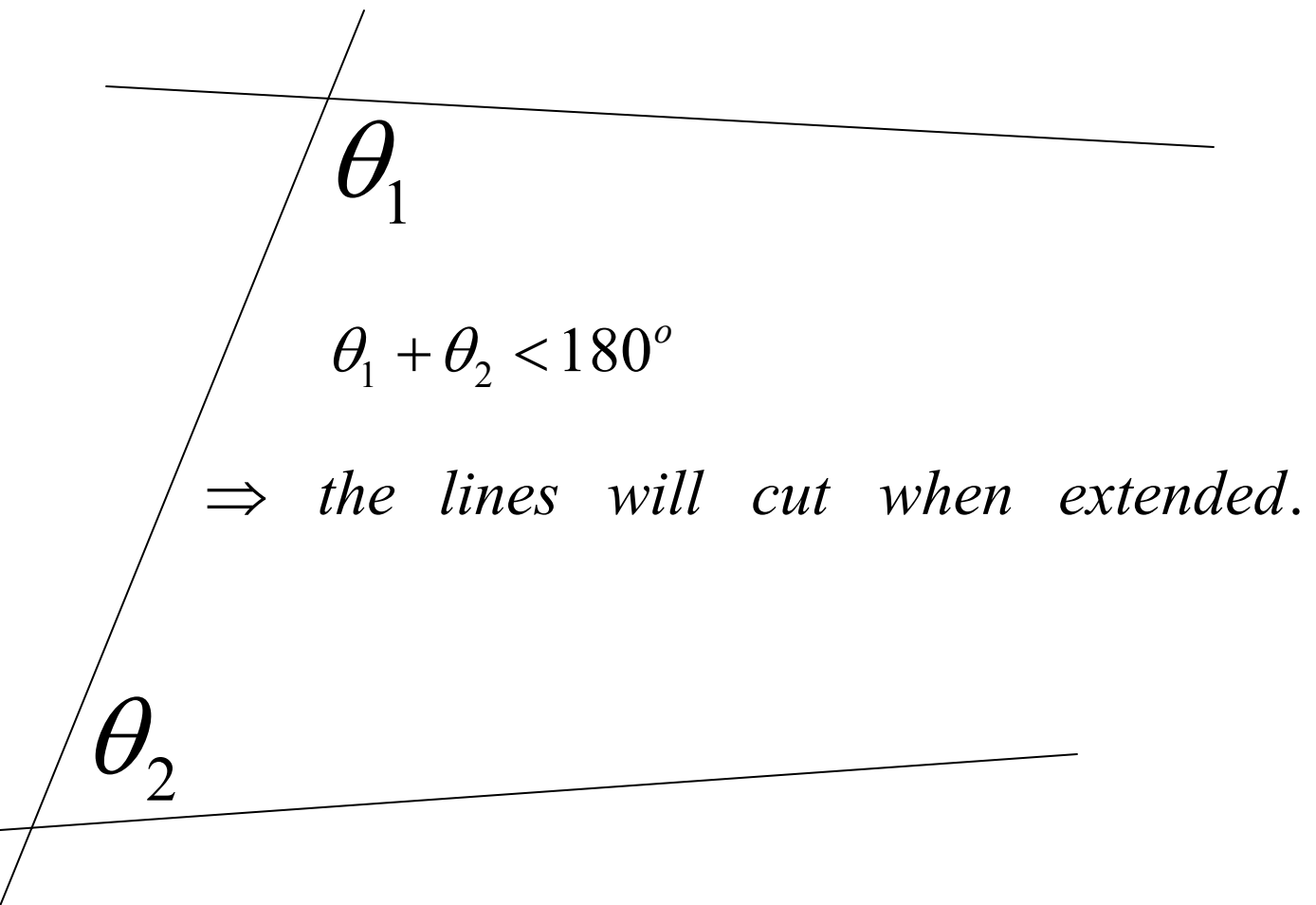


Euclid's 5th Axiom:

