We have learnt last time the Pythagoras Theorem
\[ a^2 + b^2 = c^2. \]

Observe that in the proof everything is perfectly fit to each other - ideal. (Another proof.)

What are the differences between the proof of Pythagoras’ theorem and mere calculations?

1. Generality – it covers all possible cases.

2. Reliability

3. Based on known results.
Area of a square with side \( a = a^2 \).

Area of a triangle
\[ = (\text{base}) \times (\text{height}) \div 2. \]

Hence two triangles with same base and height have equal area.

Area of a triangle remains the same under a rotation of 90 degrees.
As any proof is based on certain known statement(s), which, if you like, we can consider its/their proof(s).

It is obviously futile to try to prove everything.

\[ \ldots P_{-2} \implies P_{-1} \implies P_o \implies P_1 \]

This phenomenon is observed more than 2000 years ago by Greek philosophers and mathematicians.
The School of Athens by Raphael.
Vatican Museum.
In Book II of Plato's *The Republic*, we are told that Gyges worked as a shepherd for the king.

An earthquake opened up the ground where Gyges' flock was feeding. Inside this opening he found a golden ring.....

**Gyges’ Ring** – reveals the true nature, or essence, of the being.
Plato (~500BC): The changing thing is manifestation of something more essential, something unchanging. The invariants are known as forms.

Theory of Forms. Allegory of Cave:
Sounds like the movie Matrix?

Related concepts: Atom. Prime numbers.
Elements by Euclid (~400BC)

Inside, you can find a collection of geometric results. More significantly, the way these results are presented is rather unique.

The book starts in chapter one with definitions.

Definition 1.
A point is that which has no part.

Definition 2.
A line is breadthless length.

Definition 3.
The ends of a line are points.

Definition 4.
A straight line is a line which lies evenly with the points on itself.
It continues with postulates or axioms – statements that are assumed to be true but no proofs are given. Rather, they serve as pillars for proving other statements.
**Postulate 1.**

To draw a straight line from any point to any point.

**Postulate 2.**

To produce a finite straight line continuously in a straight line.

**Postulate 3.**

To describe a circle with any center and radius.

**Postulate 4.**

That all right angles equal one another.

**Postulate 5.**

That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.
Euclidean geometry had been recognized as the paradigm of Geometry and a good description of our world for thousands of years.

But it remains that the postulates are unproven assumptions -- it is possible to consider other types of Geometry based on different axioms.

This paradigm shift occurred only in 19th century, when Gauss, Bolya et al. showed that we may replace the Parallel Axiom and obtain the Hyperbolic Geometry or Non-Euclidean Geometry.
Einstein’s General Theory of Relativity – space-time could be non-Euclidean!

Black Hole.