Critical Thinking in Elementary Mathematics: A Historical Approach

Lim H. C.[1] and Pang P. Y. H.[2]

Department of Mathematics, National University of Singapore
2 Science Drive 2, Singapore 117543

ABSTRACT

This project aims to critique the teaching of elementary mathematics in Singapore, particularly at the lower secondary level. It will attempt to address the issues of fostering critical thinking and achieving deeper understanding in the learning process. More specifically, it will examine how the use of history in mathematics can play a role in achieving the desired results. It is suggested that we can draw from historical resources, use questions that had motivated past mathematicians and can (still) motivate students to think, and further encourage students to generate questions of their own. Also, by presenting the ways in which these questions have been resolved, we hope to enhance the students' understanding and appreciation of the mathematics they learn. This project presents a case study of a workshop designed and conducted by the second author and two classroom activities designed by the first author in attempting to demonstrate the possible use of historical resources in a way consistent with the aim of fostering critical thinking and deeper understanding.

BACKGROUND

In the assessment of elementary mathematical ability by the Third International Mathematics and Science Study (TIMSS), it has been consistently shown that Singapore students rank first among an international cohort of Grade 8 (Secondary 2) students from the 28 participating countries. Yet many mathematics educators are concerned that the presentation of materials in the current elementary mathematics syllabus in Singapore does that really provide students with the necessary level of conceptual understanding that will enable them to think critically and creatively about mathematics. Hence, it is the purpose of this paper to evaluate that claim, by actually seeking out areas that require improvement in the current teaching of elementary mathematics, particularly at the lower secondary level (Secondary 1 and 2). The primary focus of this paper will therefore be on course content, which shall be extracted from the two sets of mathematics textbooks currently used in secondary schools in Singapore.

There are many ways in which one can improve on the presentation of topics in mathematics that can lead students to greater understanding of the subject matter. However, this project focuses on the use of historical resources to bring about such improvement. From historical resources, we will highlight some critical questions that have led to the development of new mathematical ideas as well as ways of resolving these questions that lead to better understanding of our current mathematical methods and practices.

---

[1] Student
[2] Associate Professor, Supervisor

PROJECT OUTLINE

This project consists mainly of three sections. The first section presents a literature review and case studies which provide the background for this project. From analysing these research materials, we arrive at what we consider to be the most appropriate approach to the integration of history with mathematics teaching, which will serve as a guideline to the subsequent design of classroom activities.

The second section on "Classroom Activities Involving the Use of History" consists of the three classroom activities designed by the first author in an attempt to draw lessons from historical materials in a way that would make mathematics more meaningful for the students. The first two activities are meant to supplement the current presentation of the same topics in the syllabus, while the last activity can be used to introduce the topic concerned on its own. Each activity will be accompanied by a brief introduction to the historical or other materials used and a discussion of the presentation of the same topic in the current sets of textbooks. The topics of interest in this project are mainly arithmetic in nature, largely due to our own preference.

The third section of this project shall consider the possible criticisms of this project and concerns of using the historical approach. It contains a transcript of an interview conducted with the co-author of the textbook *New Syllabus Mathematics*, Mr. Looi Chin Keong and also the results of a survey on teachers' attitudes towards the use of history in teaching mathematics conducted during a teachers' seminar by Prof. Siu Man Keung. There will also be suggestions for improvement, both in relation to this project and to the current mathematics syllabus.

THE CLASSROOM ACTIVITIES

A common theme underlying the case study and all the designed activities is the introduction of a central question for the students to think about, a question that was of historical importance, and the attempts made at resolving them. This is an attitude of inquiry that we wish to promote in the students, and we also present to them how they can proceed to resolve the problem. It is an encouragement for students to do likewise. This idea is consistent with this project's objective: to foster critical thinking. The following is a brief description of the case study and the activities:

Case Study: A Comparison of Fractional Quantities

This activity attempts to present to the students the central question of why we need both fractions and decimals to represent quantities. The consideration for this question came from an observation of the common practice at the upper primary and lower secondary levels where students are taught to convert fractions to decimals and vice versa. The concern is that many students seem to think that fractions and decimals are just two representations of the same number and it is not clear to them why they need two different representations at all. In particular, this activity discusses why fractions and decimals are different and also the advantages and deficiencies of each. References are made to ways in which these deficiencies were dealt with in the history of mathematics.

This activity demonstrates that looking into the historical development of mathematical ideas can give meaning to our current practices in mathematics, for example, why we need both fractions and decimals.

Activity 1: On the Multiplication and Division of Fractions

Central question: What is the multiplication and division of fractions?

This activity aims to give meaning to two of the arithmetic operations of fractions: multiplication and division. Historical resources, in particular, the *Jiuzhang Suanshu* (translated: *Nine Chapters of the Mathematical Art*) demonstrate that people in the past have contemplated the meaning of arithmetic operations and their connections with other mathematical concepts. In *Jiuzhang*, annotator Liu Hui acknowledged that the division of integers, ratios and the multiplication of fractions are actually equivalent. Nearly two thousand years later, a study conducted by Dr. Ma Liping in China and the United States, showed that teachers who have a profound understanding of fundamental mathematics (PUFM) not only display exceptional computation ability but are also able to make explicit connections between mathematical concepts and elaborate on the various physical interpretations of the mathematical operations they perform. This activity draws from these resources in an attempt to help students understand the meaning of the multiplication and division of fractions.