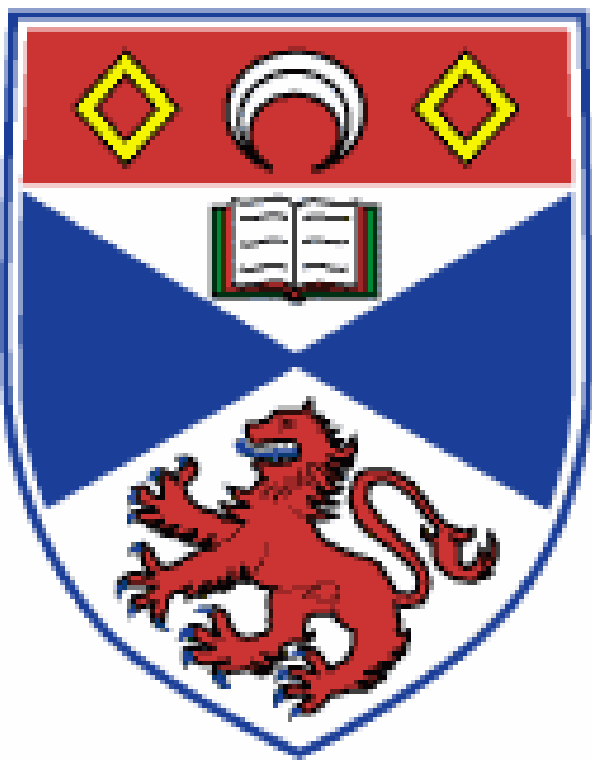
That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.



**Playfair,
an 18th century
Scottish scientist,
formulated the
axiom:**



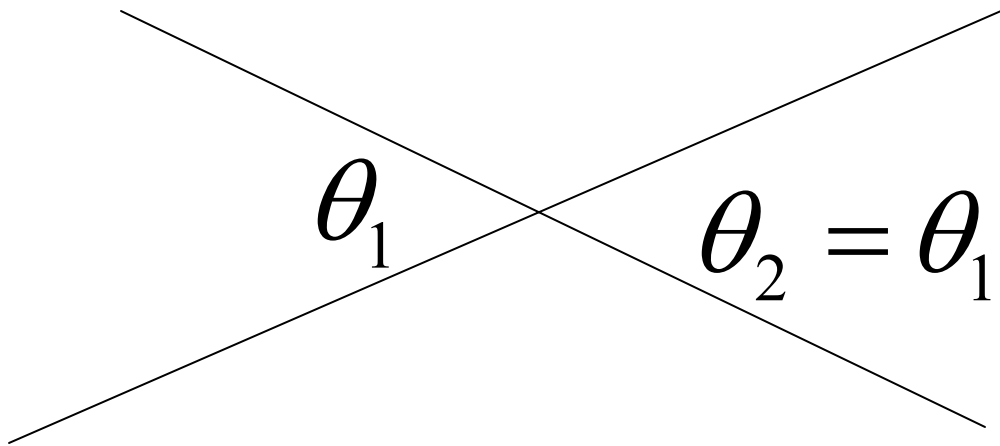
Given a line and a point not on the line, it is possible to draw exactly one line through the given point parallel to the line.



