NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

2002/2003 **MA 2213 Numerical Analysis I** **Semester I**

Laboratory Assignment 1

In 250 BC, the Greek Mathematician Archimedes estimated the number π as follows. He looked at a circle with diameter 1, and hence circumference π. Inside the circle he inscribed a square. The perimeter of the square is smaller than the circumference of the circle, and so it is a lower bound for π. Archimedes then considered an inscribed octagon, 16-gon, etc., each time doubling the number of sides of the inscribed polygons, and producing even better estimates for π. Using 96-sided inscribed and circumscribed polygons, he was able to show that $223/71 < \pi < 22/7$. There is however a recursive formula for these estimates. Let p_n be the perimeter of the inscribed polygon with 2^n equal sides. Then p_2 is the perimeter of the inscribed square, $p_2 = 2\sqrt{2}$. In general

$$p_{n+1} = 2^n \sqrt{2(1 - \sqrt{1 - (p_n/2^n)^2})}. \quad (1)$$

(i) Write a program to compute p_n for $n = 3, 4, \cdots, 40$.

(ii) Explain why in actual computation the recursive formula in (1) fails to converge to π.

(iii) By writing p_{n+1} as

$$p_{n+1} = 2^n \sqrt{r_{n+1}},$$

where

$$r_{n+1} = 2(1 - \sqrt{1 - (p_n/2^n)^2}), \quad r_3 = 2/(2 + \sqrt{2}).$$

Show that

$$r_{n+1} = \frac{r_n}{2 + \sqrt{4 - r_n}}. \quad (2)$$

(iv) Write a program using the last iteration in part (iii) to compute r_n and p_n for $n = 3, 4, \cdots, 40$.

(v) r_3 cannot be calculated exactly. Suppose that there is an error ϵ_0 in calculating r_3. Would this error dramatically affect the computation of r_n when n is large? (i.e. is the algorithm (2) for computing r_n stable or unstable?) Explain.