.10*10^6 152.10*10^6];
Moon_dist = [363.3*10^3 405.5*10^3 363.3*10^3 405.5*10^3];
%Constant
%==========================================
R = 695500;             %radius of the Sun
r = 6377;               %radius of the Earth
r2 = 1737.4;            %radius of the Moon

for i=1:4
        D = Sun_dist(i);          %distance from the center of the Earth to center of the Sun
        h = Moon_dist(i);         %distance from the center of the Earth to center of the Moon
        gamma = (acos((R-r)/D))*180/pi;
    zeta = acos((r+r2)/h)*180/pi;

    theta = gamma - zeta;

    if mod(i,2)==1
            s2 = 'Perigee';
        else
                s2 = 'Apogee';
        end
        if i < 3
                s1 = 'Perihelion';
        else
                s1 = 'Aphelion';
        end
        disp(sprintf('Sun at %s and Moon at %s', s1, s2));
        disp(sprintf('Theta = %3.5f deg\n', theta));
end
```

Appendix D: cal_lunar_rho.m

```
%calculation of rho in Lunar Eclipse
%===================================
disp('Calculation for rho in Lunar Eclipse');
disp('====================================');
h = [363.3*10^3 405.5*10^3];  %Distance from the Earth to the Moon
theta = [1.01139 0.87818 1.02023 0.88702];
for i=1:4
        if mod(i,2)==1
                e=h(1)*sin(theta(i)*pi/180);
                j=h(1)*cos(theta(i)*pi/180);
        else
                e=h(2)*sin(theta(i)*pi/180);
                j=h(2)*cos(theta(i)*pi/180);
        end
        k=e/tan(5.15*pi/180);
        rho=(pi/2-acos(k/j))*180/pi;

        if mod(i,2)==1
                s2 = 'Perigee';
        else
                s2 = 'Apogee';
        end
        if i < 3
                s1 = 'Perihelion';
        else
                s1 = 'Aphelion';
        end
        disp(sprintf('Sun at %s and Moon at %s', s1, s2));
        disp(sprintf('Rho = %3.5f deg\n', rho));
end
```

Web References

Constants and definitions used in the ephemerides for the physical observations of the solar system bodies
http://www.bdl.fr/ephem/ephephys_eng/ephephys_constantes.html

Sun Fact sheet
http://nssdc.gsfc.nasa.gov/planetary/factsheet/Sunfact.html

Earth Fact sheet
http://nssdc.gsfc.nasa.gov/planetary/factsheet/Earthfact.html

Moon Fact sheet
http://nssdc.gsfc.nasa.gov/planetary/factsheet/Moonfact.html

Photo on Saro series cycle.
http://www.mreclipse.com/

Figures used in defining terms under Saro Cycle.
http://www.hermit.org/Eclipse/

Types of Solar Eclipse
http://www.eclipse.za.net/html/eclipse_types.html

De Eclipsibus
http://www.worldastrology.net/eclipses.html

Animation on Eclipses
http://members.aol.com/tfroberg/eclipse/types.htm

Eclipse of the Moon
http://micro.magnet.fsu.edu/primer/java/scienceopticsu/lunar/index.html

Information on Eclipse by BBC
http://www.bbc.co.uk/science/space/solarsystem/sun/solareclipse.shtml

Book References

Eclipse, by Duncan Steel (A Joseph Henry Book)

Eclipse!, by Philip S. Harrington. Published by John Witey & Sons,Inc.

Totality. Eclipses of the Sun, by Mark Littmann, Ken Willcox and Fred Espenak. 2nd Edition.

Eclipse, by JP McEvoy

Total Eclipses, by Pierre Guillermier & Serge Kouthmy

The Moon Book, by Kim Long