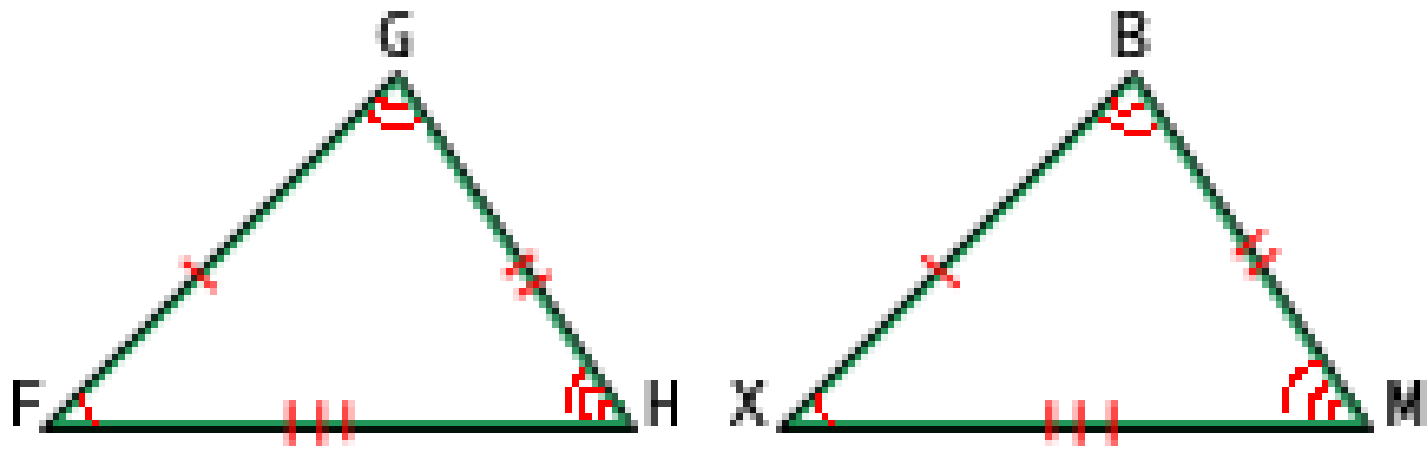
**We'll show that
this is in fact
equivalent to
Euclid's Fifth
Postulate.**

First, we remark that, under the first four axioms of Euclid, the following statements hold.

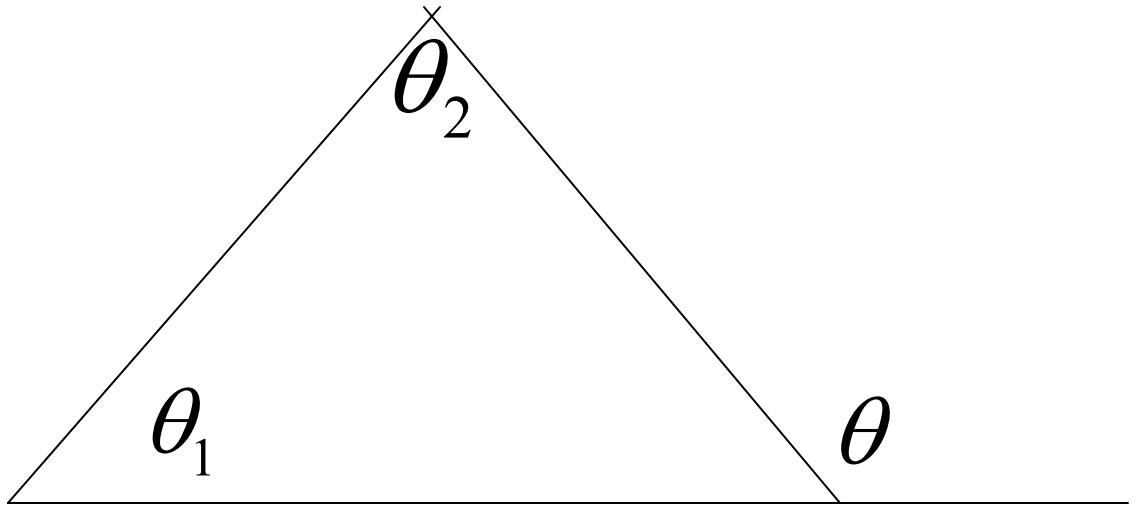
(1) Opposite angles are the same.



(2) If two triangles have both sides equal to each other and the angles between the sides are also equal, then they are congruent.



