Homework Set 2

Hand in independently written solutions by 19th September.

(1) A real-valued function f defined on a real interval I is said to be *convex* if
$$f((1-c)x_1 + cx_2) \leq (1-c)f(x_1) + cf(x_2)$$
whenever $x_1, x_2 \in I$ and $0 \leq c \leq 1$. Show that a differentiable function $f : \mathbb{R} \to \mathbb{R}$ is convex if and only if f' is increasing on $\mathbb{R}$.
(Use Mean Value Theorem.)

(2) Use Taylor's Theorem to prove the following: For any $\varepsilon > 0$, there is a polynomial $P(x)$ so that $|\sin x - P(x)| \leq \varepsilon$ for all $x \in [0, 2\pi]$.
(You may assume known properties of the sine function, including its derivatives.)

(3) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a three-times differentiable function. Express $D^3 f$ in terms of its partial derivatives.
(This is Q38(a) in the Exercises for §5. We will refrain from discussing this question during tutorials, at least before this Homework is due.)
(Let $e_1 = (1,0)$ and $e_2 = (0,1)$. First compute $D^3 f(x)(e_i, e_j, e_k)$ for any $i, j, k \in \{1, 2\}$.)