

MA3111S Semester 2 08/09

Exercises for §1

- (1) Express the following numbers in rectangular coordinates.
- $\frac{-1+3i}{\sqrt{2}-i}$;
 - $(-\frac{1}{2} + i\frac{\sqrt{3}}{2})^4$;
 - $(-i)^{1/9}$;
 - $[(-1 + i\sqrt{3})^{1/3}]^3$;
 - $[(-1 + i\sqrt{3})^3]^{1/3}$.
- (2) Use the fact that $[1, n\theta] = [1, \theta]^n$ to obtain an identity for $\cos n\theta$ in terms of $\cos \theta$ and $\sin \theta$.
- (3) Find a formula for $(a + ib)^{1/2}$ by solving for c and d in the equation $(c + id)^2 = (a + ib)$.
- (4) Graph the solution sets of the given relations.
- $|\frac{z-1}{z+1}| \leq 1$;
 - $|z^2 - 1| = 1$. [Use polar coordinates.]
- (5) For any complex numbers u and v , show that

$$|u + v|^2 + |u - v|^2 = 2(|u|^2 + |v|^2).$$

- (6) (a) Let u and v be complex numbers. Show that

$$|u + v|^2 = |u|^2 + 2 \operatorname{Re}(u\bar{v}) + |v|^2.$$

- (b) If $|z_1| = |z_2| = |z_3| = 1$ and $z_1 + z_2 + z_3 = 0$, show that z_1, z_2 and z_3 are the vertices of an equilateral triangle in the complex plane.

[Part (a) can be used to prove part (b)!]

- (7) (a) Let u and v be nonzero complex numbers. Find all possible values of $\operatorname{Arg} \frac{u}{v} - (\operatorname{Arg} u - \operatorname{Arg} v)$.

[Give examples to show that all your answers are indeed possible.]

- (b) Suppose that $|z| = 1$. Show that

$$\operatorname{Arg}\left(\frac{z-1}{z+1}\right) = \begin{cases} \frac{\pi}{2} & \text{if } \operatorname{Im} z > 0 \\ -\frac{\pi}{2} & \text{if } \operatorname{Im} z < 0. \end{cases}$$

[Part (b) takes a little bit of plane geometry.]