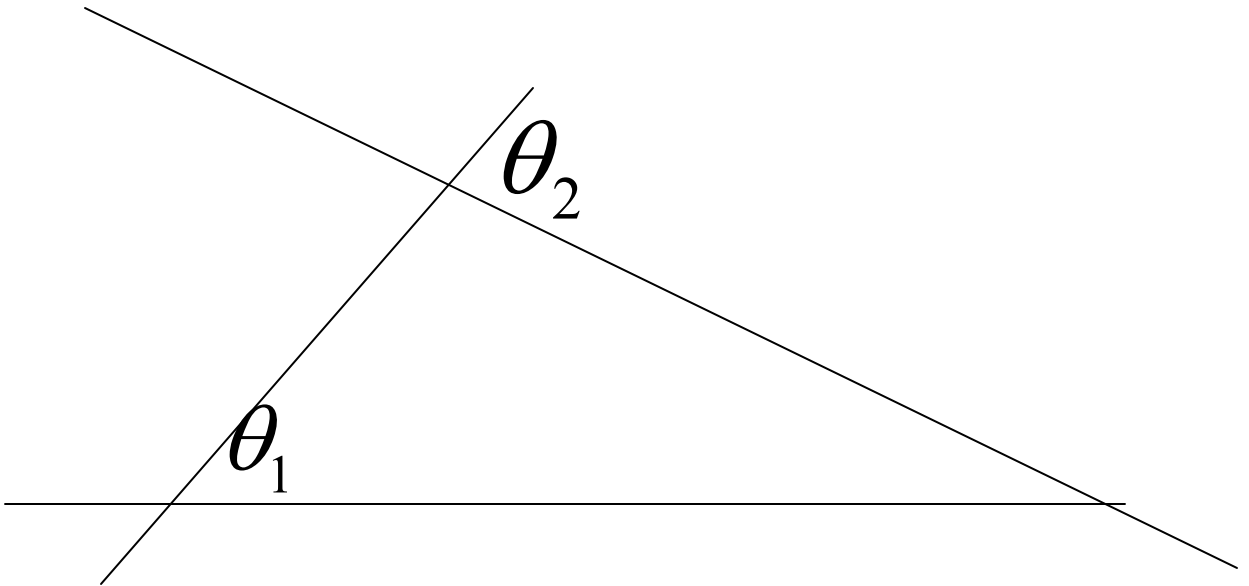
Congruent triangles



(3) In any triangle, if one of the sides is produced, the exterior angle is greater than either of the opposite angles.