This activity demonstrates that looking into historical resources can give both teachers and students directions for inquiry: one can both question the meaning of operations and contemplate the interconnectedness of concepts.

Activity 2: Pythagoras' Theorem, its Applications and a Problem

Central question: Do all right-angled triangles obey the relationship we now know as Pythagoras' Theorem?

This activity arises from a conflict between a comment made by the famous science writer Jacob Bronowski and an observation made by the first author. In his book "The Ascent of Man", Bronowski described the Pythagoras Theorem as "the most important single theorem in the whole of mathematics." The conflicting observation is made from our current textbooks, in which the theorem, constituting a small chapter on its own, appears to most students to be a trivial result unrelated to most of the other topics. This activity aims to help students realise that the result is both powerful and important: powerful because it is a generalisation, perhaps from observations of the many right-angled triangles that obey this relationship, true for ALL possible right-angled triangles; it is also very important, and is in fact linked to our realisation of the existence of irrational numbers!

The activity is therefore an attempt to present the theorem in a way that would help the students appreciate why a statement as such should merit a chapter of its own, not just our current textbooks, but also in textbooks of old, such as the *Jiuzhang Suanshu*. In this activity, we consider the historical response to generalise a piece of useful information (i.e., to prove that all right-angled triangles obey Pythagoras' Theorem). Also, a problematic implication of this theorem leads us to discover a 'new' type of numbers: irrational numbers. This activity demonstrates that history can help us order our topics of discussion in a meaningful manner.

CONCLUSION

A preliminary survey conducted on students during the case study on the comparison of fractional quantities yielded encouraging results, as students seem to welcome this inclusion of historical materials which help give meaning to mathematics. A corresponding survey on teachers however, yielded concerns that although the history of mathematics is itself relevant,

the tight schedule in our Singapore curriculum may not allow for its inclusion. Furthermore, teachers feel that there is a lack of teacher training for them to make these presentations. It is therefore concluded that although the inclusion of the history of mathematics to teaching is a meaningful enterprise, we may need to make allowances for it in the curriculum for it to be implemented.

REFERENCES

Avital, S., "History of mathematics can help improve instruction and learning", in Calinger (see below), 3 –11.

Calinger, R., ed., *Vita Mathematica: Historical Research and Integration with Teaching*, Washington, D.C: The Mathematical Association of America, 1996.

Dewdney, A.K., *A Mathematical Mystery Tour: Discovering the Truth and Beauty of the Cosmos*, New York: Wiley, 1999.

Fowler, M., http://www.phys.virginia.edu/classes/109N/lectures/thales.html

Garner, M., "The importance of history in mathematics teaching and learning" from http://www.aug.edu/dvskel/Garner1SU97.htm, 1996.

Heiede, T., "History of mathematics and the teacher", in Calinger (see above), 231 243.

Katz, V. J., ed., *Using History to Teach Mathematics*, Washington, D.C: The Mathematical Association of America, 1996.

Ma, L. *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*, Mahwah, N.J: Lawrence Erlbaum Associates, 1999.

O'Connor, J.J. and Robertson, E.F., *The MacTutor History of Mathematics Archive*, http://turnbull.dcs.st-and.ac.uk/history/Indexes/HistoryTopics.html, University of St Andrews, Scotland.

Ofir, R. and Arcavi, A., " Word problems and equations: a historical activity for the algebra classroom", *Mathematical Gazette*. **76**(1992), no. 475, 69 – 84

Radford, L. and Guerette, G., "Second degree equations in the classroom: a Babylonian approach", in Katz (see above), 69 76.

Shen, K.S., Crossley, J.N. and Lun, A.W.C., *The Nine Chapters on the Mathematical Art: Companion and Commentary*, New York : Oxford University Press, 1999.

Sin, K.M., *Exploring Mathematics: Workbook.* 4 vols., Singapore: Pan Pacific Publications, 2001.

Siu, M.K., "The ABCD of using history of mathematics in the (undergraduate) classroom", in Katz (see above), 3 – 10.

Swetz, F. et al., *Learn from the Masters*, Washington, D.C: The Mathematical Association of America, 1995.

Swetz, F., "Mathematical pedagogy: a historical perspective", in Katz (see above), 11 16.

Teh, K.S. and Looi, C.K., *New Syllabus Mathematics*, 5th ed., 2 vols., Singapore: Shinglee Publishers, 2001.

Tucker, A.C., ed., *Models That Work*, Washington, D.C: The Mathematical Association of America, 1995.

Wong K.Y. and Sin K.M., ed., *Exploring Mathematics*, 4 vols., Singapore: Pan Pacific Publications, 2001.