$$\theta_1 < \theta, \quad \& \quad \theta_2 < \theta.$$

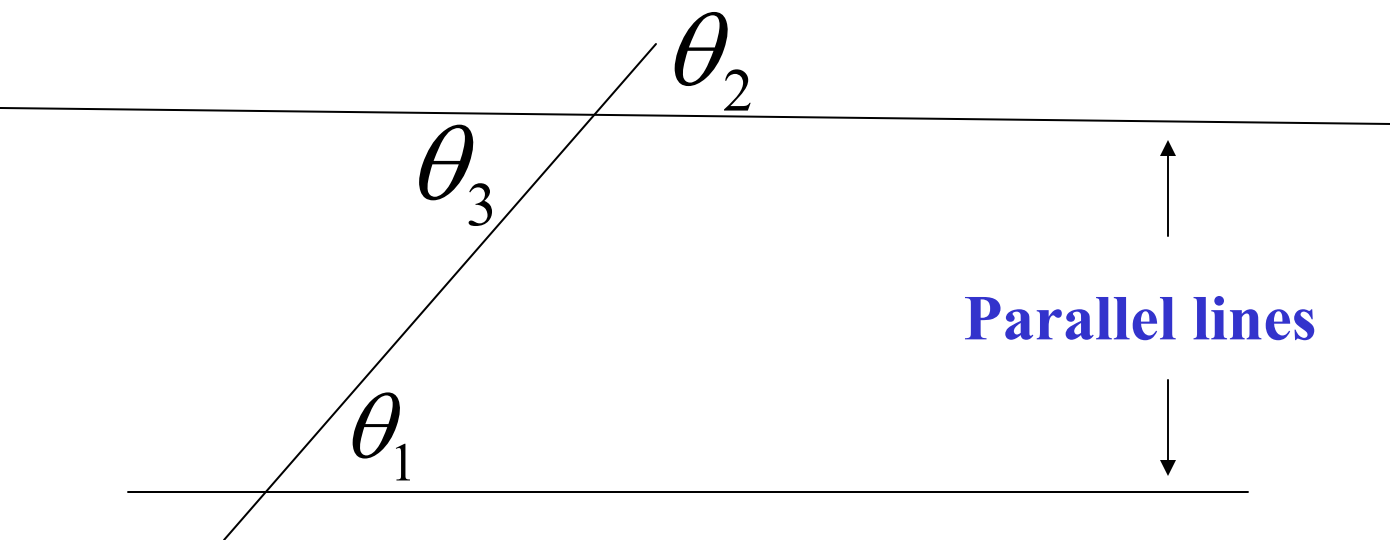
(4) If a straight line falling on two straight lines make the alternate angles equal to one another, the straight lines are parallel to one another.



$$\theta_1 = \theta_2.$$

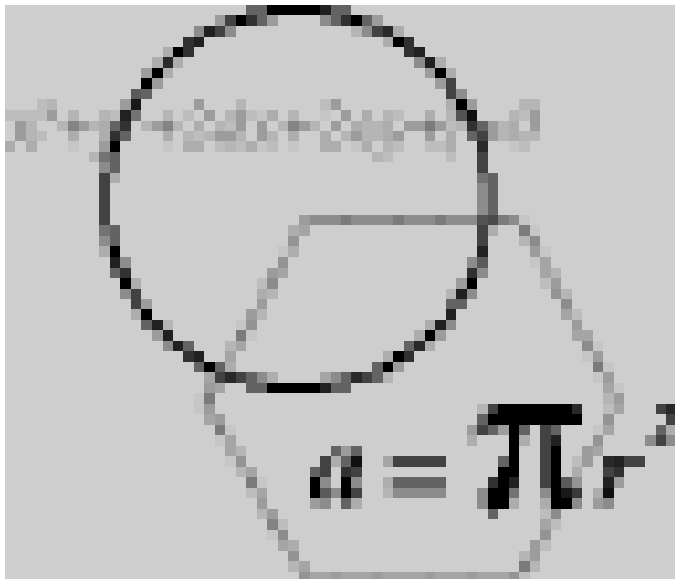
Contradicting (3).

Together with the 5th axiom of Euclid,
or equivalently, the Playfair postulate,
we have the following.



$$\theta_1 = \theta_2 = \theta_3.$$

The fifth axiom implies that the sum of the interior angles of any triangle is equal to two right angles, that is, 180 degrees.



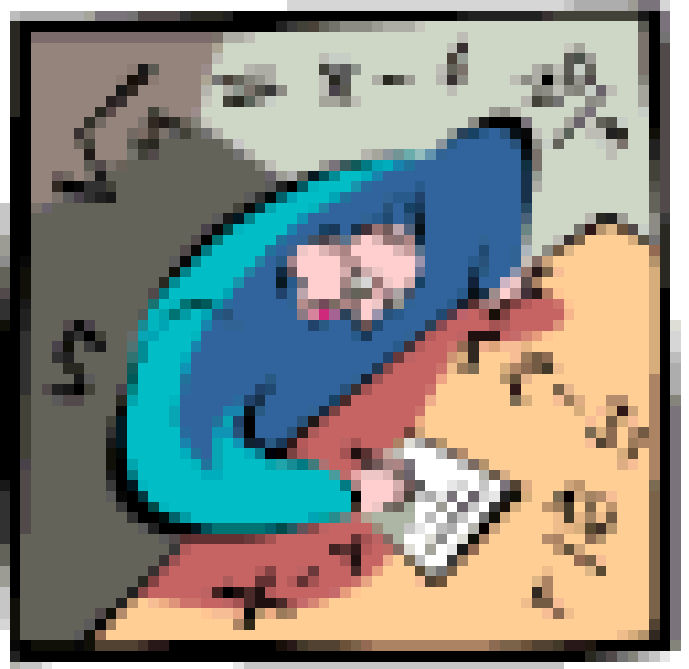
**Indeed, Euclid's fifth axiom,
the Playfair axiom,
the Pythagoras' theorem,
and the statement that
the sum of the interior angles of a
triangle is equal to 2 right angles,
are all equivalent.**

**That is, we won't change the
Euclidean geometry if we replace
the fifth axiom by anyone of the
other statements.**

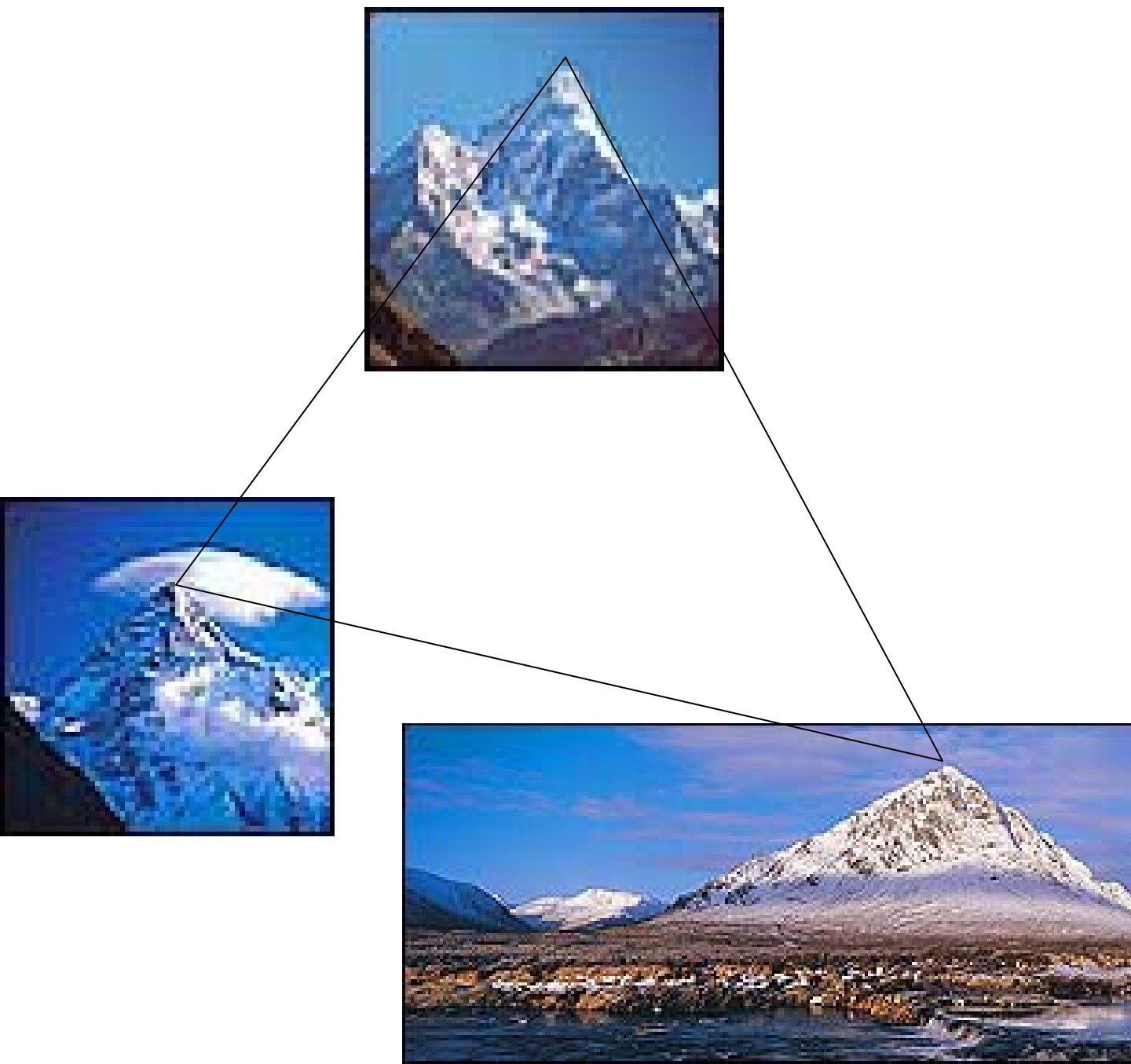
The details can be found in

Reference: <http://www.cut-the-knot.org/triangle/pythpar/PTimpliesPP.shtml>

Because the parallel axiom, or the Playfair axiom, appears to be so natural and intuitive, many had tried, unsuccessfully, *to derive it from the first four axioms.*



Despite of this, for two thousand years nobody really doubted that the parallel axiom can be changed or replaced. It was held as an absolute truth.



One of the most famous stories about Gauss depicts him measuring the angles of the great triangle formed by the mountain peaks of Hohenhagen, Inselberg, and Brocken for evidence that the geometry of space is non-Euclidean.



Gauss was apparently the first to arrive at the conclusion that no contradiction may be obtained this way. In a private letter of 1824 Gauss wrote:

“The assumption that (in a triangle) the sum of the three angles is less than 180° leads to a curious geometry, quite different from ours, but thoroughly consistent, which I have developed to my entire satisfaction.”

pauca sed matura
(few, but ripe)

Lobachevsky was first to publish a paper on the new geometry. His article appeared in *Kazan Messenger* in Russian in 1829 and, naturally, passed unnoticed. Trying to reach a broader audience, he published in French in 1837, then in German in 1840, and then again in French in 1855.



As a rector of Kazan University, Lobachevsky was awarded a diamond ring by Tsar Nicholas. However, *not* until years after his death, his name was associated with the discovery of non-Euclidean geometry. His last government citation Lobachevsky received just a few months before his death. He was cited for the discovery of a new way of processing wool.

Lobachevsky and Bolyai built their geometries
on the assumption:

Through a point not on the line there exist
more than one line parallel to the line.

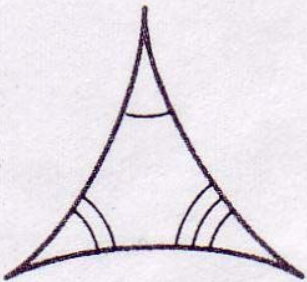
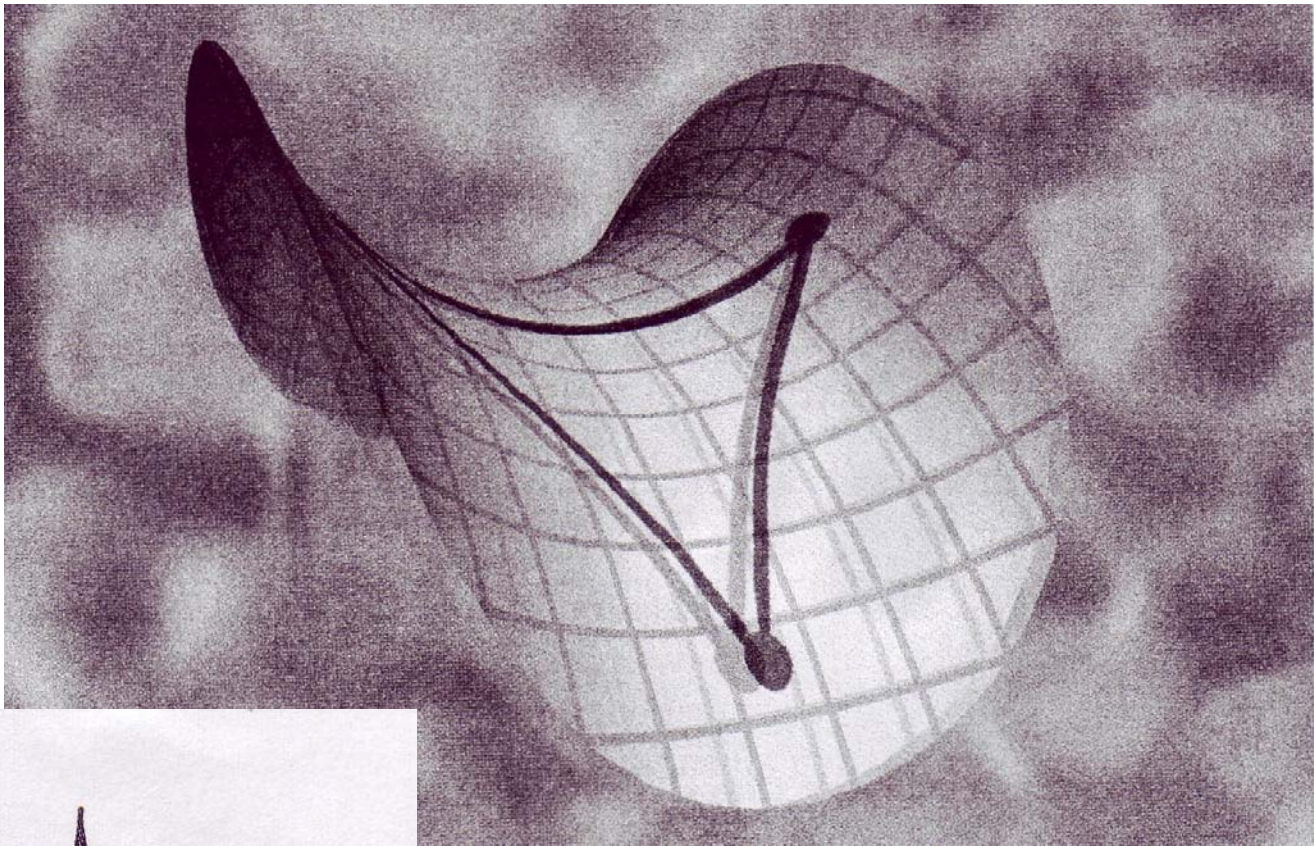
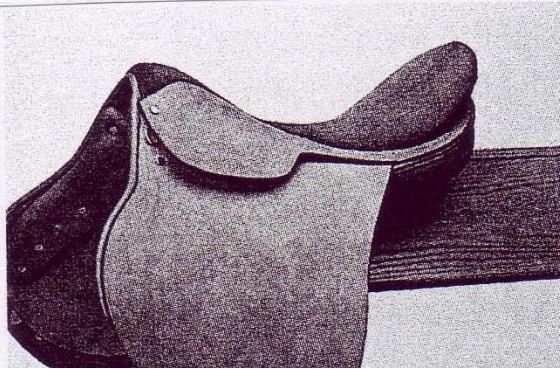
This is equivalent to Gauss' assumption that
the sum of angles in a triangle is less than
180 degree.

Rightfully, the new geometry created is called
Non-Euclidean geometry.



Portrait of Wolfgang Bolyai by János Szabó

GEOMETRY ON A SADDLE



Sum of the angles
is less than 180° .

*The related result is used by
Einstein in his General Theory of
Relativity – space-time is
non-Euclidean!*

Black Hole.

